Kenneth Arrow Contributions to Economics
by John B. Shoven

Kenneth Arrow is one of the greatest living economists. He is a leading member of the strongest cohort of economists ever. The group of economists born between 1911 and 1921 includes George Stigler, Milton Friedman, Paul Samuelson, James Tobin and Kenneth Arrow, five of the giants of the profession. It is my theory that the Great Depression attracted the best and brightest to work on the economy and these five are the best of that cohort.

Kenneth Arrow stands out even in this company. His contributions conservatively deserve two Nobel Prizes and a case could be made for three or four. Kenneth still is the youngest economist to win the Nobel Prize, having won it when he was 51. At least as impressive is that five of Ken's students have won the prize. Ken Arrow went on to win the National Medal of Science in 2004, becoming the ninth economist to win that highest of distinctions. Interestingly, all nine National Medal winners have also won the Nobel Prize and the list includes Stigler, Friedman, Samuelson and Arrow.

Now, to the contributions. Ken shared the Nobel Prize in 1972 with Sir John Hicks. The citation for the prize emphasizes Ken's contributions to general equilibrium theory. General equilibrium refers to the fact that one cannot treat markets separately, one at a time, as is so often done in Economics 1. The demand for restaurant meals doesn't just depend on the menu prices, but also on the price of gasoline, the price of food bought in grocery stores, the wage rate, hotel prices, and on and on. In fact, to some extent, the demand for any particular good depends on all other prices in the economy. Even if goods are unrelated in terms of their use, they interact through the budget constraint. If the price of something you consume goes up, you likely have less money to spend on everything else. We were reminded of this a year or so ago, when the demand for just about everything was affected by the price of oil. Similarly, the supply of any particular product depends on the entire array of prices.

The general equilibrium model, with all prices affecting the demand and supply of each and every good, was formulated by Leon Walras in 1874. If there were n goods in the model, there were n prices. By equilibrium, we mean a set of prices for which there is no pressure for them to change. This is the same concept of equilibrium used throughout the sciences. In economic terms this means a set of prices where the demand is less than or equal to the supply for all goods, with equality for goods with a non-zero price. While the general equilibrium model was recognized as the ultimate model of the market, even a proof that such an equilibrium set of prices exists proved tremendously challenging. Walras seemed to be content that the model had as many equations as unknowns, but such counting isn't convincing except in the case of independent linear equations. It took the profession 75 years to solve this problem and prove the existence of an equilibrium (i.e. market clearing) set of prices. Kenneth Arrow of Stanford and Gerard Debreu of Berkeley, in communication but working separately, accomplished this monumental task using developments in mathematical topology. Interestingly, their proof of existence did not lead to a method for calculating the set of market clearing prices. That task took another fifteen years and was accomplished by my thesis supervisor at Yale and former Stanford faculty member, Herb Scarf.

Ken's general equilibrium contributions went way beyond the existence proof. He made major extensions to the fundamental theorems of welfare economics, based on Adam Smith's conjectures of 235 years ago. The first theorem describes the circumstances under which a

competitive general equilibrium results in a Pareto optimal allocation of resources. This form of optimality is quite weak, in that in order to be Pareto optimal a situation must be such that it is impossible to make one person better off without making someone else worse off. The second fundamental theorem of welfare economics describes the circumstances under which every Pareto optimal outcome for the economy (and there are many) can be achieved with a competitive equilibrium and an appropriate set of transfers. Arrow extended these theorems to include the case of corner solutions. The traditional analysis, which used the calculus, was applicable only for the case where everyone consumed at least a little of everything. But, many of us consume absolutely zero of many things. Arrow proved the fundamental welfare theorems in a much more general way using the concepts of convexity.

Ken Arrow's contributions to general equilibrium theory and welfare economics clearly merit the Nobel Prize. The proof of the existence of equilibrium alone was a monumental accomplishment. But, these contributions weren't necessarily Ken's most important. He pretty much single handedly founded the important field of social choice theory. His PhD thesis proved the so-called Arrow Impossibility Theorem. Economists model individuals as rational in a narrow sense. Rationality implies that preferences display transitivity. That simply means if I prefer situation A to situation B, and I also prefer situation B to situation C, then it follows that I prefer A to C. This is so elemental, that it wouldn't even be necessary to ask me about A and C, if you already knew my ranking of A vs. B and B vs. C. The question Ken tackled is whether there is a way to aggregate individual preferences to form social preferences over public choices in such a way that the social preferences display transitivity and therefore this most basic trait of rationality. It was already known that simple majority rule voting can violate transitivity. But, what about other ways of aggregating individual preferences to make social decisions? Ken imposed only four relatively simple conditions on the aggregation process. Ken showed, in a brilliant proof, that under his simple and sensible four conditions it is impossible to aggregate rational individual preferences into social preferences which maintain transitivity. The work was so original that even his thesis supervisors at Columbia felt unqualified to judge it. Ted Anderson, later to be a Stanford faculty member, was then one of the more mathematical economists at Columbia. The committee asked Ted to take a look at Ken's thesis and determine whether it was any good. Ted assured him that it was more than good.

The publication of *Social Choice and Individual Values* in 1951 started the whole field of social choice. There have literally been thousands of publications in this field, all of which emanate from Ken's seminal work. If you were to go online tonight, and go to the website of Bauman Rare Books, you would find that a first edition copy of this book, signed by Kenneth, is available for $5,200. Just to put that figure into perspective, the two volume edition of Robert Frost's poems, published in 1950 and signed by Robert Frost, will set you back $3,600. For you sports fans, the 1950 biography of Ted Williams, again signed, is available for $1,500.

So, now we have the material for two Nobel Prizes, one in general equilibrium theory and one in social choice theory. But, Ken wasn't done. He had another brilliant idea. Let's go back to the general equilibrium model with n commodities. What exactly is a commodity? You might think that a black umbrella is a commodity. It had already occurred to people that an item in a different location or available at a different time is essentially a different commodity. So, an umbrella in Las Vegas in August 2008 is a different commodity from an umbrella in Las Vegas in February or certainly than one in Boston in March. Ken had an idea that greatly expanded the use of the general equilibrium model to situations involving risk and uncertainty. He used

the concept of a state of the world, which fully described the realizations of all uncertain variables. The state of the world would include the weather, the unemployment rate, whether or not an earthquake had occurred and on and on. The particular state of the world or set of outcomes would arise from a vastly bigger set of possible states of the world. Ken's breakthrough was to realize that the same item in a different state of the world is really a different commodity. Umbrellas on rainy days are different from umbrellas on sunny days, for instance. This led to the concept of contingent commodities, commodities defined contingent on the state of the world. The next step was to define a financial security that paid off one dollar in a particular state of the world, and zero otherwise. Such a financial security is called an "Arrow security." Every risky financial contract, a stock or bond or option or, as we have recently learned, a house, is a complex security consisting of a large bundle of fundamental or Arrow securities. This concept of capturing risk and uncertainty is used widely throughout finance. It is the basic building block for dealing with uncertainly. Just like computers and DVDs and almost everything electronic are digital and ultimately boil down to individual bits that can take one of two values…. Arrow securities are the bits for the modern analysis of risk and uncertainty. This was a transformational idea. Should this be Nobel three?

Ken wasn't done. In fact, Ken isn't done. Ken wrote the seminal paper that started the whole field of health economics. Ken starts by asking why medical care is different from other commodities. He notes that there are unusual problems of information and uncertainty. Doctors know much more about "the product" than the ultimate consumer, the patient. There is little if any price competition. Ken writes about the value of insurance in this market, but also about the moral hazard issues connected to insurance of having patients face very low marginal costs for care. In the time allotted, I cannot do justice to this paper other than to mention that it was commissioned by Victor Fuchs and that it started a very important field of study.

Ken wrote a series of papers on information as a commodity, innovation, patents and intellectual property. His 1962 analysis of learning-by-doing in production is another classic. The benefits of production in a particular manner are not only the output produced but also the greater experience of the firm and the workers in the techniques of production. Through production, workers and firms learn how to produce a given output with fewer inputs. Some of this knowledge may be kept within the firm, but some of it may provide an external benefit to the economy as a whole. To the extent the firm cannot capture all of the benefits, on-the-job experience will be underprovided relative to an economically efficient allocation.

But as the ads say, "Wait, there's more!" Ken was a co-author in developing one of the most widely used specifications of production functions, the constant elasticity of substitution (CES) production function. He wrote a series of papers with his friend and colleague at Stanford, Mordecai Kurz, introducing control theory to the theory of the firm and the household and to public finance. He wrote several papers on racial discrimination. One question addressed is what racial attitudes are necessary on the part of employers, workers and customers to allow racial discrimination in compensation to persist. Such analysis is essential in order to address the serious problem. Ken not only wrote on linear and non-linear programming, he was a distinguished professor in the Stanford Operations Research department.

Ken certainly is considered a highly abstract mathematical economist. However, his contributions also revolutionized the study of policy issues in any number of areas. He was on the staff of the Council of Economic Advisors in the Kennedy Administration (James Tobin was

a member of the Council – so two of "those five").   In 1970, he was amongst a group of economists who opposed massive government support for the development of a supersonic passenger plane.   Interesting, that group included Paul Samuelson and Milton Friedman (3 of the fab five).   Most recently, Ken chaired a National Institute of Medicine committee that advocated global subsidies to make antimalarial treatments affordable to the poor in Africa and Asia.

Ken's collected papers were published in the mid-1980s and they consisted of six volumes. He has written more than one hundred articles since then. He has continued to work on information economics, on capturing attitudes towards risk, on income distributions and inequality, and on social choice theory. A lot of Ken's recent work has focused on global climate change, sustainable consumption, and the well-being of distant future generations. It all adds up to 22 books in which Ken is the author or co-author, 24 books in which he was the editor or co-editor, 41 nontechnical articles, and 266 research papers. But, it adds up to much more than that… it adds up to Kenneth Arrow.

Ken knows a lot about everything.   He knows a lot about mathematics, politics, music (especially Gilbert and Sullivan), drama (Shakespeare in particular), Chinese Art, Poetry, the history of economic thought, you name it. I can remember almost thirty years ago I went to a department seminar given by a woman who we were considering hiring. Her research was on the 19th century British steel industry. Ken attended the seminar. Ken was a theorist, what could he possibly know about the 19th century steel industry in Britain? Well, as it turned out, he knew a lot. He knew the key research findings and was more than ready to talk about the degree of increasing returns to scale in that industry at that time in that location. I could hardly believe how much he knew about that particular topic.

Now, I am going to conclude by relaying a story told by Eric Maskin, one of those five students of Ken's to win the Nobel Prize. Ken's junior colleagues had all observed that on almost any subject, Ken turns out to know a lot more about it than they do. So, they concocted a plan. They would all study up on one of the most arcane topics they could think of – the breeding habits of grey whales. On the appointed day, they gathered in the coffee room and waited for Ken to come by. Then they started talking about whales and the elaborate theory of a marine biologist named Turner on how grey whales found their way back to the same breeding spot year after year. Ken was silent… they had him at last! With a sense of delicious triumph, they continued to discuss whales, and Ken looked more and more perplexed. Finally, he couldn't hold back. "But I thought that Turner's theory was entirely discredited by Spencer, who showed that the hypothesized homing method couldn't possibly work."

With all of these prodigious accomplishments, Ken is humble, warm, and a great friend to all of us at Stanford. I have learned when he starts a question with "I don't understand…." he is being polite. He usually could have started the question with "You don't understand…."   So, it is an honor to have summarized Ken's economics contributions… I have highlighted some fantastic accomplishments, but I am positive that I have missed some real gems.[1]