Zvi Griliches and the Economics of Technology Diffusion: Linking innovation adoption, lagged investments, and productivity growth

by

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ABSTRACT

This paper considers the scientific legacy of Zvi Griliches’ contribution to the economic analysis of the diffusion of technological innovations. It first examines the relationship between Griliches’ pioneering empirical work on the introduction and adoption of hybrid corn and the subsequent development of theoretical models and econometric research on the microeconomic determinants of diffusion. Next, it formalizes the way that the dynamics of diffusion observed at the aggregate level is shaped by structural conditions at the micro-level – on the supply and the demand sides of the market for products embodying technological innovations, both of which were addressed by Griliches (1957). It notes also the reflections of those processes in lagged behavior of aggregate investment in durable capital-embodied innovations. Thirdly, it extends the latter perspective to make explicit the micro-to-macro relationships affecting the total factor productivity (TFP) growth rate. These three dynamic phenomena --diffusion, lags, and TFP growth – were the topics of Griliches’ three most widely cited among journal articles, respectively. The connections among them remained implicit his writings until late in his career when he emphasized the diffusion-productivity nexus as a key proximate determinant of the pace of economic growth. Directing attention to the microeconomics of technology adoption underlying the ‘transitions’ during which the diffusion of major innovations generate surges in innovation-embodying capital formation, and waves in the TFP growth, figures prominently among the Griliches’ important contributions to modern economics.

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1. Introduction

This essay considers the scientific legacy of Zvi Griliches’ contribution to the economic analysis of the diffusion of technological innovations by examining the relationship between his pioneering empirical work on the introduction and adoption of hybrid corn and the subsequent development of theoretical models and econometric research on the microeconomic determinants of diffusion. Those developments have exposed the ways in which structural conditions at the micro-level on the demand side of markets for new process innovations, and dynamic feedbacks affecting the supply side of the markets for innovative products can interact to shape the specifics of diffusion phenomena that are observed at the level of industries and sectors. Further elaboration of this analytical perspective brings into clearer focus the micro-to-macro connections between diffusion and the lagged behavior of aggregate investment in capital-embodied innovations, as well as the impact of diffusion dynamics on the pace of growth of aggregate total factor productivity (TFP). Directing the attention of empirical and theoretical research to examine the microeconomic mechanisms that underlie technological ‘transitions’ driven by the diffusion of major innovations has revealed processes that can generate wave-like surges of innovation-embodying capital accumulation, and corresponding waves in the growth rates of industrial and sectoral total factor productivity (TFP). The impetus his pioneering study of hybrid corn imparted to subsequent research aimed at identifying the roles of structural conditions and dynamic linkages among the population of potential adopters and the suppliers of innovation-emboding producer goods, deserve recognition among the most important enduring legacies of Griliches’ contributions to modern economics.

1.1 Diffusion, distributed lags and the growth of measured TFP: Zvi Griliches’ three biggest journal publication “hits”

The three most widely cited journal articles by Zvi Griliches deal, respectively, with the economics of the diffusion of technological innovations, the econometrics of distributed lags and the sources of measured changes in total factor productivity (a collaborative paper with Dale Jorgenson). Obviously, citation statistics are but one means of gauging the intellectual impact of research contributions – and a limited one at that. Nevertheless, it is testimony to the significance of the main subject of this essay that the 1957 *Econometrica* paper on the introduction and acceptance of hybrid corn among U.S. farmers comes first in the rank ordering of the many journal articles whose cumulative annual journal citations have been compiled in Diamond’s (2003) survey of Griliches’ contributions to the economics of technology and growth. Moreover, the remarkable fact that the annual flow of citations to this particular article has continued to trend upwards throughout the forty-five years following its publication – and most probably for the full half-century -- bears striking testimony to the lively interest among economists in the phenomena that it brought within the purview of their

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The three papers top the all time citation ranking of 110 journal articles whose annual citation counts from 1956 to 2002 were compiled from the Web of Science journals, supplementing in some cases by material covered in the Social Science Citation Index. The respective total citation counts reported in Diamond (2003: Table II) for these articles are: Griliches (1957) –548.8; Griliches (1967) –214;Griliches and Jorgenson (1967) –106. Because Diamond (2003) interpolated monthly counts for September-December of the final year by applying multiplier of 1.5 to the data for the months through to August 2002, his procedure generated some fractional totals, such the one that appears for the first paper in this list. On the general topic of the limitations of publication citations as indicators of scientific importance, van Raan (1988) remains an excellent place to start.
discipline, and to the sustained influence of that path-breaking study.\footnote{Corroborative reassurance is available, from the \textit{New York Times} (5 November 1999: p. 11) obituary by Michael Weinstein, which reports Dale Jorgenson’s reference to this study, along with the 1958 \textit{JPE} article (measuring the social rate of return on R&D in hybrid corn) as the best known and most-mentioned of Griliches’ contributions. Diamond (2004) also notes Ariel Pakes’ (2000) description of the 1957 \textit{Econometrica} article as “seminal.”} Although these three top-cited publications generally have been viewed as unrelated contributions (so much so that their joint salience in economics literature owes virtually nothing to cross-citations among them), there are significant substantive connections among these topics. That contention motivates the discussion pursued in the following pages.

Necessarily, it begins by reviewing the specific details of Griliches’ path-breaking paper on the introduction and adoption of hybrid corn, and its influence upon subsequent research by others on the diffusion of innovations. This did not stimulate explicit considerations of economic relationships among the three dynamic phenomena that have just been bracketed here; the connections among them remained largely unexplored in the literature, including Griliches own research where they remained implicit, at best. Indeed, this reflected the fact that by the mid-1960’s his attention had turned away from the study of specific technological innovations and the phenomena associated with their diffusion into use, and he did not return to the subject empirical studies of diffusion – apart from Griliches’(1980), the brief (but nonetheless significant) comments prompted by Dixon’s (1980) re-examination of the available U.S. data on the case of hybrid corn. Nevertheless, clear recognition of the broader importance of understanding the dynamics of technology adoption resurfaced in Griliches’reflective writings. That the nexus diffusion-productivity growth nexus figured prominently in his approach during the 1990’s to understanding the main proximate determinants of the pace of economic growth, and there for warranted closer attention from economists, is evident from the following passage in Griliches’ \textit{R&D and Productivity: the Econometric Evidence} (1998):

> “Real explanations [of productivity growth] will come from understanding the sources of scientific and technological advances and from identifying the incentives and circumstances that brought them about and that facilitated their implementation and diffusion.”\footnote{Emphasis added. This passage is reprinted in Griliches, 1998: pp.89-90 from Griliches (1994) \textit{American Economic Review} article on productivity, R&D and “the data constraint.” His text goes on to say that recognition of these connections would – or at least should – lead economists “back to the study of the history of science and technology and the diffusion of their products, a topic that we have left largely to others.”}

On the strength of that endorsement, this paper undertakes to counter-balance the tendency of the endogenously growth model literature to gloss over the dynamic processes through which the diffusion of innovations is implicated in the realized pace of aggregate total factor productivity growth.

\subsection*{1.2 Organization and overview}

The organization of the discussion proceeds, accordingly, “from micro to macro.” Chapter 2 begins with a review of the approach that Griliches (1957) adopted in studying the micro-level economic determinants of the introduction and adoption of diffusion of hybrid corn on U.S. farms. The discussion (in sect. 2.2) considers the impetus that was imparted to economists’ subsequent empirical research on the diffusion of process innovations by the
emphasis Griliches’ commitment to the rational-actor model in which profitability considerations were the main determinant of the speed with which the innovation had been accepted within different farming regions. It traces the way that the particular empirical strategy that he had employed directed the subsequent course of theoretical model-building toward accounting for the logistic shape of diffusion paths in terms of “contagion” effects, although these were not featured in his 1957 discussion of the role of information propagation lags in the adoption process. Section 2.3 returns to re-examine the controversy with rural sociologists who had been provoked by Griliches’ treatment of the subject in terms that allow little if any explanatory importance to the aspects of it on which their approach (informed by social psychology and sociometry) had focused, and the respects in which that stance underwent some accommodating transformations in the course of responding to criticisms from that quarter (Griliches 1960, 1962). As a result, what is seen to have emerged from the controversy was substantive agreement between the emphasis accorded by sociologists and economists to social communications as the central dynamic process driving of technology adoption, accompanied by a growing disjunction between the conceptual and empirical frameworks within which work on the subject proceeded in the two disciplines. As

As Section 2.4 shows, the paradoxical state of affairs just referred to became manifest in the literature with the rising popularity among economists of the formal version of the contagion model that was introduced by Mansfield (1961), and the evident commonality between the latter’s econometric strategy and that which Griliches (1957) had introduced. Section 2.5 takes up other matters that had not been subjects of disciplinary controversy but were nonetheless recognized to be problematic -- yet not readily resolved within either Griliches’ (1957) explanatory framework or that of the rural sociologists who viewed the structures channeling social influence to be the key to understanding adoption behaviors. Central among these was the existence of heterogeneity in the circumstances of potential adopters, and its connection with the behavior of the suppliers of innovation-embodifying inputs -- such as hybrid corn seeds, which Griliches’ study had undertaken to explain also in terms of rational profit-seeking action. Both of aspects of Griliches’ original work were marginalized in the literature by the growing popularity of the “information contagion” model, but emerged during the later 1960’s in an alternative, the basic “threshold model.” This approach to explaining diffusion (David 1966, 1969, 1971) was based on recognition that many innovations were biased in their relative factor-use vis-à-vis the established production technologies, and therefore explicitly considered the way this would interact with heterogeneities among potential adopters to affect their choices among the alternative available production processes. It opened a new route through which empirical models could account for aggregate-level lags in diffusion lags by allowing for the effects of supply-side technical change that altered the relative price-performance ratio of innovation-embodying products, and thereby induced sequential demand-side switching from old to new process technologies.

Chapter 2’s discussion closes by considering (in 2.6) the implications for empirical research of the resulting proliferation in the literature of “diffusion models” that appear to be observationally equivalent, all being capable of generating the same form of aggregate diffusion curves. To the contagion, and the threshold models was added which is here seen to be an incipient evolutionary, replicator model that appeared in Nelson (1958) – although not construed explicitly in evolutionary terms until the early 1980s. All three models are seen to be capable of generating the canonical logistic form of S-shaped (ogive) diffusion curve to which the attention of model-builders had been directed by Griliches and Mansfield. There it
is shown that their observational equivalence at the aggregate level is superficial, as it masks the absence of micro-level observations that would permit identification of the quite distinct underlying specifications of in these models of behavioral responses, structural conditions and related time constants of the dynamic processes governing the temporal distribution of adoption decisions taking place among the population of potential adopters.

Chapter 3 presents a more complete account of the “threshold” approach to the microeconomics of technology adoptions based upon explicit modelling of the role of population heterogeneities. Griliches (1980), in a brief reflective comment contrasting treatments of diffusion, characterized this recognition of the implications of heterogeneities among the potential users economic and environmental circumstances as a useful “moving equilibrium” alternative to the paradigm of a “disequilibrium” transition governed by information constraints. The exposition in section 3.1 is based mainly upon David and Olsen (1984, 1986), which expands the basic “threshold model’s” analysis of an informed rational agent’s choice-of-technique decisions (between a novel, evolving technology and an familiar, mature technology), by taking into account the dynamic effects of anticipated feedbacks from changes in supply conditions affecting the costs of adoption. The latter formulation absorbed Stoneman and Ireland’s (1983) introduction of supply-side learning effects in the “threshold” framework, but went further by considering the implications of learning externalities or “spill-over effects” on the pricing of durable production assets when potential users (as well as producers) could anticipate the future trajectory of the relative prices of innovation-embodying goods. Whereas Griliches’s (1957) viewed the supply of innovation-embodying inputs as affecting only the timing of the first significant commercial introduction of the new technology, in the more fully elaborated models of the 1980’s the behaviors of agents on both the supply-side and demand-side are seen to continuously shape the dynamics of diffusion path. Moreover, together with conditions in the market for the goods what it is used to produce, these endogenous forces can generate a moving upper ceiling, or saturation level in the extent of diffusion -- towards which the actual path of diffusion’s convergence drives the ceiling upwards until the niches into which the innovation can spread are exhausted.

The sources and forms of the distributions of the putative heterogeneities among potential adopters that can be admitted in models of this general class, and their relationship to the time-constants of the micro-level diffusion process are discussed in Section 3.3. This serves to make explicit the connections between the structural conditions underlying such processes and the distributed lag dynamics that diffusion could generates in the volume of aggregate industry-level or sector-level investments in capital equipment and structures that embody the innovation. There it is noted that the explicit dynamic analysis of adoption decisions presented in Chapter 3 underscores a proposition that in a sense is the corollary of the point emphasized in section 2.6: fully specified models of diffusion belonging to the large class that posit the existence of heterogeneities among the population of potential adopters can generate very distinctive differences in aggregate level diffusion dynamics, even where the hypothesized economic micro-level mechanisms are identical.

A forward-looking message that emerges in section 3.4 is that econometric testing of these and still richer theoretical models of technology diffusion will not be possible without much greater concerted efforts to develop extensive bodies of consistent micro-level cross-section data and aggregate time-series observations that allow controlling for confounding effects on each of the distinct subprocesses that are recognized by the array of alternative diffusion mechanisms. It is noted that the inability to meet that exacting challenge undermined early socio undermined sociometric research by Coleman, Katz and Menzel
(1957, 1966) that was long taken by sociologists to empirically support the claims of the “social learn thorough information contagion” explanation of diffusion phenomena – and to an extent that has only recently been appreciated. Clearly, panel data providing observations not only on adoption behavior but intervening non-pecuniary interactions, including social communications among actual and potential users of the innovation, are what will be required empirical identification of the micro-level processes and measurement of their characteristic time constants and relative impacts on the course of changes in the aggregate extent of diffusion.

Chapter 4 builds upon the analysis of the general threshold model of diffusion presented in Section 3, and examines the implications of its underlying determinants for the rates of growth labor productivity and total factor productivity at the aggregate level of an industry in which a new, capital-using process technology is displacing a pre-existing method producing a consumer-good using only labor. The main structural features and rationalizing assumptions of the simulation model that is constructed for this purpose are outlined in section 4.1 (relegating the formal details to the paper’s Appendix). Section 4.2 examines the main points that emerge from the simulation exercises, which exhibit the distinct ways in which underlying structural parameters affect the amplitude and durations of wave-like movements that the process of the innovation’s diffusion generates in the aggregate level growth rates labor productivity and TFP.

Having completed the announced task of showing the connections between micro-level adoption behaviors, the dynamics of distributed lags in industry level investment, and the diffusion-driven sources of aggregate TFP growth, the paper concludes in section 5 by commenting briefly on a possibly important set of connections that may warrant investigation and inclusion in a future elaboration of the simulation structure presented here. These concern the linkages between diffusion processes and the behavior of R&D investment in the industries that supplying the innovation-embodied capital goods, and translate knowledge gained from experience in producing those goods, as well as from the firms that adopted previous vintages of their innovative products into newer and more effective inputs that can gain acceptance among the market niches that the innovation has not yet penetrated. Little appears to be know about this aspect of the nexus that may indirectly connect R&D and productivity growth, and such an exploration of the diffusion-R&D connection might usefully round out the program of research that became the main focus of Zvi Griliches’ sustained contributions to the empirical study the sources of economic growth in the modern era.

2. The Nature of the Legacy — Economics and Technology Diffusion

The approach taken here to assess the nature of Zvi’s Griliches’ (1957) seminal contribution to the literature on the economics of diffusion must consider not only what had been accomplished in that work, and how it shaped the ensuing development of the literature devoted to this subject, but also what had left to be done by later contributors. This section therefore focuses upon both the achievement and the limitations of this famous study of the introduction and acceptance of hybrid corn by U.S. farmers.

2.1 The hybrid corn study — an econometric paradigm is born
Zvi Griliches’ early econometric study of the commercial introduction and diffusion of hybrid corn in the U.S. was influential for two reasons that were somewhat in tension with each other. First, he construed the phenomenon of diffusion in economic rather than sociological terms, and so opened a new avenue to examining the economics of technological change. Second, the quantitative approach he adopted was primarily inductive, rather than dependent upon formulating a particular theoretical model from which deductive propositions could be derived and subjected to statistical tests. Instead, his work had given others a simple way to quantitatively characterize three aspects of the generic phenomenon of technology diffusion observed at the aggregate (population) level – the length of the introduction lag, the speed of the innovation’s adoption (or “acceptance”), and the terminal extent of its diffusion. This providing an intuitively appealing and tractable empirical methodology that proved to be widely applicable, and attractive in leaving room for many alternative explanatory hypotheses concerning the underlying behaviors of the adopting agents.

Both aspects of Griliches’ approach are reflected in his implicit characterization of the diffusion process as one that occurred successively within distinct geographical regions, and in his selection of a statistically convenient descriptive specification for the diffusion path – namely, the cumulative logistic distribution. He could then proceed immediately to hypothesize that the speed of diffusion (and hence the overall shape of the S-shaped path determined by the slope parameter in the diffusion function) would reflect economic conditions having to do with the innovation’s profitability for a representative adopter. Similarly, economic factors could be supposed to affect the location of the onset date for the diffusion process under examination, and therefore to be reflected statistically in the value of that second (scaling) parameter of the logistic. 4

Griliches thereby was able to characterize the diffusion path in terms of two readily obtained parameters – the slope coefficient of the logistic function, and the ‘intercept’ coefficient (which sets the initial or conventionally perceptible) proportion of adopters from whence the observed diffusion process proceeds. 5 The elegance of this simplification had a major initial influence in stimulating econometric research on diffusion: characteristics of various technological innovations, or of the industries and markets into which these had been introduced, could simply be entered as regressors that might account for inter-innovation variations of the logistic slope coefficients.

As will be seen, this soon was seized upon by economists as an “obvious” way proceed in quantitative studies of the role of demand-side conditions in determining the adoption of new production techniques and new goods. But, an equally novel aspect of Griliches’ paper in *Econometrica* (1957) was concerned with factors operating on the supply side of the market for hybrid corn seed in the U.S. Like many other innovations, hybrid plants are most efficient as elements of a production system when they have been designed for a specific environment. In some cases the relevant “environment” is economic, in the sense of

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4 When the upper asymptote of the diffusion path is taken to be unity (universal adoption), signifying the assumption made by Griliches’ (1957), there are only two free parameters to fit econometrically for the logistic distribution. But the conventional approach is to estimate a slope parameter and an intercept constant from a (linear) model of the log-odds ratio. See discussion below, in this section.

5 Griliches (1957:p.504, esp. n.10) obtained these parameter estimates by unweighted least-squares regression, based on the log-odds transform of the logistic growth curve, rejecting the weighting methodology proposed by Berkson (1955) to correct for heteroskedasticity, on the grounds that the bio-assay context in which Berkson’s procedure had been proposed was of doubtful applicability in the (time-series) context in which he was working.
being defined by the structure of relative prices of the array of inputs used by the production system; in others, it is the physical environment to which the process required being adapted. In the case of hybrid corn, Griliches noted, local variations in soil types, climate, and pests called for the suppliers of seeds to develop particular varieties that would be best suited to the requirements of farmer in the various sub-regions of the U.S., ranging southwards from Wisconsin and Iowa, to Texas and Alabama (see Fig. 2).

Consequently, considerations of the profitability of the incremental “development” investments this entailed – such as the extent of the existing corn-acreage in the sub-region, and the typical size of the farm that the company’s seed-salesmen would have to visit – would be expected to affect the spatio-temporal sequencing of the innovation’s introduction. In this part of his analysis, Griliches clearly had anticipated the efforts of later builders of microeconomic models to incorporate the effects of incremental supply-side adaptations upon the diffusion process. Yet at the time this aspect of his path-breaking study, about which I will want to say something more, attracted comparatively little notice.

Much more notice, and even some cross-disciplinary controversy that heighten economists attention, was focused on aspect of his work that supported the conclusion that adoption behavior exhibited a “rational” response to the availability of a superior method of production, one that was “consistent with the idea of profit maximization.” Although the lagged adoption of hybrid corn by the farmers in a particular region of the country reflected the fact that “the spread of knowledge is not instantaneous” (1957:p.522), the speed with which the region’s imperfectly informed farmers came to “realize that things had in fact changed” in the technology of corn cultivation, and adjusted their methods accordingly.

Griliches’ classic (1957) paper on hybrid corn did not undertake to develop a formal theoretical justification for its reliance on the logistic specification in the econometric analysis; nor did it emphasize an economic (or sociological) explanation for the failure of the innovation to be taken up instantaneously and universally as soon as it was introduced in any particular region. It is reasonable to suppose that such emphasis was not thought to be necessary, in view of the compelling empirical patterns exhibited in that famous chart of the S-shaped curves tracing the rising proportion of corn acreage planted with hybrid seed in each of the major-corn growing states. There are, however, several passages that clearly indicate Griliches’ view that the uncertainties surrounding the decision as to whether to adopt a novel method of production, such as hybrid corn, could not be immediately dispelled; it would take time for farmers individually to accumulate information that successively reinforced confidence the innovations’ alleged beneficial (profitable) properties. Here is the way he puts it (Griliches, 1957, p. 516): “Also, in a world of imperfect knowledge, it takes time to realize that things have in fact changed. The larger the shift the faster will entrepreneurs become aware of it, “find it out,” and hence they will react more quickly to larger shifts.”

At the end of that sentence appear a footnote that has been largely overlooked by most of the commentaries on this frequently cited paper, but which is nonetheless significant in reveal that Griliches envisaged the individual process of decision making under uncertainly as inherently time consuming, because sufficient confirmatory data had to be accumulated in order to justify abandoning an established routine. He explicitly refers to the statistical method of sequential sample analysis for quality control decision, and the procedure for this (based on the Average Sample Number) that Wald (1947) had devised, taking that as support for his interpretative hypothesis that there would be an inverse association between the size of
the stimulus (in the form of differentially greater profitability) and length of time that would elapse before the a deliberating farmer responded by accepting the new method. The Average Sample Number as the expected number of items that would need to be sampled for a given batch before being able (with a specified level of confidence) to reject the batch as belonging to a lower average quality population than the standard that was desired. Griliches thus likened the farmer’s decision under uncertainty to a quality control decision, in which information would have to be accumulated sequentially in order to attain the requisite ASN for discarding open pollinated corn in favor of the hybrid alternative:

“This is analogous to the situation in Sequential Analysis. The ASN (average sample number) is an inverse function of, among other things, the difference between the population means. That is, the larger the difference between the two things which we are testing, the sooner we will accumulate enough evidence to convince us that there is a difference.” Griliches (1957: p. 516, n.33)

A number of notable points emerge from closer consideration of Griliches’ somewhat arcane statistical analogy. First, this explanation for the phenomenon that new and supposedly superior methods are not adopted instantaneously is fully consistent with Griliches’ approach to adoption phenomena as reflecting rational decision-making – in this case under conditions of uncertainty. Secondly, insofar as there is a decision model, it is framed at the individual level and therefore invokes a representative agent’s behavior to explain why the observed transition to hybrid corn by a typical farmer would be found to proceed more quickly where the profit differential offered by the innovation was bigger. Notably, there is no indication that the arrival of confirmatory information is a variable that could depend upon interactions or communications among neighboring farmers; in the sequential analysis set-up the decision is being made completely independently of others, and based entirely on the receipt of information that is generated autonomously – perhaps, in the actual case at hand, being broadcast by the USDA agricultural extension service.

Thirdly, micro-level explanatory framework accounts for the delay in adoption, and for differences in the length of that delay following the introduction of an adapted (and hence profitable) hybrid in the region. But this does not explain the phenomenon of a continuous rise in the proportion of farmers that have joined the ranks of the adopters. Something is missing, without which the time path of diffusion a given corn-growing region would take the “bang-bang” form of adjustment: nothing would happen following the introduction of the innovation, and eventually all the farmers would adopt at once – supposing there was no constraint on the available supply of the hybrid seeds.\footnote{Strictly speaking, this would not necessarily be the case, given Griliches’ choice of the proportion of regional corn acreage planted with hybrid corn seeds as the measure of the extent of diffusion. The adoption decision could be to plant some of the farm’s acreage with hybrid corn, and adjust this upwards over time. Therefore the binary decision framework does not necessarily apply in these circumstances, whereas it would if the index of diffusion measured the proportion of farms or farmers on which hybrid seed had been planted at all. Mansfield (1963a, 1963b) distinguished between inter-firm and intra-firm diffusion, with appropriate different measure of the extent of adoption.}

Fourth and last, the proximate source of this problem is the representative agent approach that has been seen to be implicit in Griliches’ interpretation of his empirical results. If the decision to adopt an innovation poses a binary choice, then, in order to account for the
continuity of the observed transition to a widespread acceptance of the new technology among the population, it is necessary to posit some heterogeneity among the agents. Within the framework suggested by Griliches’ discussion of sequential analysis, one could readily introduce differences in the rate at which information would reach different farmers; or suppose that the loss functions, or confidence levels characterizing different agents would result in a continuous distribution of average sample numbers, so that if broadcast messages about the benefits of switching to hybrid corn were reaching every one at the same rate, the dates at which different farmers attained their respective ASNs would be distributed continuously through time. But, the pull of representative agent modelling was strong, and the path towards models based on explicit recognition of heterogeneities in the population of potential adopters was not taken immediately by the economists who followed Griliches’ lead.

2.2 Toward behavioral interpretations: the “contagion model” emerges

Not long thereafter, Edwin Mansfield’s (1961, 1963a, 1963b, 1966, 1968b) inquiries into the diffusion of industrial innovations began to erect an impressive empirical edifice by applying the descriptive statistical approach that Griliches (1957) had pioneered. Mansfield (1961) ventured beyond Griliches’ work, by proposing a formal economic rationale for his econometric studies’ employment of the logistic form in characterizing the time path of measures of the proportionate extent of diffusion -- explicitly hypothesizing that information imperfections effectively constrained adoption, but would be gradually eliminated as knowledge about of the innovation became more and more widely disseminated by social communication within the relevant circle of potential adopters. The supposed mechanism of dissemination was a social contact, ‘word-of-mouth’ transmission of the relevant knowledge, rather than one based upon economic consideration of the benefits and costs of searches for information firms for which the new industrial processes were potentially relevant. This was viewed as natural, and essentially costless, unlike the broadcasting of information about the benefits of the innovation by the producers and vendors of the inputs that would enable the installation and operation of the of the required new facilities and method’s operation – knowledge intermediaries such as the publicly funded agricultural extension agents, and the marketing personnel of the private hybrid seed companies.

Rather than trying to explain why such information seemed to flow along particular channels within a network of social communications -- of the sort to which studies by rural sociologists’ tended increasingly assign importance, (1961:pp.746-748) produced a version of the so-called ‘contagion’ model of information diffusion, by starting from a general conceptualization in which the probability of adoption is a function of perceptions of the magnitudes of the greater profitability enjoyed by users of the innovation interacted with information about the extent of its adoption among others (“competitors”) in the industry, and working his way toward the well-known separable form of the differential equation for the logistic “information propagation” process. This classic “social contagion” or random

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7 This derivation involved beginning with a Taylor series expansion of an general multivariate function, and proceeded by successive arbitrary suppressions of higher-order terms, and an equally ad hoc imposition of limiting initial conditions, to arrive eventually at a deterministic expression for the increased number of adopters whose separable form expressed the probability of an incremental adoption as the product of two components: the probability of a randomly drawn agent becoming informed, and the conditional likelihood that an informed agent would become an adopter. The rationale for this cumbersome procedure nowhere is stated by Mansfield. But its palpable effect is to convey the appearance of arriving at the logistic regression model from a completely general theory of the representative firm’s adoption behavior, rather than from the behavioral hypothesis that the
“word-of-mouth” process of information dissemination, separates the expected effect upon a non-adopter of learning about the relative advantages of an innovative method from the likelihood of a non-adopter (in a symmetrically interactive and completely inter-penetrating population of agents) becoming informed by others about the innovation’s putative benefits. By positing that information is transferred (as an infection) through random social contacts between those who had already adopted the innovation and those who had not, one arrives almost immediately at the differential equation: \( \frac{dP}{dt} = \nu \{ P(1-P) \} \), where \( P \) is the mean probability that a randomly drawn member of the population of potential adopters will already have adopted the innovation and hence possess information as to its benefits. Under conditions of complete and random social inter-mixing, the product term \( \{ P(1-P) \} \) is the probability of a random dyadic contact at each “moment” that will alter the non-adopter’s information state to that of “sufficiently informed.” If the constant \( \nu \) is the mean probability that a newly informed non-user will join the ranks of adopters, the left hand-side of the equation represents the resulting increase in the expected increment in the share of adopters among the population. Then \( P(t) \), thus defined, is a measure of \( D_n(t) \) the extent of the innovation’s diffusion at time \( t \), and so, by integration of the differential equation for \( dP/dt \), one may arrive immediately at the result that \( D_n(t) \) is a logistic function of \( t \), with slope parameter \( \nu \).

Mansfield then could follow Griliches’ (1957) empirical strategy and his general emphasis on the centrality of profitability considerations in adoption behavior, hypothesizing that where expected profitability of adoption was higher, there would be a higher mean probability that an informed non-adopter would immediately accept the innovation. He first found \( \nu \) as the slope parameter of the logistic equations fitted by weighted least squares regression to the time-series observations on the actual extent of diffusion \( (D_n(t)) \) for each of 12 process innovations that had been introduced in the period between 1900 and the immediate post-World War II years, and eventually were adopted by all the major firms in 4 industries -- ranging from bituminous coal mining, to beer brewing, iron and steel and railroads).

8 This and other more complicated, mixed broadcast and social contagion processes originated and were elaborated in the field of mathematical epidemiology -- notably by Bailey (1957), upon whose work Mansfield (1961) had drawn. See also, Bailey (2nd Ed., 1975) for further refinements. The implicit contrast with “completely interpenetrating” is a population that is stratified by social class, income-related status, or other ascriptive bases for preferential social attachments, or otherwise partially “balkanized” within essentially disconnected networks of communication. Coleman (1964: Ch. 17) treats the mathematics of such processes under the heading of “diffusion in incomplete social structures”.

9 If \( 0 < \nu < 1 \), as is usual in these model, in the deterministic equivalent version of these models one must allow the possibility that the average “contacted” agent will have been contacted and hence “informed” but (with some positive probability) remain a non-adopter. Implicitly it is assumed that the informational effect of the contact is immediately dissipated, so that the expected effect of the next contact is that same as that of the first, or any among those that preceded it. The foregoing text is phrased accordingly.

10 Mansfield (1961:p. 746) offered the same rationale for the hypothesized effect of profitability, referring to Griliches’ (1957) study: “As the difference between the profitability of this investment and that of others widens, firms tend to note and respond to the difference more quickly. Both the interviews [the Mansfield conducted with industry personnel from whom he obtained some of his innovation specific data, e.g., on the payback period] and the few other studies regarding the rate of imitation suggest that this is so.”
Mansfield took as a measure of the “profitability” of the i-th innovation in the j-th industry the inverse of the ratio between the average “payback” period that the major firms in the j-th industry had experienced when they installed that innovation (for all the major firms in his dataset did eventually adopt these technologies), and the average payback period that his management informants from that industry reported was used by firms in evaluating all such investment projects. This was one of a pair of right-hand variables he entered in a linear regression model to account for the variations among the innovations in the (slope parameter) estimates of their respective “speed of acceptance” (to use Griliches’ terminology) -- or what Mansfield, focusing upon the adopting firms, termed the “rate of imitation.” The second “explanatory” variable measured the relative “lumpiness” of the required investment outlay as the ratio between the average initial expenditure for the i-th innovation in the j-th industry and the mean asset size of the major companies in that industry. Mansfield (1961: p. 747) included this variable on the argument that “firms tend to be more cautious before committing themselves to such projects and that they often have more difficulty in financing them.”

To test the ability of the foregoing pair of industry-specific variables to account for the inter-innovation variations in the speed of imitation, Mansfield estimated the linear (log-odds logistic transform) regression model for the 12 “rate of imitation” (slope) parameters, constraining the coefficients on these variables to be the same for each of the innovations adopted within each of the industries. Finding a statistically significant negative estimate of the coefficient for this “investment size” variable, along with the significant positive regression coefficient for the relative brevity of the (ex post) “payback period”, Mansfield concluded that – some unexplained inter-industry variations notwithstanding – the hypothesis that process innovations diffused into use more quickly when they were more profitable, holding constant the greater risks and financing constraints associated greater required investment size, stood up to the data “surprisingly well.”

This paralleled the procedure of Griliches (1957: Tables VI, VII), in constraining the estimated effects on the logistic slope parameters of the variables (average corn acreage per farm, pre-hybrid average yields, etc.) indicative of hybrid corn’s differential profitability to be constant across the states, and across crop-reporting districts.

Describing this approach as based upon the “deterministic” version of the information contagion model, because it sought to explain the rate of imitation based upon (the logistic parameter estimated from) the actual timing of adoptions by firms in the case of each innovation, Mansfield (1961:sect. 6) tried a second econometric strategy, estimating a “stochastic version” of the model. In effect, this method obtained an estimate of the (hypothesized) a constant hazard of adoption from the mean of the observed proportions of (previous) non-adopters that joined the ranks of adopters at each successive date. Mansfield (pp. 359-360) reported that the expected number of new adopters at each date, obtained by using that value of the slope parameter, showed greater deviations from the actual observations than the residuals obtained from the weighted least-squares estimates of the (deterministic) logistic model, but that the results of regressing the estimated slope parameters for each innovation on the profitability and investment size variables were virtually identical to those he had obtained by weighted least squares regression estimation of the log-odds equation. There is a small detail of econometric methodology that differentiates Mansfield’s empirical approach from that of Griliches, but appears to have passed without notice in the subsequent literature. Whereas Griliches (1957:p. 505) explicitly justified his use of the logistic curve simply as a descriptive device (reducing an extensive body of data to three sets of parameters – origins, slopes and ceilings, Mansfield presented his regression model as based upon the “deterministic” logistic equation derived from a behavioral theory of diffusion driven by social contagion, adding a disturbance term to the log-odds transformation of the equation for the expected number of new adopters in each period. In this case, however, minimum Chi-square estimation would have been more appropriate than the maximum likelihood (weighted least squares) estimation procedure that Mansfield had employed, in view of the problem of serial correlation that would be induced by disturbances in the behavioural
2.3 Griliches and Mansfield and the ‘disequilibrium’ conceptualization of diffusion

Two sets of remarks now are in order concerning the foundational phase of empirical research on the economics of technology diffusion, represented by the works of Griliches (1957) and Mansfield (1961). The first of these concerns the role assigned to informational constraints, and the reliance upon the passage of time as a proxy for the unobserved improvements in the state of knowledge among the population of potential adopters about the economic benefits of the innovation. The second concerns the assumption that the population of firms (and farms) is essentially homogeneous in regard to their situations, except in regard to their information state, so that a representative agent model is sufficient to characterize individual behavior and the translation between the analysis of micro-level behavior and phenomena observed at the aggregate level of industries or sectors is easy and immediate. While these comments might be read as critical, they are not “fault finding” and take nothing away from the seminal achievements of the pioneers in this field. Rather, the purpose here is to indicate how the subsequent flow of analytical and empirical research contributions was channeled by the paths that had been opened during this formative phase, and those that remained to be explored.

2.3.1 Information propagation and time in the diffusion of innovations:

In his brief comments on Dixon’s (1980) “revisit” to the subject of the diffusion of hybrid corn, Griliches (1980: pp.1463-1464) remarks that since much of the relevant data describing and affecting individual adopters of new technologies are unobservable, “time was brought into the analysis as a proxy” for one or another of the forces that were viewed to be responsible for the innovation’s gradually widening acceptance. His own work is described as having featured the “‘disequilibrium’ interpretation” of the situation created by the introduction of commercially available hybrid corn seed in each region, and taking “time” to proxy for “the spread of information about the actual operating characteristics of the technology and the growth in the available evidence as to its workability and profitability.” The same approach, taking the adopters’ information states to subject to alteration with the passage of time characterized Mansfield’s research, although he explicitly cited social communications as influential sources of information transmission from (successful) adopters to non-adopters, whereas Griliches (1957) remained non-committal about the specifics of the information-transmission mode. Rather, as has been seen, Griliches’ reference to the “ASN effect” implies that the content of the information would affect the representative agent’s speed of acceptance of the innovation, whatever the source: the larger was the associated profitability advantage reported by the flow of information, the fewer would be the expected number of such reports (i.e., the average sample number), and (by surmise) the shorter would be the time required to persuade the individual to adopt the innovation in question.\(^\text{13}\)

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\(^{13}\) Uniform messaging speed, independent of the content, is therefore implied by this – and the social contagion models, an assumption that is quite plausible when considering diffusion phenomena that extend over a span of
The contrast between the way these two foundational research contributions treated the question of how the supposed information constraint upon adoption came to be removed may well have reflected a difference in the realities of the situations they studying, and consequently in the source materials that were available in each case. Footnote reference in his article footnote references leave little room for doubt that Griliches was thoroughly aware of the existence of active broadcast sources of information about the advantages of hybrid corn emanating from the U.S.D.A. agricultural extension service and the private seed companies; whereas nothing of equivalent salience is in evidence regarding “marketing” efforts by the vendors of any among the diverse of process equipment embodying the industrial innovations studied by Mansfield. But, perhaps more significantly, the latter’s proposal of an “information contagion” interpretation of the deterministic logistic equation appears to been shaped at least in part by the publication -- coincident in time with Griliches (1957) -- of Coleman, Katz and Menzel’s (1957) article on the adoption of the broad spectrum antibiotic Tetracycline among prescribing physicians in a New England city. This was the first in a line of influential quantitative sociological studies by these authors that focused on the role in the diffusion of innovations of personal communications and “social contagion” processes.

There is nothing to suggest that Mansfield equated the circumstances of the medical innovation studied by Coleman, Katz and Menzel (1957, 1966), and their conclusions about the role of word-of-mouth transmission of information, were immediately germane to the economic or social circumstances of the industrial innovations in his sample. The marked differences between the 6-month period within Tetracycline went from marginal acceptance to near universal adoption within the communities of physicians studied by Coleman, Katz and Menzel (1957, 1966), and diffusion processes that Mansfield’s data described as extending not only for years, but over decades, would be enough to raise doubts that same underlying dynamic of information percolation was responsible in all of them. Mansfield months or several years. But, when the process of acceptance stretches over decades, the time-invariance assumption becomes more dubious.

14 Mansfield (1961: pp. 745-746) referenced this work in support of his argument that adoptions would become more likely as “as more information and experience accumulate,” adding: “Competitive pressures mount and “bandwagon” effects occur. Where the profitability of using the innovation is very difficult to estimate, the mere fact that a large proportion of its competitors have introduced it may prompt a firm to consider it more favorably.” This, however, says nothing explicit about social communications, and the footnote discussion (p.746, n.8) focused on “the snow-ball effect” noted by Coleman, et al., remarking the “almost all the executives we interviewed considered this effect to be present.”

15 Coleman, Katz and Menzel (1957), and the immediate sequel in Katz (1961), represented the first fruits of an effort to substantiate the contention in Katz and Lazarsfeld (1955) that personal communications were an important influence in individuals’ willingness to try new products. A more elaborate monograph on Tetracycline’s adoption among the physicians in four small Illinois cities, was published subsequently by Coleman, Katz and Menzel (1966) under the title Medical Innovation and became widely influential. as having established the critical role information propagation by of word-of-mouth through social networks (see, e.g., Rogers, 1995).

16 Indeed, Mansfield’s (1961:p.746, and fn.8) referred to this article in the context of arguing that “the mere fact that a large proportion of its competitors have introduced it may prompt a firm to consider it more favourably,” and in the footnote commented only that “Coleman, et al. [6] noted a ‘snow-ball’ effect. For what it is worth, almost all the executives we interviewed considered this effect to be present.”

17 Even within Mansfield’s collection of innovation, there are large variations around the mean duration of x years from first introduction to adoption by all the (major) firms in the respective industries -- the population for which Mansfield’s diffusion measures were calculated. For example, from the graphs in Mansfield (1961: Fig. 1) is seen that 30 years or more elapsed before all among the major railroads had adopted install centralized traffic control, and automatic car retarders; that was the case also regard to the acceptance of by-product coke-
did not remark on that particular disparity, however, but remarked (1961: p.744, and n. 8) that while the diffusion of a new technique generally was a slow process, there were wide variations among innovations in the “rate of imitation”: on average 7.8 years elapsed following an innovation’s introduction before half the industry’s (major) firms had adopted it, but the range of inter-innovation variation around that sample mean was from 0.9 to 15 years. One might wonder how plausible it is to suppose that 15 or 16 years elapsed before the last half of the major pig-iron producers heard by word-of-mouth about the proven advantages of installing by-product coking ovens, whereas only 5 years were required for the corresponding message about the benefits of continuous mining machines to percolate into the non-adopting half of the major coal companies.

Nevertheless, Mansfield’s empirical strategy sought to account for those variations in terms of differences among the (fixed) characteristics of the innovations that would affect the profitability and, recognizing that installation of lumpy industrial facilities was required in some cases, the risks or financing obstacles involved in their installation. This supposed (implicitly, in endorsing the “contagion” model) that when an innovation was insufficiently attractive to win acceptance by substantial portion of the industry, it would take longer for non-adopters to get the news, and a small proportion of those who would act upon it positively.

The “epidemic” or “social contagion” model of diffusion has gained an enduring place in both the economic and the sociological literatures on the adoption of innovations, and, without warrant, as been seen in now casually associated with micro-level behavioral interpretations that are adduced to account for both Griliches' (1957) and Mansfield’s findings about the relationship between profitability and speed of acceptance or imitation. Moreover, in view of the foregoing skeptical observations concerning the relevance of information-constrained delays in adoption – which would have had to persist for decades within some industries before gradually being alleviated through the slow dissemination of knowledge carried by word-of-mouth -- this represents one of modern economics’ more remarkable triumphs of “nice theory” over “empirical plausibility.”

2.4 Conflicts over economic rationality and the heterogeneity of potential adopters

A second feature that Griliches (1957) and Manfield’s (1961) contributions share, in addition to (and as the dual of) their common emphasis on the slow percolation of information required to persuade rational managers of the benefits of new techniques, is that both worked within a representative agent framework. Consequently, their explanations of the diffusion phenomena paid little if any attention to the role that could be played by the existence among the agents (farmers or firm managers) populating the relevant industries of differences bearing upon the profitability of adopting a particular innovation at a given point in time.

ovens, the continuous annealing process by the major iron and steel firms: whereas 10 years or fewer elapsed before all the major firms had installed the pallet-loading machine, the tin container, and continuous mining machinery. These variations did not merely reflect inter-innovation differences in the extreme right-hand tail of the distribution of adoption lags (measured from the respective introduction dates. Half of the major pig-iron producers were using by-product coke oven within 15 years , where as half of the major coal producers were using the continuous mining machine within 3 years.
This commitment to the representative agent modeling approach lurked beneath the surface of the controversies into which Griliches found himself drawn by the reactions of rural sociologists to his *Econometrica* article. The interest of economists in the topic of diffusion of technology, and their appreciation of young Griliches, undoubtedly, was raised further by that controversy and the confident and deft way in which he deflected the points of his critics. Griliches had presented as so compelling to be beyond cavil his conclusions that “taking account of uncertainty and the fact that the spread of knowledge is not instantaneous, farmers have behaved in a fashion consistent with the idea of profit maximization.” But he may have underestimated the extent to which his plain expression of them would be read in some quarters as fighting words. Anyway, here is what he said in a footnote at the very end of the 1957 paper:

“It is my belief that in the long run, and cross-sectionally, [sociological] variables tend to cancel themselves out, leaving the economic variables as the major determinants of the pattern of technological change.”

Among rural sociologists, and especially those approaching the study of diffusion in the research tradition that had developed from Ryan and Gross’s (1943) pioneering study of Iowa farmers’ responses to the introduction of hybrid corn, those fighting words would continue to rankle for many years afterwards. They are quoted in the review of this controversy that appeared in Everett M. Rogers’ subsequent surveys of research on the diffusion of innovations— to be immediately dismissed in those pages as exemplifying a “ridiculous” subscription to the naive homo economicus conceptualization popular among members of the “Chicago School.” In truth, Rogers (1983, and in the two preceding editions of that book) completely missed the point of the position taken by Griliches (1957, 1960, 1962) in turning aside the complaints of this critics. However rashly phrased, his parting remarks in the 1957 article really were not about whether human action was or was not influenced by non-economic considerations; instead, they concerned the statistical question of whether non-economic influences upon individuals’ behaviours were sufficiently correlated over time, or in the cross-section, to exert a significant influence on the observed aggregate outcomes.

At least two aspects of what passed between Griliches and the rural sociologists in that now remote controversy over the diffusion of hybrid corn usefully be “revisited” in the present connection. The first exposes more fully points of commonality as well as the disagreements between the approach to diffusion phenomena in what became the Griliches-Mansfield tradition in economics, and the defenders of the rural sociological tradition. The second illuminates the eventual, reflective rapprochement with the sociologists’ perspective that that can be read in Griliches (1980) “Comments” on Robert Dixon’s (1980) paper “Hybrid Corn Revisited”.

In the following paragraphs these two threads are taken up in turn, and shown to be entangled in their implications for the way that economic research on diffusion has developed from the seeds planted by Griliches. There is, in addition, an important relationship between the two point, inasmuch as both were completely aligned with the same implicitly “pro-innovation” disposition that can be found in the studies by Griliches, and later by Mansfield—and, for that matter, in the research tradition on diffusion established among rural sociologists in the U.S. The common view was that the innovation in question, whether it was boiling

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See, for example, Rogers (1983: pp. 32-34, 56, 214-215).
drinking water, planting sorghum, tractor-ploughing, or the oxygen process in the iron and steel industry, was universally superior in some important objective sense vis-à-vis the “traditional” techniques that were in used farms or firms under examination. Further, that superiority was taken to be established from the first moment of the innovation’s introduction, and to persist thereafter. Implicitly this assumed away objective differences in the circumstances of potential adopters that might have a bearing on their (rational) decisions.

The issue in contention between the rural sociologists and Griliches’ supporters and followers, therefore, was simply whether or not the index of that superiority was comparative profitability. Yet, there was nothing in the structure of Griliches’ (1957) presentation of the descriptive, statistical model of logistic diffusion itself that connected relative profitability directly to the pace of diffusion as measured by the estimate of the slope parameter of the logistic curve. The logistic form was an \textit{a priori} specification that had yet to be statistically against other possible diffusion paths. Mansfield’s proposed interpretation, which suggested that relatively higher profitability would increase the likelihood that informed potential adopter would actually accept the innovation, after learning about it from a current user, also was not tested empirically; although it might have been, by considering whether the order of adoption of a given innovation was positively associated with its relatively profitability for the adopters.\footnote{Instead, Mansfield (1968) studied a number of different innovations, and sought to relate the speed of (logistic) adoption to characteristics of the typical firms in the industry that could be viewed as having a bearing on the profitability of the innovation in question.}

2.4.1 ‘Congruence’, ‘compatibility’ and variations in adopter’s objective circumstances

The second point to be noted concerns the awkward implications that follow from acknowledging, as Griliches’ rejoinder had done, that an innovation’s profitability might be affected by its “congruence” and “compatibility” with other elements in the established farming regime – as the sociologists had contended. In other words, the comparison in the case under debate might not be properly characterized merely by looking at the costs and yields of hybrid corn seeds and the open-pollinated alternatives.

Hybridization in effect had created a more efficient pump for nutrients, and, as the agricultural extension officers of the day were explaining to farmers – albeit in different terms – this pump would not deliver what it could unless it first was properly set up. Chemical fertilizers would have to be supplied, and that meant fertilizer tanks would need to be purchased and installed; more water would be required to go along with the fertilizer, and that might mean digging new wells, or otherwise improving irrigation capacity. Providing more nutrients would heighten the problem of weeds, and so chemical or mechanical means would need to be introduced to suppress these competitors for the expensive nourishment that the hybrid plants were supposed to pump up. Even that was not the end of the matter. In addition to the direct financial costs of those fixed inputs, access to working capital would be critical when a wholesale switch was made to hybrid seed, because if bad weather or pests spoiled the harvest, the wherewithal to purchase new seed for following year would become a critical condition for the farm family’s survival on the land.

Looked at from this angle, the “representative agent” version of Griliches’ “profitability counts” story about hybrid corn appears rather too facile. Objective economic differences existed among the farms of the Midwest in this era, quite noticeably in regard to
their current and their expected future corn acreage, the terms of their access to bank finance, their family labor supply situation, and also in the educational attainments of the farms’ operators – which might well affect their capabilities to grasp and manage critical aspects of the new, more intricate system of cultivation.\textsuperscript{20} Surely the heterogeneity of the population in these respects might be expected to show up in cost, realized yield, and farm revenue differences. Hence, by the very same argument that Zvi had used to deflect the criticisms from his sociological antagonists, the determinants of perceived profitability might well be said to govern the extent to which the innovation would be adopted within a farming community. But profitability was not simply a function of seed yields and prices that were essentially the same for everyone.

Supposing, then, that awareness of the requirements for commercially successful deployment of hybrid corn had become thoroughly disseminated as a result of the efforts of those agricultural extension officers, there could nonetheless be “rational non-adopters.” If that was the case, something else in the objective situation would have to change in order for there to be the further expansion in the proportion of acreage under hybrid corn. That, at least, was the way it appeared to this city-boy when, without ever having set foot on an Iowa corn-farm, he started to think about the determinants of the diffusion of grain harvesting in the antebellum Midwest, and came upon the debate that had gone on between Grilichesi and his critics.\textsuperscript{21}

There was another bothersome matter — not unrelated to the one just noticed, but having to do with the neat empirical strategy that Griliches had devised. The family of diffusion paths exhibited in the 1957 papers clearly is an artifact of the U.S. Department of Agriculture’s statistical reporting practices: total corn acreage under cultivation, and acreage under hybrid corn were collected at the county level, aggregated and published for the states. Surely these political units had to be viewed as rather arbitrary aggregations from an economic standpoint; there were not even any apparent state-level farm policy issues that would render it of economic interest to concern ourselves with the dynamics of the diffusion of hybrid corn on a state-wide basis. But, what was the theoretically appropriate level of aggregation? Supposing that the data could be obtained on a county-by-county basis, would that be a more ‘natural’ population unit within which to examine the course of diffusion? Or should the county-level data be re-aggregated to form some economically distinct larger regions within which there could be said to be substantial homogeneity? Would such a thing be feasible, let alone appropriate for analytical purposes?

The answers were not obvious. It was not even clear that the idea of regional differences could be specified clearly, and if so, whether or not it should be independent of fixed features – such as climate, or soil types – that might have a bearing on micro-level adoption decisions. What did seem clear is that if all the state data for corn farmers was aggregated, it would exhibit an adoption path which was far more protracted than those

\textsuperscript{20} The significance of human capital intensity among the sources of the growth of U.S. farm productivity would emerge as a notable finding in Griliches’ (1964), an aggregate production function study.

\textsuperscript{21} The influence of those doubts about the sufficiency of the ‘contagion’ model found its way into the approach taken in David (1966). But, as that publication was meant to be a contribution to economic history – in a festschrift for Alexander Gerschenkron, the advisor of my yet unfinished doctoral dissertation — and not about diffusion theory, its pages contained no explicit references to Griliches’ study of hybrid corn and the controversy it had ignited. Such matters would wait until my incipient heterodoxy could be formalized for a different audience, in David (1969).
shown for the individual states, a point commented on by Griliches (1980: Further, because that diffusion curve would have a unique inception date, the slope parameter estimated from the logistic regression would need to describe the more protracted time path, and consequently would be smaller than that for many of the sub-regions. On the Griliches-Mansfield interpretation, however, differential profitability of hybrid corn would affect the slope coefficient of the logistic and consequently govern the speed of the contagion process; hence, for the larger aggregate if would have to be said that the differential profitability of the innovation for farmers was weaker in the aggregate (and \textit{a fortiori} weaker in the late adoption regions) than was the case in the early adopting sub-regions.

But Griliches (1957) had provided a different explanation for the separation between early- and late-adopters: it was supposed to have stemmed from the differences in the \textit{innovation-suppliers’} expected profitability, which gave rise to the sequence of introduction dates. Looking at the process from the aggregate level, it now seemed that the heterogeneity of corn-growing conditions across the U.S. played a role in the diffusion process that was not acknowledged when Griliches focused his empirical analysis at the state level. Yet, if this was a valid conclusion, might it not also be one that would hold in regard to the diffusion process at the state level? And then, why not also at the county-level, and below?

Perhaps not altogether surprisingly, rural sociologists – even before they reached the fighting words at the end of Gliches’ \textit{Econometrica} article – were finding it difficult to square the complex and nuanced picture that had been projected by Ryan and Gross’s (1943) influential study of the response of Iowa’s farmers to the introduction of hybrid corn,\textsuperscript{22} with Griliches’ stark emphasis upon the differential profitability of hybrid corn as the principal systemic factor affecting the speed of its diffusion. Ryan and Gross (1943) had reported a complicated array of objective and subjective considerations affecting the social communication of influential information within Iowa farming communities as having shaped the reception of hybrid corn cultivation as a substitute for the traditional farming regime based on open-pollinated corn-seed. Furthermore, the critical responses to Griliches (1957) that appeared in \textit{Rural Sociology} from Babcock (1960), and Havens and Rogers (1962) in defending the research tradition stemming from Ryan and Gross’s pioneering study actually went beyond it and emphasized the roles of the “congruence” and “compatibility” of the innovation with the pre-existing farming regimen and beliefs about their efficacy of those practices. These underlying structures were held to be the real determinants of the alacrity with which individuals embraced.\textsuperscript{23}

\textsuperscript{22} This had been a landmark contribution to the methodology and substance of interview-based research on the adoption of innovations. Indeed, the tradition among quantitative sociologists springing from the work of Ryan and Gross (1943) continues to shape empirical studies in the field of marketing, as well as academic sociological inquiries into the adoption of technological and other innovations among rural communities in the developing economies. There was no reference to this work in Griliches’ \textit{Econometrica} article, although it is implausible to suppose he could have been unaware of its existence. The following quotation from Ryan (1948: p.273) was offered by Griliches (1957, p. 516 (n.31) to support his contention that the available supply of the hybrid seed in the 1930’s was not a binding constraint during that early stage of its diffusion: “the rapidity of adoption approximated the rate at which farmer’s decided favorably upon the new technique.”

\textsuperscript{23} See, for example, Babcock (1962), Rogers and Havens (1962). Rogers and Shoemaker (1971) and Rogers and Kinkaid (1981) have carried forward, and considerably elaborated and generalized the emphasis that Ryan and Gross (1943) placed on the importance for the dynamics of technology adoption of structure social interactions among the potential adopter population that affected access to persuasive information about hybrid corn’s advantages.
Griliches’ (1960, 1962) sequential rejoinders to Babock and Havens and Rogers, respectively, were spirited but actually avoided counter-attacking; his rebuttals adroitly sought to deflect points they offered in criticism of his emphasis on profitability as the determinant of the speed of the diffusion process in any given farming locale. Essentially, he pointed out that the considerations they raised (which will be examined more explicitly in section 3) would affect both the reality and the perception of the innovation’s profitability; therefore, to portray their influences as distinct from that of profitability was a “false dichotomy”, and “another false dichotomy.” On one face, this could be read as something of a retreat from the blunt dismissal of the relevance of “sociological factors” in Griliches’ 1957 text, while on its other face, it served to enlarge the tent of the “profitability” approach to explaining farmers’ behaviors – so that everything could be taken in beneath it, and no real concession need be made to the critics. Both sides withdrew, and, among themselves the sociologists and the economists alike each declared total victory.

One strand in the subsequent development of models of innovation can be traced back to the lively controversy with rural sociologists that Griliches had sparked by conceptualizing the adoption of innovations as a matter of rational, profit-seeking individual behavior, rather than as a social process. That cross-disciplinary encounter in itself promoted the interests of economists in extending the application of their familiar paradigm of microeconomic behavior into a new area of empirical and theoretical analysis, thereby contributing to the emergence of “the “economics of technological innovation” as an important emerging topic for the profession during the latter 1950s and the early 1960s.24

2.4.2 Reflections on the legacy

Zvi Griliches’ 1957 article on the adoption of hybrid corn established his position as the path-breaking economist in the area of empirical research on the diffusion of innovations and the source of inspiration for much of the following research on the subject. The literature eventually moved beyond his initial contribution, both in the theoretical analysis of technology adoption and kindred phenomena, and in the specifics econometric tools that have been employed, is only to be expected. In the history of science certainly it is not uncommon for the formative legacies of a major breakthrough to be rapidly outgrown as others are attracted to a newly opened, promising line of inquiry. But, in this case, progress towards the integration of more intricate theoretical analysis with more sophisticated empirical studies has turned out to be more modest than might have been expected at the outset.

24 See for example, the seminal article by Nelson (1959) on the “appropriability problem” in R&D investment, and the wide range of new research on the rate and direction of “inventive activity” represented in the NBER conference volume edited by Nelson (1962). Much of the novel focus of that NBER conference reflected the recent formation of an informed and influential audience that had formed among economists (outside the sub-filed of agricultural economics), and who were especially appreciative of the step represented by Griliches’ (1957, 1958) publications. This audience, fortuitously for the young Griliches’ career, had been created by the major program of research on the economics of technological innovation conducted at RAND (in Santa Monica) during 1942-1962. This early work in the early 1950s featured formative memoranda on topics such as the information-theoretic formulation of the economics of research, and the implications of empirical “learning curves” for cost-functions in the airframe industry – drawing contributions from Kenneth Arrow, and Selma Schweitzer Arrow, Armen Alchian, and many others who went on to have distinguished careers in other fields, as did Richard Nelson who came to RAND later in that decade. But for the discoveries of Hounshell (2000) in the RAND archives, little would be known about this important intellectual episode – apart from the personal recollections imparted by some among the participants.
Griliches himself commented upon this, suggesting that however important the phenomenon of “diffusion lags” might be as a practical matter, it was an awkward anomaly for economic analysts accustomed to thinking rational agents and collectivities of agents that were operating at, or in the near neighborhood of equilibrium. In an interview conducted by Krueger and Taylor (2000: p.181), he went further suggested that economists’ over-riding preoccupation with models of equilibrium had contributed to limiting further development of his initial empirical work of the diffusion innovations:

“We never have had a good theory of transitions. And the field, by and large, moved toward an interpretation where everything was in equilibrium, all the time. So the diffusion story, as such didn’t seem like the model people wanted to develop….Most of the economy is quite far away from the boundaries of the current state of knowledge. Some of it is because it is equilibrium—it’s not profitable at the existing cost structures. But some of it is because it’s new and it hasn’t been fully developed yet. It’s in the process of being adopted.”

But another explanation for this “disappointment” would seem to lie in a different problem, about which Griliches was outspoken in other research contexts. It soon became apparent that there were very exacting, and costly data requirements if consistent econometric studies were to be pursued at the aggregate level at which his hybrid corn research had been pitched, and at the microeconomic level toward which both theoretical modelling and empirical research on adoption behavior began to shift soon thereafter.

Whereas Griliches had been able to construct diffusion detailed region-specific time-series from the available voluminous data on total U.S. farm acreages planted with corn, and the acreage planted with hybrid corn – and using some unpublished data obtained from Department of Agriculture, Mansfield set out to build a dataset trace the penetration of the range of new techniques through a variety of trade publications, and private industry sources. Much of the impact of Mansfield’s (1963a,b, 1966, 1968b) later publications resulted from the sustained data collection effort he mounted, focused on the adoption of new technology both within firms and across firms in the manufacturing and transport sectors, and much of what came to be known about the central tendencies and variations in the patterns of diffusion of industrial processes stemmed from this research program, in which several generations of Mansfield’s graduate students at the University of Pennsylvania were enlisted.25

With the exception of a few outstanding contributors, the economics profession responded weakly to the “data constraint” challenge. The result has been that systematic econometric research on diffusion continues constrained by the lack of suitable micro-level data, and the gap between theoretical modelling and empirical studies has tended to widen.26 One of the manifestations of this gap has been the profusion of models that have multiplied, unconstrained by the data. The other side of the “data constraint” on econometric work in the field of diffusion has been comparative absence of the imposition of “data discipline” upon theoretical elaborations of models that are designed to be observationally equivalent within the limited sphere of “stylized facts” that substitute extensive and varied datasets about actual

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25 In the U.K. Nabseth and Ray (1974), and Davies (1979), similarly contributed to building the empirical foundations for the study of diffusion, but this effort has not continued.

26 Recent exceptions prove the rule: see Forman, Goldfarb and Greenstein (2003); Bresnahan and Yin (2007).
diffusion experience. The following section is meant to underscore this point with concrete examples.

2.5 Multiplying models of logistic diffusion

Mansfield’s (1961) invocation of a random contact process of contagion was only one among the multiplicity of interpretations that might equally have been attached to the phenomenon of logistic diffusion. At the reduced form level at which econometric work in this area was conducted, most of the alternative formulations were observationally equivalent. In the following paragraphs I demonstrate this explicitly without attempting to be exhaustive.

2.5.1: An ‘evolutionary’ economic interpretation?

Consider, for example, a variant that has a distinctly evolutionary flavor; not surprisingly, perhaps, as it has its roots in the mathematics of population genetics. Lucca Cavalli-Sforza and Marc Feldman (1981) provide a genetic model based upon random population inter-mixing, in which the proportion of the population (again, we may label it $P$) carrying the mutant trait evolves according to a logistic function of time. They show that the slope coefficient of the logistic (again, call it $\nu$) in this case is simply $\nu = \ln(k_m/k_o)$, where the ratio $k_m/k_o$ measures the Darwinian fitness of the mutant gene relative to the old gene. Translating this into more familiar terms, we could say that the slope coefficient is a log-transform of the ‘fitness’ measure of the innovation’s advantage vis-à-vis the established (cultural) trait.

To go from this metaphor to a formal “evolutionary economics”-style model of diffusion some further bits must to added, making the replicator dynamics explicit. This is not so hard: suppose that the innovation (mutant cultural trait) is a production method that reduces unit production costs for the adopting firms, and that the latter are operating in a competitive market. Next, suppose the rate of capacity growth via investment in the facilities required by the innovation is equal to the profit rate. Since the firms enjoying the lower unit costs will get more profit per unit of capacity profits, the capacity of the firms adopting the innovation will grow relative to that of the non-innovating remnant by $k_m/k_o$. The result will be a logistic path for the proportion of (full capacity utilization) output that is produced with the new technique. Viewed from this unaccustomed angle, Zvi Griliches’ intuitive identification of the slope parameter of the logistic with some measure of the relative profitability of the innovation in question might entitle him to further esteem – surprisingly in this case, as a pioneering evolutionary economist in spite of himself!\footnote{If we are entertaining evolution, why not also have a Cambridge-Pasinetti style theory of savings, in which the capitalists save everything and the workers nothing?}

\footnote{This Molière-like denouement should not obscure the credit for the replicator dynamic in this formulation, which was introduced much later by Nelson and Winter (1982), who created a stochastic version of a system with this structure and examined its path of adjustment (through selection) in response to the recurrent emergence of mutations characterized by varying degrees of “relative fitness.”}
The ‘evolutionary economics’ overtones of the foregoing sketch-model notwithstanding, it follows the work of Griliches and Mansfield faithfully in its assumption that the coefficient of relative fitness for the innovation (i.e., the new ‘cultural trait’ in question) is inherent in the innovation itself. An obvious justification for this supposition is to dismiss as inconsequential the possible variations of the environments encountered by those who acquire the trait. More complex models of population genetics subsequently have abandoned that simplification, allowing both heterogeneity of environments and their dynamic transformation as a consequence of the diffusion of the new (behavioral) trait within the population.

In a sense, the literature on the microeconomics of diffusion began to move in the same direction (although not consciously regarding the evolutionary parallels). This development turned upon a more formalized acknowledgment of the implications of heterogeneities in the adopter population, and biases in the properties of innovations. Together, these empirical realities posed a challenge to the casual assumption that innovations were universally dominant vis-à-vis pre-existing technologies. They therefore pointed to the possibility that diffusion lags were not necessarily explained by incomplete information. This, in turn, raised questions about the rationale of policy programs designed to promote technology adoption by providing demonstration programs and identifying efficient channels for the propagation of information about the innovation in question.29

The approach finding its way into corners of the economics literature focused less upon information contagions, and more upon the implications of population heterogeneities, combined with fixed costs of adoption and variable input-saving biases in process innovations. It allowed for the possibility that expected scale of operations might enter into investment decisions involving choices between new and old techniques, such, that given the relative prices of the fixed and variable inputs, there would be a “threshold” output scale below which adoption would not occur, so long as the decision agents were myopic cost-minimizers.

2.5.2 Moving equilibrium: threshold models and logistic diffusion

My initial and subsequent contribution to the modelling of diffusion innovations introduced and generalized that approach for the case of process innovations. But, this hardly is the occasion on which to review the details of all those papers.30 Rather, the point in bringing up the matter here is simply to underscore the previous assertion that there are many different models that will account for the phenomenon characterized of logistic and logistic-like diffusion at the macro-level. The class of so-called ‘threshold models’ — in which a variety of formulations is subsumed — can perform that trick without having any recourse to imperfections in the information states of the agents. Furthermore, their simpler formulations suppose that adopters and non-adopters alike at each moment are in profit-maximizing (or, at least, cost-minimizing) equilibrium. The essence of the approach is to view the diffusion path itself as a moving equilibrium, the dynamics of which can have exogenous or endogenous drivers, or both.

29 See e.g., Rogers and Shoemaker (1971); Rogers and Kincaid (1981) for programs of that genre, and the implicit critique by Stoneman and David (1986).

In this light, one might re-phrase Griliches’ comment (quoted above) about the economic profession’s penchant for thinking about equilibrium rather than about transitions: the profession’s predilection for modelling the behavior of agents in equilibrium terms posed an obstacle to explaining macro-transitions simply in terms micro-level disequilibrium. Models of temporally extended diffusion processes that allow feedback from the process itself to provide dynamic drivers for a moving equilibrium therefore could restore the conceptualization of the macro-phenomenon as a transition. But the cost to the neoclassical world view of pursuing the latter interpretation was the acceptance of externalities (such as various forms of “learning”) as a vital source of the system’s dynamics.

In order to fix ideas here we may start with the basic formalization of the “threshold” model of technology selection, and develop a simple specification that allows this model to mimic the time series implications of the basic contagion model — by generating a logistic diffusion path. It is best at this stage not to burden the exposition with mathematical formalism, so the notation here is kept to a minimum — even at the cost of leaving some loose ends that can be tidied up afterward.

The basic notion of an adoption “threshold” is that there is a variate $z$ that enters the discrete choice problem of individual agent $i$ who is characterized by the value $z^i$, such that the agent will select the novel option — the innovation — over others when $(z^i) \leq z^*$. Thus, $z^*$ is implicitly defined as the “threshold adoption level” of the key variate. Let us assume that the technology choice is for all intents and purposes irreversible, perhaps because the decision to adopt the innovation entails acquisition of a highly durable and indivisible physical asset.31

If we then suppose that the critical variate $z$ has a continuous frequency density function in the population of potential adopters, $f(z)$, the proportion of the population among which the condition for adopting the innovation is not fulfilled will be just the value of the corresponding (stationary) cumulative density function, $F(z^*)$. Therefore, we have as a measure of the extent of diffusion the proportion of the population that should adopt:

$$D(z^*) = 1 - F(z^*) .$$

Under the assumption that $F(z)$ is stationary, $D(z^*)$ can increase if and only if $z^*$ becomes smaller. In other words, the threshold point has to pass downwards through the $z$-distribution.

If we know the shape of $F(z)$, and can characterize the dynamics of $z^*(t) = g(t)$, it is straightforward to deduce how the latter’s motion must re-map $F(z^*) \Psi F(t)$, thereby generating a ‘moving equilibrium’ path for the diffusion index in the time domain, $D(t)$. It’s really no more complicated than that!

All sorts of diffusion paths, and all manner of micro-level processes generating aggregate level time-series measures of diffusion can be rationalized in terms of this basic framework. Putting it the other way ‘round, by specifying $f(z)$ and $g(t)$ one may derive the shape of the diffusion path. Thus, let us posit that the $z$-distribution is the log-logistic, and

31 See Sect. 3, for discussion of the specific contexts in which this formalization first developed, and subsequently was elaborated.
that \(g(t)\) declines exponentially with time at the instantaneous rate \(\delta\), and see what happens.

We now may write

\[
D(z) = 1 - F(z^*) = \exp\left(-((\ln z)) \right) \left[1 - D(z)\right] = (z)^{-\left[1 - D(z)\right]},
\]

and

\[
z^*(t) = g(t) = z^*(0) \left[ \exp\left(-\delta \right) \right].
\]

Upon finding \(\ln(z^*(t))\) and substituting this in the expression for \(D(z=z^*)\), we immediately obtain a logistic function in \(t\), the slope parameter of which now is revealed to be \(\nu = \delta\). This is readily confirmed by forming the resulting expression for the familiar log-odds ratio:

\[
\ln\left\{\frac{D(t)}{1-D(t)}\right\} = (\gamma \lambda) t - \gamma \ln\{z^*(0)\}.
\]

This model of logistic diffusion affords an interesting interpretation of the coefficient of \(t\) that may be estimated by linear regression methods; it reflects both the rate at which the threshold point is falling, and the shape of the underlying \(z\)-distribution. Given an extraneous estimate for \(z^*(0)\) – the value observed for the initial adopters – both of the model’s structural parameters can be recovered from the intercept and slope coefficients of this linear expression. Formulated as a regression model by the addition of a disturbance term, the parameters of interest may be estimated by linear regression methods.\(^\text{32}\) Obviously, different specifications of the underlying heterogeneity, i.e. of the distribution function for \(z^*\), would generate different regression models, and correspondingly different interpretations of their coefficients.\(^\text{33}\) It may be noticed that although profitability considerations obviously can enter into the marginal agent’s micro-level decision functions, the estimated parameter \(\nu\) says nothing at all about the relative profitability of the innovation.

### 2.6 A multiplicity of models and their implications for empirical research

The specifics of the underlying adoption process for the innovation cannot be uniquely identified simply from the shape of the diffusion path that emerges at the aggregate level. The three simple models just reviewed are observationally equivalent if one is restricted to looking at the times-series of the changing proportion of adopters in the population, or, as Griliches pioneering study of hybrid corn has done, at the proportion of aggregate output produced with the novel technology in a given region. This spotlights the formidable challenges that are posed for empirical researchers who seek to identify the

\(^\text{32}\) A word of caution is in order for those who would follow common econometric practice and estimate the log-odds equation by OLS. methods. In this time-series relationship the problem of auto correlated disturbances suggests relying instead on minimum Chi-square estimators for the slope coefficient.

\(^\text{33}\) The threshold model developed in David (1969) featured expected output as the key variable \((z)\) in the adoption of a fixed input-using technology, and specified this to be distributed lognormally within the population of potential adopters. With the threshold falling at a constant exponential rate, \(\delta\), a one obtains a probit regression model. The slope coefficient of that linear relationship in \(t\) is simply \(\delta/\Phi\), where \(\Phi\) is the standard error of \(N(0, \Phi)\), the standard normal distribution of \(\ln(z)\).
specific mechanism, or groups of mechanisms that are responsible for the temporally distributed uptake of particular innovations in one or another historical time and place. The requirement to collect consistent micro-level cross-section and aggregated time-series observations not only on adoption, but on the variables hypothesized to be critical determinants of individuals’ decisions regarding their choices among available techniques, certainly imposes a heavy burden on the individual researcher who would undertake an econometric investigation of this kind. In this light we may recall Zvi Griliches’ (1994) expression of concern that the economics profession’s internal reward structures continue to inhibit our collective ability to ease the “data constraint” on empirical research into important questions affecting economic growth: 34

“We ourselves do not put enough emphasis on the value of data and data collection in our training of graduate students and in the reward structure of our profession. It is the preparation skill of the econometric chef that catches the professional eye, not the quality of the raw materials in the meal, or the effort that went into procuring them.”

But the “observational equivalence” of alternative mechanisms generating diffusion paths at the aggregate level is not the whole of the identification challenge. Consider one class of these mechanisms, the “threshold model”, and assume that a dynamic driver sets a specific time-path for the fall in the “break-even” variate \( z^*(t) \). It will then be seen that different distributions of \( z \) -- the critical heterogeneity in the population of potential adopters -- may generate distinctive mappings of diffusion in the time-domain. When the threshold variate is falling exponentially, it has been seen that a log-logistic distribution of \( z \) in the population of potential adopters generates a diffusion time-series that is logistic and symmetrical. Yet, were \( z \) to be distributed log-normally the same exponentially decreasing threshold \( z(t)^* \) gives rise to a transformed cumulative normal (TCN) time-path for the proportion of adopters in the population.

Alternatively, one might posit that the \( z \)-distribution was log-Gompertz, so that the time-series observations on proportion of adopters in the population would turn out to be the cumulative Gompertz distribution, and the time profile of the incremental increases in the extent of diffusion (\( D(t) \)) follow the innovation’s introduction would resemble distributed lags of the Koyck form. Without being able to directly examine the hypothesized \( z \)-vari ate’s distribution, it would not be possible to attribute the variant diffusion paths to the differing specifications of the same structure, rather than to possibly fundamental differences in the underlying adoption mechanism.

To establish the connection between technology diffusion seen as a continuous time-series process at the aggregate level, and as a quantal response at the level of the individual adopting agents, one must an appropriate characterization of the production technologies and both kinds of data. To be consistent, moreover, the time series should be aggregated from a

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35 The TNC diffusion curve does not have a traceable closed-form expression, as it involves evaluation of a definite integral, but, on the other hand, the proportion of adopters in the population at successive points in time lies along a straight line when plotted on normal probability scales, and the underlying parameters describing the log-normal distribution and the rate at which the threshold is falling can be estimated by probit regression analysis. These implications are derived in David (1969: section III.3), and Davies (1979) subsequently -- but apparently quite independently -- made extensive use of probit regressions his empirical studies of diffusion. David (1969) shows that if the critical variate \( z \) is output scale, the measure of the extent of diffusion \( D_v(t) \) -- denoting the share of total industry output that is produced using the innovation -- need not take the familiar ogive form of the cumulative normal. When the diffusion process commences with adoption by production units whose output scales are much above the mean of the output scale in the industry, it is possible that \( D_v(t) \rightarrow 1 \) without exhibiting an inflexion.

36 Unlike the logistic, the Gompertz distribution does not generate symmetric S-shaped diffusion paths, and Dixon (1980), following the empirical strategy devised by Griliches (1957), pointed to the observed lack of symmetry in the updated regional time series of the extent of corn acreage under hybrid corn as his grounds for estimating the parameters of the Gompertz function rather than the logistic. Griliches (1980: p. 1463), however, questioned this on the grounds that it was merely curve-fitting to accommodate the observed “slow upper tail”, which, he suspected, was due to delayed supply-adaptations in hybrid seeds --required for them to be profitably use in the certain areas that were ill-used to the initially available varieties.

37 Tempting as it may be to adopt a “representative agent” framework and introduce mathematically convenient aggregative production relationships to finesse the need to understand aggregate level phenomena at the underlying microeconomic level, using appropriate micro-level data, succumbing to that temptation yields at best a “simulated understanding” of the economic realities. An example of what from the present perspective must be seen as an error of aggregation in representing the economics of the transition to a major technological innovation is provided by a recent attempt to apply a “representative agent” model of capital accumulation in characterizing the diffusion of mechanized production as a moving equilibrium process. Manuelli and Seshadri (2003) construct a simulation model based on a one-sector neoclassical production function which they use to to portray the growth of the stock of farm tractors (and the decline of that of horses and mules) in U.S. agriculture during 1880-1930 as “friction-less technology adoption” -- i.e., pure factor substitution -- in response to the effect of rising real wages. There can be no objection here to the choice-of-technique framework, or the
succession of cross-sections of the population. But, if what is wanted is a time-series measure of diffusion expressing the proportion of aggregate output produced with the novel techniques, or with each of the relevant new and old techniques to which the agents had access, then there is a further requirement: one must have a measure of the outputs of the production facilities for which choices among the available techniques are being made. Griliches (1957) was working with a measure defined in reference to the corn acreage planted in each region, but, from that alone it would not be possible identify an economic diffusion mechanism that involved the propagation of information, or other influences transmitted in the interactions among the farmers. The plots of arable land do not switch from being under open-pollinated corn to being planted with hybrid seed because they see what’s going on in the adjoining acreages, or pass messages to one another. Perhaps that “data constraint” served to restrain Griliches from explicitly discussing the views that had emerged from the local studies of rural sociologists, concerning the importance of social communications among the sources of information influencing farmers’ acceptance of new production methods.

Consequently, it is important to stress that at the start of his econometric research career, in his study of the diffusion of hybrid corn, Griliches was (characteristically) careful in observing a distinction between two kinds of empirical research strategies. One is to establish the nature of the regularities that are present in the aggregate level outcomes of economic agents’ behaviors, and offer one or more hypotheses that are in effect “conjectured interpretations” of the sources and significance of those established statistical regularities. A quite different research strategy undertakes to specify alternative hypothesized mechanisms governing the behaviors of the individual actors, and, where appropriate the temporal propagation of influences among them that connect the sequence of those actions. The goal of the latter, structural approach is to make it possible to uniquely identify which among the alternative mechanisms had generated the observed phenomenon – in principle, at least, given the requisite data whose relevance is implied by the model(s). Clearly, Griliches’ (1957) path-breaking contribution opened up a path that would lead others to follow his work and that of Mansfield in taking the second, structural modelling approach in econometric studies of the diffusion of process technologies. But he did not embark upon it himself.

abstraction from short-term lags due to “frictions” in the transmission of information about the advantages of tractors. But Manuelli and Sashadri’s analysis, unfortunately, remains un-informed by any of the recent research on the subject of “tractorization” in the U.S. -- e.g., Sargen (1979), Ankli (1980), Fite (1980), Whatley (1985, 1987). Consequently, it takes no notice of the wealth of micro-level information about technical changes in input usage and input prices (especially relative prices of horse fodder and tractor fuel), as well as such critical matters as the constraints that climate imposed on the duration of available time intervals for post-harvest ploughing and the comparative speeds of tractors vs. horse-teams (on the northern plains); or about the impediments to mechanization posed by labor market institutions (in the pre-WWII cotton South). The result is a “explanation” that “simulates” the growth of the aggregate stock of tractors without being in anyway enlightening about the processes that underlay that development in different regions of the country during different periods of the half-century under examination.

38 See e.g., Trajtenberg (1990), Karshenas and Stoneman (1992), Stoneman (2002), Bresnahan and Yin (2007), to name only a few leading exemplars.
3. Heterogeneous and the Microeconomic Equilibrium Approach to Diffusion: Theoretical and Empirical Implications

An inclusively broad and diverse class of diffusion models that possess a common fundamental structure is opened up by abandoning the representative agent approach and characterizing the population of potential adopters as heterogeneous in one or more respects that impinge upon the outcome of a rational micro-level assessment of the economic benefits and costs of selecting among available production techniques. Analysis of those decision as irreversible investments in the durable facilities required to install a novel process of production – whether in a business firm, or a household – calls for explicitly dynamic specification of the potential adopter’s opportunity set, and the way in which the latter may be changed as a consequence of feedbacks and interactions with suppliers of the innovation-embodying goods, and interactions with other adopters.

3.1 Generalizing and Elaborating the Threshold Model

Such considerations, already touched upon in the exposition of the basic “threshold model” of diffusion (in section 2.5.2), have carried the theoretical literature well beyond the framework of the pioneering contributions to the economics of technology diffusion represented by the work of Griliches and Mansfield. The new conceptual and analytical departure of this approach consists in explicitly assuming that the population of potential adopters is heterogeneous in some respect that is relevant to the individual agents’ rational and informed choice-of-technique decision regarding the adoption of a novel production technology, and seeking to characterize the dynamic conditions under which there will be an equilibrium path for that innovation’s diffusion.

The existence of differences among potential adopters in a variable (denoted by $z$ in section 2.5b) affecting decisions as to whether or not to adopt the novel automation equipment, and will has the effect of partitioning the population at each moment in time between those for whom accepting the innovation is optimal, and those for whom it is not. If the critical value $z^*$ at that moment -- the so-called adoption “threshold” -- remains unchanged, the process of diffusion would halt. But, with a secularly falling threshold, the effect of the heterogeneity of the population (with regard to $z$) is to spread out the dates on which different members of the population choose to install the new production facilities. In the hypothesized circumstances, therefore, following the “shock” of the innovation’s introduction the dynamics of diffusion will have the appearance of a distributed lag process of investment in automation equipment by the ensemble of “progressive” (innovation accepting) firms in the consumer-goods sector.

In the simplest formulation of this model the innovation-embodying capital equipment is assumed to be an infinitely durable “machine” that is indivisible and sufficiently “lumpy”, having sufficiently large capacity to obviate the need for any of the adopting firms to make subsequent further investments in equipment of the same type. Consequently, the lags observed in real gross investment at the industry level would reflect the time distribution of the advancing extensive margin of the innovation’s acceptance in the industry. The shape of that distribution, like the shape of the diffusion path, would reflect the the underlying
frequency distribution of $z$ in the population, and the dynamics of the threshold value, $z^*(t)$. As has been previously noted, with $z^*(t)$ falling at a constant (exponential) rate and $z$ being distributed in the population as log-Gompertz, gross investment in the industry would exhibit a Koyck lag distribution following the innovation’s initial commercial introduction.\(^{39}\)

To fix ideas here it is convenient to give a brief exposition of the generalized “threshold approach” to modelling technology diffusion by following David and Olsen’s (1984, 1986) presentation, in which the dynamics of transition to a new technology is envisaged as a process of capital-using “automation” of production methods in a consumer-goods industry. This model, like that developed contemporaneously by Ireland and Stoneman (1986), introduces an endogenous source of dynamics by allowing for incremental supply-side modifications that lower the costs of the required innovation-embodying capital assets (or, equivalently, increase the savings in unit variable costs of production with equipment of constant price). Recognition of the supply-side of diffusion phenomena is very much in the spirit of Griliches’ (1957) examination of the role of the hybrid corn seed companies, which sought to expand their market by successively re-adapting their product to suit the conditions (of soil, climate and insect populations) in different corn-growing regions.\(^{40}\) These

\(^{39}\) A further point may be noted in this connection. If one supposed that $z(i)$ represented the $i$-th firm’s unit transport cost to ship the adopting firms’ respective outputs to the nearest market, then, as will be seen from section 3.2 (below), the distribution of the expected (and realized) volume of sales would be determined endogenously by the profit maximizing adopters’ choices of production methods. Output would shift upwards as a result of the reduced unit cost of production for infra-marginal adopters, and (ceteris paribus) would also follow the underlying $z$ distribution. Because the effect of the innovation would be seen in a rise in the level of total industry output (as well as average firm output), and if average transport costs were falling secularly, the time-series of the level of output in the industry and the gross flow of capital formation (need to implement the innovation) would be positively correlated. Since, if $z$ were distributed as log-Gompertz, gross investment would follow a Koyck lag-distribution, casual econometric analysis might well yield the appearance of support for a “representative agent” formulation of the familiar capital stock adjustment model. But appearances may be misleading, and in the case envisaged, the underlying microeconomic reality is that the sequential decision to adopt the innovation and undertake the required investment was causing the investment.

\(^{40}\) In the case of hybrid corn, the prices of sees were much the same for all regions, although the performance characteristics of the newer seed were improved vis-à-vis those that had been introduced earlier (see Griliches (1957, p. 507, n. 19). But the resemblance to the situation envisaged in David and Olsen (1984, 1986) and Stoneman-Ireland (1983) models is not complete in two respects. Firstly, each modification of hybrid corn-seed was intended to suit the needs of farmers in a specific geographical environment other than those where it had previously found acceptance, and secondly, Griliches’ (1957: esp. pp.507-509) depicts these changes as the result of distinct “introduction” decisions by commercial seed producers as to the most profitable order in which to enter the different geographical markets. Therefore, each of these successive “innovations” – based on publicly funded experiment stations’ research and private R&D by the seed companies – would have had little if any “spillover” consequences for corn-growing in other regions (except, perhaps, through the effects on the eventual market price of corn itself). Both the David-Olsen and the Stoneman-Ireland models specify learning effects in supply following the commercial introduction of the new technology. As these lower the unit cost of producing the capital good, even with imperfect competition on the supply side the prices in the market for the e innovative-embodying input would tend to fall relative to the output of the final goods producers. This drives the diffusion of the innovation among them, and hence continues the accumulation of the vendors’ experience in producing the capital-good. Stoneman-Ireland assume supply is monopolized, the monopolist follows a optimal inter-temporal price-discrimination strategy, reducing the market price over time; whereas David-Olsen assumes learning effects are not internalized and spillover among the firms in a competitive supply, forcing the supply-price to fall as diffusion proceeds. (The so-called “Coase conjecture” – questioning the viability of intertemporal price-discrimination by the monopolist vendor of a durable good, such as is is assumed by Stoneman and Ireland (1983), prompts David and Olsen (1992) reformulation the model under the supposition that (like IBM, and the United Shoe Machinery Company) the monopoly supplier rented, rather than sold the innovative producer durable.
incremental innovations are assumed to be embodied in successive vintages of a new class of indivisible capital equipment (“machines”), the first vintage of which is treated as having been introduced by an exogenous discrete innovation. The “moving equilibrium” model of adoption of the novel method of production particularizes the general observation that innovations seldom remain in their original form, and that improvements by the suppliers play an important part in widening the field of their application and eventual adoption (see, e.g., Rosenberg, 1972).

The form in which the model will be set out here takes a further step, however, by linking the process of incremental improvements in the innovation’s formulation with an endogenous “learning process” in the sector that supplies the “automation equipment”, i.e., a durable reproducible asset that embodies the basic innovation. It is assumed that this novel type of capital-good is competitively supplied, and the improvements that are made on the basis of accumulating experience with “field operation” of this type of equipment, for with its construction, is not privately appropriated by the “learning entities”. Instead, the knowledge is a “spill-over”, being shared by all the vendors of the innovative capital goods and therefore is fully translated into reductions of their unit price-performance ratio. Since experience-based learning effects of this kind depend upon the cumulative processes of producing (and/or installing and using) the new equipment, they represent a feedback effect from the innovation’s diffusion and therefore constitute an endogenous driver of the sequential adoption of the innovation by the heterogeneous firms in the machine-using industry.

Endogenous generation of incremental innovations is conventionally represented as a learning process that results in a continuous reduction of the unit reproduction cost of “machines of a constant kind.” This reduction proceeds at a pace governed by the rate of accumulation of collective experience among the ensemble of firms engaged in the business of supplying those machines. Assuming conditions of perfect competition prevail in the latter industry, these real cost reductions equivalent to a relative decline in the hedonic price of the

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41 Rosenberg (1972) emphasizes the generality of this phenomenon and its importance in the eventual displacement of formerly dominant technologies by new ones that, in their original formulations often were crude, and harboured defects that circumscribed their sphere of effective commercial application. This perspective in effect blurs the conceptual distinction been dynamic processes of diffusion and innovation, although it is not inconsistent with insistence on distinguishing between the micro-level adoption decision (involving a choice among existing products or methods of production), and the innovator’s improvement of the performance attributes of an existing product or production technique.

42 It is important to emphasize that while the induced improvements in the price-performance ratio of the innovation-embodying equipment may be regarded as positive feedback effect that is an externality of the adoption decisions of the firms in the consumer-goods sector, the latter is a pecuniary externality. This avoids complications would arise were the feedback effects of adoption to take the form of non-pecuniary externalities, such as class “network effects” or interdependences among agents’ preferences due to “demonstration effects.” ces arise in an externality of capital. As will be noted below, the presence of non-pecuniary externalities, when against are fully informed, can give rise to a multiplicity of equilibrium diffusion paths, including paths that exhibit discontinuities rather than a continuously rise in the extent of diffusion.

43 The ubiquitous nature of the latter phenomenon supports the plausibility of supposing that a major technological breakthrough would establish a potential for many subsequent incremental improvements whose cumulative effect upon production costs might well overshadow that of the initiating innovation. See, e.g., Enos (1962, 2001) on petroleum refining; Hollander (1965) on rayon. But, see also Cohen and Klepper (2001) on the role that purposive R&D – rather than experience-based learning—may play in the generation of incremental technical progress.
indivisible capital goods embodying the new technology. Because the pace at which this kind of learning can proceed remains limited by the rate at which the new technology is being adopted, expectations about the future trajectory of incremental innovations and the continuation of diffusion process itself, in effect, become hostage to one another.

Treating this complication due to the feedback from use-experience in the micro level model makes it essential to take account of the effects of anticipations (or expectations) of continued technological innovation upon the timing of adoption decisions, as will be seen from the following formalization of the generalized threshold model.

3.2 The generalized threshold model

Consider an industry comprised of firms producing a homogeneous final output, denoted by $X$, and marketing it competitively at price $p$. Further, there is a capital-goods industry that supplies machinery used in the production of this final good under conditions of perfect competition. To simplify matters, we may assume that the remaining sector of the economy is large in relation to the former two, and its product is the numeraire of the system; machinery from the capital-goods industry is not used by this (residual) sector.

The machines embodying the innovation in this setup are taken to be supplied only as large and indivisible units of capacity, at the unit purchase cost $k$. Although only one unit of this automation equipment need be installed by any firm in order for it to acquire access to the latest production technology, in this general formulation one may allow the possibility that the acquiring firms are able to operate it over a wide range of output scales.

We may abstract from the possible effects of imperfect information and uncertainty upon the process of diffusion, and assume instead that all firms in the final goods industry have identical (costless) knowledge concerning the benefits and costs associated with use of the new production technology. The population of firms in the industry is taken to be fixed, implying that only firms already established when the new equipment first becomes available will have access to the innovation. Rather than complicate the presentation by introducing realistic considerations such as the depreciation of capital goods, and the possibility of replacement of obsolescent equipment at some future date, one may make the following further assumptions: (a) all investment is irreversible, which is to say that there is no market for used machinery of any type; (b) capital equipment is infinitely durable; and (c) the recently introduced line of machines is the only major technological innovation relevant for the final goods-producing industry within the foreseeable future.

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44 Another connection may thus be noted – between studies of diffusion dynamics and the theory and application of hedonic prices, to which Griliches made pioneering contributions (see the assessment by Pakes (2003).

45 See Rosenberg’s (1976) discussion of the latter in a context where “learning effects” are kept in the background.

46 This assumption is not a serious restriction upon the analysis, inasmuch as the extant firms are left free to vary their respective production scales; yet, it greatly simplifies matters, by equating the stock of the newest type of capital equipment (measured in standard machine units) with an index of the proportion of the firms that have adopted automation.

47 Without radically altering the David and Olsen (1984) model, the more glaring unrealism of simplifications (b) and (c) can be avoided, putting in their place the assumptions of stochastic depreciation of the “one-horse shay” variety, and stochastic technological obsolescence, both following exponential processes. Under the one horse shay assumption, depreciation occurs completely and instantaneously. It thus takes exactly the same form
3.2.1 Production technologies and the adopter’s investment decision

Each firm in the final goods sector must decide if and when to adopt the new technology, and will make this decision on the basis of the net present value of the investment represented by a newly installed automated plant. Suppose a firm has decided to invest at date $T$. It should then produce so as to maximize instantaneous profits at each point in time, using the old technology (which may be embodied in existing capital goods) before date $T$, and the new technology thereafter. Assume that instantaneous net operating revenue functions are well defined for both technologies—at least over the range of output volumes considered here. For convenience the latter can be referred to as “profit functions”, remembering that the profits in this case are gross of fixed cost charges.

This specification of a profit function, denoted as $R_i(\cdot)$ for the $i$-th technology, implies decreasing returns to scale in the utilization of variable inputs in the production processes. Let the instantaneous profit functions for the old ($i=1$) and new ($i=2$) technologies, respectively, be

$$R_i(p) = \max_x \{px - C_i(x)\} \text{ for } i = 1, 2 \quad (2.1)$$

where $C_i(\cdot)$ denotes the respective variable cost functions and $p$ is the product price. Note that the firm’s optimal output $x$ is now given for each technology by the derivative of the profit function:

$$x_i(p) = R'_p i (p) \text{ for } i = 1, 2 \quad (2.2)$$

These cost functions remain stationary over time, by assumption. This implicitly imposes the simplifying assumption that factor input prices are time-stationary. For the sake of concreteness and convenience, let us refer to the new technology ($i=2$) as “computer automation.” Then, enhancements in the efficiency of automation equipment will be treated as equivalent reductions in the unit reproduction cost of “machines of a constant kind”, i.e., machines characterized by time-stationary variable costs of operation.

The automation technology can only be of interest to the firm if the profit difference

$$B(p) = R^2(p) - R^1(p) > 0 \quad (2.3)$$

for at least some range of future prices. This difference (the undiscounted gross benefit from adoption) is taken to be positive for all output prices considered here. It does no great violence to the engineering realities to posit also that this gross benefit function is as technological obsolescence due to the sudden availability of a superior type of machine. If the stochastic processes governing these events yield exponential distributions of the depreciation and obsolescence dates, and if those distributions are independent, then the constant hazard rates for both events may be added to find the hazard rate for the termination of the benefit stream associated with a given piece of capital equipment. Assuming risk neutrality, the expected present value of the benefit stream may then be found simply by using the latter (constant) hazard rate as a “risk premium” added to the (riskless) time discount rate, leaving the analysis otherwise undisturbed. Ireland and Stoneman (1983), use this approach to model the effect of variations in obsolescence risks. The difficulty in treating physical depreciation the same way is that replacement demands break immediate correspondence between cumulative sales of the newest type of machine and the diffusion index.
increasing in \( p \). The latter is equivalent to supposing that the marginal cost schedule for technology 2 lies everywhere below that for technology 1, and that the firm’s optimum supply (holding the market structure unaltered) thus is higher under the regime of automation. This last implication follows directly from

\[
B_p(p) = x^2(p) - x^1(p) > 0.
\]

(2.4)

As in all equilibrium models of diffusion under perfect competition, it is essential for this analysis that there be some objective, identifiable heterogeneity in the population of firms which results in the benefit function varying across the firms of the industry. This proposition was demonstrated by David [1969]). Possible sources of heterogeneity will be mentioned shortly, but for the present it is sufficient simply to note that the profit function(s) will be indexed by a firm-specific characteristic, \( z \).

Now consider the investment decision facing the firm of type \( z \) at date \( t \), when the cost of installing a new, automated plant is \( k_T \). The latter price is not indexed by \( z \), since at any moment of time a uniform price prevails in the (competitive) market for capital equipment. The problem to be solved by the \( z \)-th firm therefore is to choose an adoption date \( T \) that will maximize the net present value function

\[
V(T, z) = \int_T^\infty \{ B(p_T, z) e^{-rT} dt \} - r k_T e^{-rT},
\]

(2.5)

in which \( r \) is the rate of time discount.

Assuming smooth price paths, the necessary first order condition is

\[
V(T, z) \equiv - B(p_T, z) + r k_T - k_T^* = 0.
\]

(2.6)

This has a straightforward and familiar interpretation: the cost of marginally delaying the adoption (i.e., investment) date beyond \( T \) is the loss of instantaneous profits equal to \( B() \), whereas the marginal gain is the sum of the averted costs, namely the rend costs, \( r k \), and the anticipated capital loss, \( - k_T^* \), that otherwise would be incurred due to the instantaneous drop in the reproduction cost of the new, automated plant following date \( T \).\footnote{David and Olsen’s (1984, 1986) formulation of the potential adopter’s problem as the maximization of \( V(T, z) \), defined as a net present value function, represented a departure from David’s (1966, 1969) treatment of the demand problem as that of short-run profit maximization by competitive agents in the machine-using industry, thereby reducing the choice-of-technique problem to that of myopic cost-minimization. The introduction of expected changes in the price of the capital good in the derived demand function for the innovation at each point in time distinguished David and Olsen’s (1984) analysis from that of Stoneman and Ireland (1983, and 1986), which follows David (1969) in modelling the demand side of the market.}

Implicitly, the inclusion of allowance for a change in the value of the fixed asset required by the innovative process presupposes that rational adopters would form expectations regarding the future course of the price (and hence the reproduction cost valuation) of that asset.\footnote{In addition to the theoretical merit of this belated improvement upon basic threshold model formulated in David (1966, 1969), Lew (2000) suggests it may be empirically relevant. In his study of the diffusion of tractors on the Canadian prairies during the late 1920s, the parameterized version of the basic threshold model over-predicted the rate of adoption. Lew suggests that this discrepancy resulted from the model’s failure to...}
Additional sufficient conditions for date \( T \) to be optimal for the \( z \)-th firm are straightforward:

\[ v(t, z) > \alpha T \quad \text{for} \quad t > \alpha T \tag{2.7a} \]

and

\[ v(T, z) \geq 0 . \tag{2.7b} \]

Evidently, because rational decisions regarding adoption of the new technology would be influenced by expectations regarding the future real costs of the innovation-embodying capital equipment, and anticipated decreases in its price-performance ratio – whether due to an autonomous trend of technological advances affecting firms in the machine-building industry, or specific “learning effects” in the production of novel capital-equipment in question -- would tend to delay adoption by marginal firms. But, where incremental improvements of this kind can be anticipated to arise as feedback from the diffusion process itself, further adoption would depend upon the balance between negative effect of anticipated capital losses and the positive effect of a realized reduction of the fixed capital input’s price vis-à-vis that of the variable input required by the novel process.\(^\text{50}\)

The present formulation follows (1983) in explicitly recognizing the role in diffusion of post-innovation developments on supply side of the market for the embodied innovation. In this respect these models bear some kinship with Griliches’ original analysis of case of hybrid corn, but with a significant difference. In the latter situation the sub-markets were spatially as well as temporally separated, and anticipations of eventual “improvements” in seed-quality would not exert the same delaying efforts on adoption, because they wouldn’t be pertinent to the conditions of the farmers in the initial regions where the innovation had already been introduced.

### 3.2.2 Modelling “learning effects” as a supply-side driver of the dynamic process

Introducing learning-by-doing externalities among competitive firms that supply the innovation-embodying capital good implies that the marginal cost of producing the latter, \( c(t) \), will be a monotone increasing function of the cumulative production experience of that industry. Experience, however, may be taken to be indexed by \( S(t) \), the cumulative volume of the industry’s shipments (measured in “machine units” from the first unit sold at \( t = 0 \)). Thus, we have a learning function \( c(t) = L \{ \} \), which, under the assumed competitive

incorporate farmers’ expectations of future prices, and the volatility in wheat prices and falling tractor prices are shown to make delaying a tractor purchase a rational decision for wheat farmers in Saskatchewan in the late 1920s. On the other hand, it must be noted that the success of using the threshold model (in any version) to simulate changes in the (aggregate) extent of diffusion or the growth of the tractor stock in a region during a specific time interval should depend also on the accuracy with which the pertinent heterogeneity in the adopter population is characterised – which in this case would be the relevant portion of the distribution of arable wheat acreages on Saskatchewan farms at the time.

\(^{50}\) See Rosenberg (1976) for a discussion of the role of expectations in decisions regarding the adoption of innovations, which considers a wide range of anticipations, including those of changes in the performance or user-costs of substitute and complementary technologies. David and Olsen (1986) show conditions under which the combination of a high initial user-cost of capital, and too rapid an expected initial rate of fall of the price-performance ratio of the innovation-embodying capital good – due to the initial steepness of the learning curve - - would prevent the diffusion process from starting automatically.
conditions of supply in the innovative capital-goods industry, determines the dynamics of its product price:

\[ k_t = c(t) = L \{ S(t) \}, \text{ where } L_S < 0. \quad (2.8) \]

Given the infinite durability of the innovation-embodying “machines,” the index of experience \( S \) also indexes the stock of automation equipment which adopters collectively have installed, one per firm, by assumption. Since the size of the potential adopter population may be held constant at \( N \), one may write

\[ S(t) = NX \{ D(t) \}, \quad (2.9) \]

\( 0 < D \leq 1 \), being the measured extent of the innovation’s diffusion among the firms composing the final goods sector. Now, following the representation of the the threshold model in in section 2.5, we have \( D(t) = [1 - F(z(t))] \), when \( z = z^* \). Substituting this in eq. (2.9), the function determining \( k_t \) can be expressed as

\[ k_t = L \{ [1 - F(z^*(t))] \}. \]

Given a mapping \( z^*(t) = y \), where \( y \) indexes the marginal adopter at \( t \), this relationship can be written more compactly as:

\[ k_t = F(y), \quad (2.10) \]

so that the supply price of the innovation-embodying capital good would tend to decline relative to that of the output of the sector adopting the new technology (assuming no equivalent technological progress affecting other inputs in the final goods sector. With the real interest rate \( r \) constant, the real “user-cost” of the innovation-embodying capital, \( r k_t \), must be falling as the threshold level moves downward in the distribution of potential adopters to firms of lower rank \( y \).

3.2.3 Heterogeneity and the “z-distribution” of firms

If firms were identical, they all would choose the same adoption date, and at that date new plants would go into production as rapidly as they could be installed. Insofar as any diffusion path was observable, it would only reflect the sequence of temporary, “rationing equilibria” in the market for automation equipment. To create the possibility of market-clearing equilibrium diffusion paths in the present model, David and Olsen (1984) make a crucial assumption: firms in the final goods sector can be ordered according to a single parameter, or index, \( z \), such that the gross profit difference \( B(.) \) is a continuous and monotonically decreasing function of \( z \):

\[ B_z (p, z) < 0, \quad \text{for all prices } p \text{ in the relevant range.} \quad (2.11) \]

One possible interpretation of the \( z \)-parameter is that it indexes an intangible attribute affecting costs, such as managerial efficiency. But \( z \) also may be taken to represent inter-firm differences in more objective, and directly verifiable conditions impinging upon

\[ 51 \text{ In this case we should call it “z-efficiency”, by analogy with Harvey Leibenstein’s famous “x-efficiency”.} \]
operating profits, such as transport costs differences affecting the f.o.b. prices of their final product. The firm-specific variate \(z\) also may be interpreted as the price of one of the inputs used in the firm’s production process, thereby permitting recognition of factor market imperfections as a source of the heterogeneity among the population of potential adopters.

It is of course a drastic simplification to suppose that firms can be rank-ordered along a uni-dimensional scale. But while working with multivariate distributions is straightforward conceptually, it requires further specification of the co-variances among the several sources of heterogeneity, and these – however relevant to the fine-grain empirical details – soon begin to clutter up the analysis. Nevertheless, it is worth remarking also that the \(z\)-parameter in the foregoing case would turn out to be correlated with the scale of output under each technological regime. Hence, there could be positive rank correlations between the order of adoption among firms and their \textit{ex ante} or \textit{ex post} (output) size, just as in the scale-constrained models presented by numerous empirical studies.

Indeed, a closer examination of the results obtained in Mansfield’s (1961, 1968) studies of the diffusion of industrial innovations suggests that the information-contagion rationale offered for his econometric specification notwithstanding, the statistically significant “profitability effects” on the rate of adoption that he reported could have altogether different underlying causes. The statistically significant effects found in Mansfield’s regression estimates of the logistic slope coefficients are generated by the subset of industry cases where the innovations in question were fixed-capital using, and the indexes of firm characteristics (supposedly bearing on the innovation’s profitability included measures that in all likelihood were positively correlated with differences in expected output scale. That might imply that adoption decisions occurred where scale was sufficient to bring the firms across the break-even threshold level, rather than as a response to higher prospective rates of return on the required fixed capital outlay.

In the model just presented, however, \(z\) may be a more fundamental source of heterogeneity that explains observable differences in output scale, including the adjustment of production scale in response to the availability of the innovation itself. Moreover, the factor-use bias of technological change may switch between one major wave of innovations and the next, so that the firms which enjoyed input-price advantages causing them to be largest in scale of output under the old technology would not necessarily be first to adopt the new.

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52 The differential transport cost interpretation of \(z\) appears to be quite germane in discussions of the diffusion of automated assembly technology, in view of the necessity of concentrating production in plants that will be intensively utilized through multiple shift-working.

53 For this interpretation, note that when \(z\) is a factor’s price, Hotelling’s lemma tells us that \(B_z(p, z) = R_{z2}(p, z) - R_{z1}(p, z) = [L_z(p, z) - L_z(p, z)]\), where \(L_i(p, z)\) are the input demand functions for that particular factor under the alternative technologies. Thus, if the new technology results in the firm expanding its demand for the factor at the prevailing price \(z\) —at least over the relevant range of output price \(p\) —condition (A8) will indeed be satisfied.


55 Instead, the size ordering of firms in the industry may undergo non-monotonic transformation in the course of the diffusion process. To appreciate this, one would have to look more closely at the product market equilibrium conditions.
Under the more general conditions David and Olsen (1984, 1986 and 1992) obtain for the existence of a rational foresight equilibrium path of diffusion, the time-profile of the proportion of firms that have already installed equipment of the new type—the measure of the extent of diffusion denoted by \( D_n(t) \) — may exhibit the classic ogive, or S-shape. Nonetheless, there are circumstances in which the diffusion curve would be concave over its entire range. With this class of models, universal adoption or “complete” diffusion is by no means a necessary, foreordained outcome that resembles the gradual but inevitable filling up of a bottle.

Although there are solutions (dynamic equilibria) in which the diffusion and learning will get under way and continue until it approaches universal acceptance -- the situation depicted in Figure 1, it is also quite possible for the diffusion process to be brought to a stop short of universal adoption, which the case depicted by Figure 2.

[Figures 1 and 2 here]

The reasons for this are several. (i) The innovation may shift the using-industry aggregate supply schedule relative to the market demand schedule, leading to falling marginal revenue and the disappearance of a positive gap between anticipated benefits and anticipated costs of adoption at the extensive marginal. (ii) It is possible that production of the capital good embodying the innovation requires an exhaustible resource input, whose rising marginal supply price checks the fall in the relative price of the capital good, stopping the break-even threshold for adoption from moving downward through the z-distribution. (iii) Partial diffusion is more likely to arise, therefore, if the dynamic driver of the threshold level is entirely endogenous, and dependent upon positive feedback from the movement of the extensive margin of adoption—as in the learning model.

In the full information threshold model the heterogeneity of the population requires that we abandon the notion that at time t there are potential adopters beyond the extensive margin: if the threshold ceased to fall, the process would be at a stationary equilibrium. This is what Griliches (1980) pointed to when he commented that a model of “a moving upper limit” – such as that in David (1969) – could replace his specification of an ex ante fixed upper limit that could be observationally identified. In addition, and of possibly greater interest, is the result that in some conjunctions of initial supply-side and demand-side conditions, the start of a diffusion-cum-learning process driven by perfect competition may remain blocked. This can be the case even where the positive-feedback process driven by learning effects could take over once the level of adoption and capital goods prices had been brought to a critical “take-off” point, presumably by non-market interventions. More generally still, under full-employment conditions, optimum social management of a new technology’s adoption in the presence of learning externalities may call for faster diffusion than would occur even with complete information (perfect foresight) and perfect competition prevailing in all the relevant markets. 56

Complications of this nature begin to take on greater economic policy significance when one turns, as I do now, to consider the connection between the microeconomics of technology adoption and the macro- industry-level course of productivity growth.

56 The implications of this latter point for patent policy as a second-best public strategy are examined in David and Olsen (1992).
3.3 Sources of adopter heterogeneities and time-constants of diffusion processes

Although the emphasis of the foregoing has been placed upon the common structure of the explanation of diffusion phenomena provided by the threshold adoption models and the consequent homomorphism that is present across the array of specific diffusion processes, it is important not to close this discussion without calling attention to the empirically observed differences in the time-constants that are an important identifying variable. This is point that has tended to be overlooked in the economics literature.

Finally, it is appropriate to conclude by recalling that the generic structure of the adopters’ heterogeneity approach to understanding diffusion phenomena is so inclusive that unobservable psychological resistances or other behavioral impediments to action under uncertainty can be formulated within this framework. If one starts with the supposition that such directly unobservable impediments, often described in the rural sociology literature as “resistances to novelty” or “psychological inertia” differ among individuals, then shape of the distribution of such resistances then becomes a relevant structure affecting the dynamics of the novelty’s acceptance in the population (see, e.g. David 1969). For example, Young (2005) proposes to use the shape of the aggregate diffusion paths to identify whether the process underlying the diffusion of hybrid corn was simple contagion model involving information transmission (generating a logistic), and or by a “social learning process” of the kind first suggested by Granovetter (1978) – which has been noticed can be formulated a a member of the family of “threshold models.”

The information propagation process can be specified in both cases as proceeding by a random contacts in a completely intermixed population, so that the only structural difference is the models is whether there is a uniform degree of “a priori resistance” among the agents, or a distribution of resistances that must in each case be overcome by a correspondingly stronger signal regarding the net benefit of adoption. Given the specification of a random contact process, if the experience of adopters were held to be uniform it could be supposed that some cumulative number of contacts by adopters would be required to overcome resistance in each potential adopter’s case.

While a much in this vein is possible, the relevant practical question is whether the speed of these information transmission processes is not so rapid, compared with the transition rates involved when irreversible investment decisions about new transport

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57 But one may ask whether it really is plausible that information propagation lags and social learning about a given, universally superior technology could account for these observed lags?. Even in the case of Griliches hybrid corn study, the differences between Iowa and other states, e.g. Kentucky is 4 vs 9 years to go from 10% to 90% adoption. A simply contagion process of the kind proposed by Mansfield would have to explain why the time rate of contacts between adopters and non-adopters was so much higher in Iowa than elsewhere. The alternative ASN mechanism for the transition of individual farmer’s to the new method would encounter the same problem: if individuals need repeated exposures to reports of the innovation’s economic advantages before becoming convinced, why is the support of the distribution of psychological “resistances” so much more compact among the farmers in some regions than elsewhere. Griliches comments on heterogeneities among the crop reporting regions within states as a source of lower acceptance rates at the state level. Within his framework of explanation those difference would have to correspond to differences in the observed profitability of adoption, and the sources of the latter remain identified but it is reasonable to suppose that this would be of the same ve magnitude as the differences in the observed rates of acceptance. (Given the power of the test and the error level for false positives, the ASN is an inverse function of the difference between sample means.)
infrastructures, or durable and lumpy advanced wafer fabrication equipment (“steppers”) in the semiconductors are at stake, that the fine grain differences among them will not substantially affect the impact of the diffusion process upon productivity growth. It is to empirical questions of this kind that the following section is directed.

4. Diffusion and Productivity Growth: From Micro to Macro

A portion of the comparative neglect of empirical research on the microeconomics of technology choice and innovations’ adoption histories may be attributed to the fact that the connection between diffusion and aggregate productivity growth has not been developed formally. As has been pointed out elsewhere (David 1986; and also David & Foray 1995), the political economy of growth policy has promoted excessive attention to innovation as a determinant of technological change and productivity growth, to the neglect of attention to the role of conditions affecting access to knowledge of innovations and their actual introduction into use. The theoretical framework of aggregate production function analysis, whether in its early formulation or in the more recent genre of endogenous growth models, has simply reinforced that tendency. To try to correct that imbalance by explicitly modelling the connections between productivity growth and diffusion dynamics is something that Zvi Griliches might well have done, had had he continued to pursue his early interest in the diffusion of process innovations rather than focusing his research to empirical studies of the relationships productivity change, R&D investment, and patenting activity.

The model considered here suffices to capture the direct and indirect effects of the diffusion of a major or “radical” process-innovation upon the measured growth of input productivity. While it serves to highlight several general propositions that are simple but often overlooked about the relationship between the pace of productivity growth and the pace at which productivity-enhancing innovations are adopted,\(^{58}\) this heuristic exercise’s main value is to identify explicitly key micro-level determinants of the dynamics of diffusion among the proximate factors governing the aggregate productivity growth rate in the adopting sector of the economy.

The components of the model are reduced form relationships that embed a set of underlying the microeconomic conditions governing firms’ decisions to adopt a new, fix-input using and variable (labor) input-saving technology for consumer-goods production, and the induced processes through which suppliers and users of the new process-equipment modify its design and mode of application on the basis of field experience. Being based on “learning by doing” and “learning by using,” such modifications are driven by the innovation’s diffusion and, in turn, contribute both directly and indirectly to enhancing productivity in the consumer-good’s sector. Their direct impact is due to the reduced labor-input requirements with the new technology with that technique, which are assumed to result from labor-training effects and organizational changes that spill-over throughout the “progressive,” innovation-adopting segment of the consumer-goods industry, thereby raising

\(^{58}\) These relationships have previously been explored in the specific historical context of the effects of the diffusion of the electrical dynamo (and secondary electric motors in particular) on the surge of U.S. manufacturing productivity growth during the 1920s. See David (1991:Technical Appendix), an OECD publication that has been cited more widely than it has been read; a pdf version may be obtained on request from the author. See also David and Wright (2003) for comparative evidence on the diffusion-productivity growth link in the case of U.S., British and Japanese industrial electrification during the Interwar era.
its average level of productivity. Their indirect impact stems from the lowered price-performance ratio of the capital equipment, which expands the extensive margin of adoption in that industry, increasing the relative weight of the industry’s “progressive” segment in total industry output, and thereby raising the industry’s overall level of labor productivity.

In the basic formulation exhibited here, the simulation structure doesn’t pretend to capture the entire range of complex interdependences that could exist between the pace of the new technology’s diffusion and the rate of (endogenous) improvements stemming from experience with the new technology, but it conveys an empirically plausible picture of a dynamic transition in the industry that is being driven endogenously, by feedback from the process of diffusion itself. Even though there may exist other sources of change affecting user-costs of the new technology, in addition to the experience-based improvements in input efficiency explicitly represented in the model as depending directly upon continued diffusion, the specifications employed in the simulations will be seen to plausibly allow for other indirect dynamic feedback effects on the growth of aggregate labor productivity and TFP in the consumer-goods industry.

Presentation of the details of the micro-to-macro simulation model is relegated to the Appendix, and text that follows complements the formal derivations found there by discussing the plausibility (if not the “realism”) of the main assumptions and specifications (in sect. 4.1), and commenting upon the interpretation of the simulation exercises (in sect. 4.2).

4.1 Assumptions, model specifications and implications for the diffusion-driven aggregate productivity growth rate:

For these purposes one may envisage a discrete process innovation embodied in an indivisible capital-good (“machine”) that is assumed to be of fixed capacity and infinite durability. The production process using this new technique is characterized by lower unit labor input requirements than are obtained with the pre-existing, purely labor-using method of producing the consumer good, and it is assumed that the respective unit labor requirement for each of the techniques are invariant to the scale of production (up to the capacity limit of the machine in the case of the newer one). Specifically, denoting the real output flow per unit of labor input using the j-th technique at time t as $\pi_j(t)$, where $j = o$ represents the “old” technique and $j = N$, the “new” technique, it is assumed: (a) that the innovative technique is characterized by higher labor productivity: $\pi_N(t) \geq \pi_o(t)$ for all $t$; (b) the old technology uses only labor, and its unit labor input requirements remain unaltered: $\pi_o(t) = \pi_o$, for all $t$.

Further, there is an “incremental improvement function” for the innovation, arising from the accumulation of experience in commercial applications of the new technique which yield “learning spill-overs” within the progressive segment of the consumer-goods industry. These “learning externalities” leave the output capacity of the machine unchanged but have the effect of raising $\pi_N(t)$, the average labor productivity of all of the facilities in the progressive segment of the industry, that is to say, of all production entities that have adopted the innovation. Because the capacity of the innovation-embodying “machines” is assumed

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59 The following text refers readers to specific equations, using the numbering that appears in the Appendix.

60 The context of the innovation’s application here refers to different production facilities, all of which are assumed to involve essentially the same production operations. This abstraction from reality is worth noting,
to be fixed, and the latter are perpetually durable, cumulative production experience grows *pari passu* with the stock of machines. Further, under the simplifying restriction that the total number of production facilities (firms) in the sector is constant, the share of the sector’s output produced by its innovation-adopting firms is the measure $D(t) = D_n(t)$, where $D_n$ denotes the proportion of adopters among N firms of this sector. As the innovation comes into more widespread use in the consumer-goods sector, this results in micro-level average unit labor requirements decreasing for all adopters, both absolutely and vis-à-vis the “old” technique.\(^{61}\)

At a given moment in time the level of average labor productivity in the sector of the economy to which the innovation is applicable can be represented as a function of the share of the sector’s output being produced by its “progressive”, innovation-adopting segment and the relative labor input requirement of the new and the old techniques. The output share of the industry’s “progressive” segment is the relevant measure of the extent of the innovation’s diffusion, $D(t)$, \(^{62}\) and the average level of labor productivity throughout the consumer-goods industry is found as the inverse of the weighted harmonic average of the unit labor requirements of adopters and non-adopters of the innovation (eqs. A.1, A.2) -- the weights being $D(t)$ and $[1 – D(t)]$, respectively. Consequently, an increase in the extent of diffusion has a direct, compositional shift effect on the growth rate of average labor productivity in consumption-goods sector, and may have “indirect effects” through the variety of channels through which more widespread use of the new technology alters its unit labor input requirements — vis-à-vis that of the “old” technology -- in all applications (eq. A.1a). Improvements that lower the unit labor requirements associated with the innovation, in term are a source of “self-reinforcing” feedback” that sustains its widening acceptance.

A further, general empirical implication may be remarked upon at this point. It is evident (from eq. A.5) that the average rate of growth of labor productivity in the innovation-adopting industry cannot be strictly proportional to the rate of growth of the output-share measure of the extent of diffusion, $D(t)$. When the time-path of $D(t)$ takes the classic, symmetric S-shaped form described by the cumulative logistic (or the normal) distribution, especially when one considers the diffusion of so-called “general purpose technologies.” Adaptation of GPTs to the requirements of different industrial applications typically has entailed significant collateral investment in technical improvements, ancillary capital formation and organizational change -- as has been documented for specific cases as different as electrification and ICTs (see, e.g., David, 1991; David and Wright (2003); Brynjolfsson (2000); Bresnahan, Brynjolfsson and Hitt (2002).\(^{61}\) Because the process envisaged is one in which $D(t)$ increases monotonically, it is not necessary to formally specify that the enhancement effects due to learning are irreversible, so that the ratio $(\pi_n'(t) / \pi_n(t))$ is non-decreasing.

\(^{61}\) The domain of $D$ is [0,1], but the upper limit is definitional and the attained limit of $D(t)$ is taken to be endogenous, for reasons discussed in connection with Figure 2, above, in Section 3, but, under the assumptions made here, production facilities using “the machine” always are operated at the latter’s fixed capacity (specified in terms of output). Further, it is convenient to assume that there is a “management constraint” on the use of labor in the old technology, so that the output of individual production facilities (firms) in the non-progressive segment is bounded at a constant uniform level that never exceeds that of the machine-using firms. These assumptions support the equivalence of the measures $D = D_n$. Note that Griliches (1957) also works with an output related measure of $D(t)$ -- the proportion of total corn acreage that was planted with hybrid corn seeds. Had he sought to define an appropriate (output share) measure for deriving average productivity in corn-production as a function of the acceptance of hybrid corn seed, it would have been necessary to allow for difference in yield per acre between hybrid and open-pollinated corn in each of the crop-reporting regions. But that was not Griliches’ purpose, and he simply accepted the available U.S. Agriculture Department statistics of acreages planted by type of seed.
the annual change in the extent of diffusion \( (dD) \) will reach a maximum (the inflection point of the curve) where \( D = 0.5 \), but the the peak in the proportional growth rate of labor productivity necessarily occurs after the extent of diffusion had passed its peak -- the “half-way” mark, under the stated symmetry conditions. Moreover, further postponement of the productivity growth peak would result where the elasticity of the innovation’s indirect effects is not constant, but instead increases as the innovation becomes the dominant technology within the sector. Such “delayed positive feedback effects” on the differential between productivity levels characterizing the new and old technologies are quite likely to be important where there are significant network externalities, and labor force training externalities that accompany more widespread adoption of the innovation.

For convenience of implementation, the simulation model is specified under the assumptions that:

(i) there is a stationary log-logistic frequency distribution of an underlying critical variate \( z \) among the firm belonging to the population of potential adopters (e.g., their respective expected output scales, or distances from the market for their output);

(ii) the threshold value for agents to select the new technique is \( z^*(t) \) at time \( t \), and it declines at the exponential rate \( \beta \);

(iii) that, since the new technique is embodied in a fixed discrete input-bundle, only one unit of which is acquired by each adopting agent, firms working with a unit of the innovative technology all will have identical and constant flow output capacity \( k_N \), whereas non-adopting firms will have constant flow output capacity \( k_O < k_N \).

These assumptions lead immediately to two results that are useful in simplifying the exposition. The index of the extent of diffusion at time \( t \), \( D(t) \), defined as proportion of the population that has adopted the innovation, will be a logistic function in the \( t \)-domain, with asymptotic saturation at \( D(\infty) = 1 \) (eq. A.6). Secondly, the instantaneous growth rate along the diffusion path will be directly proportional to \( [1-D(t)] \), where the factor of proportionality is \( \gamma \lambda > 0 \) (see eq. A.7b).

The parameter \( \beta \) , as has been noted, is the exponential rate at which the threshold variate \( z^*(t) \) is decreasing, whereas \( \gamma \) is the slope parameter of the cumulative logistic distribution, and varies directly with the kurtosis (peaked-ness) of the frequency distribution of \( z \) and inversely with its variance among the firms forming the population of potential adopters in the consumer-goods sector. Variations of this parameter pair, reflecting alternative underlying conditions affecting the microeconomics of adoption of the innovation therefore will translate into alternative shapes of the diffusion path, \( D(t) \), and thence into corresponding alternative productivity growth paths.

A further simplifying specification of the endogenous “improvement function” for the new technology results in

(iv) a constant elasticity of average labor productivity among adopting firms with respect to the extent of diffusion in the industry: \( \varepsilon(t) = \varepsilon(0) = \theta \), for all \( t \).

Because \( D(t) \) is tantamount to an index of the extent of the cumulative experience with the production and the utilization of the innovation-embodiing capital goods, this constant elasticity condition is satisfied by the classic learning curve or “progress function”
suggested by Hirsch (1952) and Arrow (1962) – as is seen from (eq. A.8). That interpretation is entirely straightforward when the “learning” involving integrating the novel capital goods into production operations and the acquisition by workers of skills in its use, as a function of operating the infinitely durable fixed-capacity machines. As was noted above, given the proportion of output represented by the capacity of the new machine stock then would vary directly with the cumulated output of the industry supplying such equipment, and also with the cumulated volume of gross investment represented by those machines.\footnote{This follows immediately from the assumptions that the “learning effect” in the use of machines is both Harrod-neutral and disembodied. The former of these assumed conditions leaves the per period output capacity of additions to the stock of the novel capital good unaltered, so that the ratio of aggregate output to the stock of machines installed remains constant, and, given the infinite durability assumption, over time the cumulative volume of production rises\textit{ pari passus} with cumulative investment in the new equipment. The latter condition implies that it “spills over”, providing an externality for all the enterprises that have adopted the innovation – in the form of the reduced unit labor-requirements in production of the consumer good (as noticed in footnote 37, above).}

It is necessary to pause here and to notice an implication of having previously specified that the micro-level “break-even” threshold was being driven downwards through the distribution $F(z)$ at a constant (exponential) rate ($\delta$). Consistency between that specification and the specification of a constant elasticity of “learning” with respect to $D(t)$ -- which, by lowering the unit labor requirements for users of the new (mechanized) technique will reduce the critical threshold ($z^*$) at a slowing pace as diffusion proceeds, implies that some other forces in addition to the assumed diffusion-induced “learning effects” must also be acting on $z^*(t)$. These auxiliary dynamic drivers would have operate so as to reduce $z^*(t)$ at a compensating pace that was\textit{ increasing} in $D(t)$ -- although less-than-proportionately – in order to maintain the over-all constancy of the specified exponential rate ($\delta$) of the threshold’s downward passage through the $z$-distribution.

Plausible candidates are available to fill that role, and the interpretation of the reduced form simulation model can be enriched by briefly describing a particular set of mechanisms that are consistent with the previous analysis of diffusion dynamics in a heterogeneous population of potential adopters.\footnote{Harking back to the multi-sector model presented in section 3, it will be recalled that the population of establishments in the consumer-goods industry may be heterogeneous with respect to their expected scales of production, and the threshold scale for adoption $z^*$ would then tend to be driven downward by increases in the ratio of the unit labor costs to the user cost of the fixed capital equipment (the indivisible “machine”).}

The accumulation of process innovations in the capital-goods sector that was supplying the machines -- deriving from the shared cumulative experience gained in machine-building, could generate cost-savings advances in machine-building at a rate that would be augmented by recombination of the resulting incremental process improvements.\footnote{On recombinant generation of technological (and other) novelties, see, e.g., Weitzman (1998), and Shapiro and Varian (1999), and the conceptualization a process of incremental “recombinant innovation” generating a rising – albeit bounded -- rate of technological change. This is assumed here to yield an accelerated pace of gains in the efficiency with which the innovation-embodying machines can be produced.}
Therefore, we may posit the operation of this second feedback loop – from the progress of diffusion to process innovations in machine-building that are cost-saving for the firms in that sector. The effect of the later will be to *increase* the rate at which the threshold for adoption in the consumer-goods sector is reduced over the course of the innovation’s diffusion. This induced concomitant of continued diffusion could combine with an exogenous constant trend rate of growth of economy-wide real wage rate due to developments arising outside the innovation adopting consumer goods industry, and thereby suffice to exactly offset the decelerating rate of fall in the threshold due to the feedback from the direct and indirect effects of the innovation’s widening adoption. Indeed, it is not difficult to find the conditions under which the joint operation of these exogenous and endogenous drivers would just suffice to maintain the constancy of the exponential rate fall in the threshold for adoption, fulfilling the simulation model’s ability to generate a diffusion path that has the classical logistic form.66

The foregoing interpretation of the reduced form labor-productivity improvement function for the innovation clearly shares something with the conventional suppositions that learning-by-producing and learning-by-using new investment have effects on productivity that are Harrod-neutral. Consequently, it is conceivable these endogenous effects could be incorporated in growth models that are consistent with the existence of a balanced, or steady-state growth path for the economy. But, as will be seen, the transition following the introduction of an innovation in a given industry or sector of the economy generates a wave in the growth rate of aggregate productivity. To maintain the economy’s aggregate productivity growth at steady rates would require a succession of fortuitously timed innovations appearing in various sectors, or a sequence of transitions driven by the overlapping diffusion of successively introduced general-purpose technologies.

Such constructions, however, lie beyond the aspirations of the present simulation exercise, which aims to establish more limited propositions the diffusion-productivity growth nexus. It should be equally obvious that the present formulation has sought to avoid the complications of allowing for less-than-universal and instantaneous learning externalities, such as would be present were one to suppose that the learning effects resulted machines that improved in quality continuously from vintage-to-vintage.67 Were the latter to be the case,

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66 From the specification of the learning function in eq. A8, and eq. A7b it can be seen that the rate at which \( z^*(t) \) would be falling due to the endogenous learning effects within the adopting industry is \( \delta(t) = \theta [ \hat{c}(\log(D(t)) / \hat{c}(t) ] \). This rate, obviously, is decreasing in \( D(t) \). The simplest specification of an offsetting endogenously generated downward force on the adoption threshold would require that rate to be: \( \delta'(t) = \delta + \theta [ (1 - \gamma \theta) \cdot D(t) ] \), i.e., a rate that is *increasing* with \( D(t) \), but less-than-proportionately. In the present context of an innovation that is relatively fixed factor intensive and variable input- (labor-) saving, the joint candidates for this role are (1) an exogenous constant upward trend in the real wage \( \delta \) at the rate \( \theta - \gamma \theta \) – due to developments exogenous to both the machine-building industry and the particular final goods industry, and an endogenous feedback from cumulative “learning effects” in the machine-building sector. The latter must continuously reduce the ratio between the price of the innovation–embodied capital-good and the wage rate throughout the diffusion process – a rate that is increasing with cumulative increase in the stock of machines that had been installed: \( \delta (1 - \gamma \theta) D(t) \).

67 Comin and Hobijn (2008) have made an effort to introduce continuous vintage improvement of embodied innovations into a one-sector model of steady-state growth, while allowing for the growth rate of aggregate labor productivity to be affected by technology-specific “diffusion lags”. As theirs is a representative agent modelling approach, the concept of diffusion departs from the definition of increasing extent of acceptance of the innovation among a heterogeneous population of potential adopters, or of the rising proportion of aggregate
however, the latest incremental reduction in labor requirements would be confined only to the current extensive margin of adopting firms in the consumer goods sector—which is not the case here.\textsuperscript{68}

The foregoing log-logistic heterogeneity specifications, and the expression for the growth rate of aggregate labor productivity as a direct and indirect function of \( D(t) \), lead to an expression (eq. A.9) for the proportional rate of growth of aggregate labor productivity in the consumer-goods sector -- denoted by

\[
\pi_{\pi}(t) = \left( \frac{\beta(t)(1-\theta)+\theta}{1-\beta(t)D(t)} \right)\left(D(t)[1-D(t)](\gamma\lambda)\right),
\]

where

\[
\beta(t) = 1 - \alpha[D(t)]^{-\theta},
\]

and \( \alpha = -\frac{\pi_0}{\pi_N(\phi)}(\kappa) \).

\( \kappa > 0 \) being a normalization parameter in the learning function.

It is evident that that the growth rate is quadratic in \( D(t) \) -- whether or not there are indirect learning effects from diffusion (i.e., for \( 0 \leq \theta \)) -- which leads one to anticipate the finding that the monotonic rise of the logistic diffusion curve will generate a single-peaked wave in the growth rate of labor productivity. Further, the product of the micro-level parameters, \( \gamma\lambda \), will acts as a scalar multiplier affecting the slope of the diffusion curve and hence the the amplitude of the wave generated in the productivity growth rate. The first aspect of this may be seen directly from the simulations displayed in Figure 3, which shows three alternative diffusion paths on the left-side of the panel, corresponding to different heterogeneity specifications corresponding to the value of the variance of the \( z \)-distribution, while holding constant \( \phi \) the rate of fall in the threshold \( z^*(t) \). On the right-side of Figure 3 appear the corresponding waves that are induced in the growth rate of aggregate labor productivity.\textsuperscript{69}

From the positive value of \( \phi \) that appears in the notes beneath Figure 3, it can be seen that these growth rate simulations allow for learning effects with the new technology. The output produced with the new technology. Curiously, therefore, “lags” in their model do not measure the time elapsed between commercial introduction and substantial saturation of the market for particular innovations. Instead, they define the “diffusion lag” unconventionally as the (constant) delay between invention and commercial introduction of each vintage of the family of innovations --including the first). The latter corresponds to what Griliches (1957), Mansfield (1962) and the following literature label as the “introduction” lag, distinguishing it from the “acceptance” or “diffusion” lag.

\textsuperscript{68}Without a complete specification of the micro-level adoption dynamics it is not possible to compare the implications for the path of aggregate productive change of a vintage model with embodied learning effects, rather than the present model of diffusion with disembodied augmentation of the benefits accruing to all the adopters of the innovation.

\textsuperscript{69}From the top panel of Figure 4 (below) it also may be seen that although the behavior of the average labor productivity growth rate is non-monotonic, the underlying diffusion rate (i.e., the proportional growth of \( D \), is undergoing continuous retardation along the logistic path. A good bit of surprise, and some confusion on this point stems from the casual supposition that the rate of productivity growth should reflect immediately reflect the rate of diffusion, whereas it is the absolute rate of change in \( D \) that matters.
timing in the peak in the aggregate labor productivity growth rate, which occurs after the maximum in the slope of the diffusion curve — at the inflexion of the logistic \((D(t) = 0.5)\), as has been seen, is a general property that holds whether or not there are indirect, feedbacks from “experienced-based learning”.

[Figure 3 here]

Figure 3 exhibits a third point, upon which Part 2 remarked: the influence of the shape of the distribution of underlying population heterogeneity upon diffusion dynamics, and hence upon aggregate productivity growth in the industry. Other things being equal, the lower the value of the logistic parameter \(\beta\), the greater is the variance (and the lower is the Kurtosis) of the frequency distribution of the population characteristic \((z)\) that enters the micro-level choice of technique decisions. Thus, with the “break-even” assumed to be falling exponentially at the same (fast) rate in all three situations, its is seen that lower values of \(\beta\) stretch out the diffusion process, lower the productivity growth profiles and displace the (attenuated) peak substantially into the future.  

Two implications follow immediately from this latter observation. First, one is only seeing half the picture by focusing on the determinants of the pace at which the threshold point \(z^*(t)\) is pushed downward through the \(z\)-distribution. Putting this more concretely, the essentially neoclassical factor-substitution story that economists today like to tell about the way that a new form of capital raises aggregate capital-intensity and thereby raises labor productivity, more often than not is an inadequate “representative agent” tale. All the emphasis is placed on the forces causing the relative fall of the real user-costs of fixed capital inputs (such as computer equipment) vis-à-vis labor inputs — as the driver of factor substitution, and hence the determinant of the growth rate of labor productivity. But if the \(z\)-distribution differed from one sector of the economy to the next, there would be quite different patterns of diffusion and correspondingly different labor productivity performance — for which the hypothesized representative agent “model” would have, at best, only ad hoc explanations.  

Secondly, it is worth noticing that the measured pace of diffusion and the dynamics of productivity may well be affected by the alteration of the underlying \(z\)-distribution, as a result of the economic pressures emanating from the adoption of the innovation by some firms in the industry. It is quite conceivable that competitive pressures on the non-adopting remnant of the industry would force out firms at the low \(z\) end of the distribution, thereby tending to raise the parameter \(\beta\) over the course of the process. The result would no longer be a strictly logistic diffusion path. To preserve the latter form, it would be necessary for the \(z\)-distribution to be transformed by a \(\beta\)-preserving upward shift in its mean. Suppose that evolution of the first moment of \(z\) proceeded at a constant proportional rate, \(\lambda\). It is simple enough to show that the slope coefficient of the resulting logistic diffusion path would then become \(\lambda(8+\cdot)\). Consequently, the working of competitive forces at the industry level can

\[70\] Note that the absolute values of \(\beta\) and \(\lambda\) used in this simulation are rather arbitrary; the same results could be obtained if the annual rate of decline in \(\lambda\) were taken to be half as fast — approximating the 15% per annum trend in the hedonic prices of computer and communications equipment — if the underlying heterogeneity distribution was half as spread out (i.e., \(\{0.6, 0.9, 1.2\}\)).

\[71\] The resemblance of this picture to the now-popular line of interpretation of the computer-revolution’s contribution to aggregate productivity (developed in the influential work of Jorgenson (2000) and his co-authors) is not entirely coincidental.
quite neatly be formally assimilated into this richer account of long-run productivity growth dynamics.

4.2 Simulating effects on micro-level determinants of diffusion on the growth rate of measured TFP

Explicit modelling of the microeconomics of diffusion decisions shed further and different light upon the sources of the “productivity residual.” Quite clearly this cannot be the whole picture, because the effects of the diffusion process in the case of each innovation (or family of improvable innovation) are transitional. When the new technique finds its way into all the available niches of use, the impetus imparted to productivity improvement is exhausted. Diffusion thus resembles evolutionary processes of selection, in being “a fire that consumes its own fuel.” This transparent consideration certainly is sufficient warrant for the attention that Zvi Griliches’ empirical research program devoted to the nexus between firm-level R&D investment and multi-factor productivity growth. But perhaps something was lost by working at that very low level of aggregation: it suppressed attention to the industry-and sector-level productivity effects that depended upon the diffusion of the novelties created in company laboratories, and by publicly funded research in universities and government mission-agencies.

Granting while sustained advances in total factor productivity do depend upon the generation of further innovations, to the degree that substantial productivity raising techniques are not being introduced continuously in time, the pace of TFP growth will be subject to wave-like impulses that reflect the dynamics of the overlapping diffusion of sequential innovations. Technological breakthroughs that yield potentially large and ubiquitous gains unit labor requirements for the adopters, and induce clusters of innovation in numerous technically related industries, consequently may well generate pronounced surges in the productivity growth rate some considerable time after the key innovation is first introduced.

The simulation model developed in the Appendix generates changes in the TFP growth rate by expressing the latter as inverse of the weighted average of the rates of change in the unit labor input and unit capital input requirements in the progressive, innovation-adopting segment, and the rate of change in the unit labor input requirements of the part of the at industry that continues to work with the old (un-mechanized) techniques. The weights for those two segments, as before, are found from the measure of the extent of diffusion $D(t)$ (see eq. A.11). In order to calculate the weighted average of the growth rates of capital and labor productivity in the “progressive” segment of the industry, however, is necessary also to weight the latter by the elasticities of output with respect to each of the factor inputs. The simplest approach available is the one that has been adopted for the present simulation exercises, namely, positing that the segment of the final goods industry comprising production facilities that have adopted the innovation is characterized by an aggregate production function of the Cobb-Douglass form, a specification that is, at least, not inconsistent with the restrictions that have been placed on the new technique of production.\footnote{A Cobb-Douglass aggregate production function for this (growing) sector would be implied if the fixed-coefficient production techniques being used by the constituent firms (each technique being characterized by a}
It follows that the elasticity of output with respect to labor input in the adopting segment of the industry will be constant over time. Under conditions of competition in the product and factor markets, this implies that the share of labor in aggregate output of the adopting segment of the industry also will be a constant, 0 < \omega_N < 1. But in the consumer-goods industry as a whole, labor’s share will be contracting as the diffusion of the relatively capital-using innovation proceeds and the aggregate capital-output ratio in the industry is pushed upwards (see eq. A.14).

There are two alternative special assumptions of interest in regard to the rate of change in output-capital ratio with the new, mechanized technology, \( U_N(t) \): under the assumption of Harrod-neutrality it is zero, whereas under the assumption of Hicks-neutrality is equality to the rate of change in labor productivity. Correspondingly, there are two alternative expressions (eqs. A.15 and A.16) that relate the rate of change of the industry-wide average productivity of capital in the consumer goods sector to the growth rate in the measure of diffusion.

Because it has been assumed that a Cobb-Douglass aggregate function describes the relationship between labor and capital inputs in the progressive, innovation-adopting segment of the portion of the consumer-goods industry, Harrod-neutrality and Hicks-neutrality are equivalent specifications for the effects of technological change within that portion of segment industry, but that is not the case for the industry as a whole. Consequently, there are two alternative simulation equations for the growth rate of TFP, in the Harrod-neutrality case:

\[
A(t) \mid \text{Harrod} - N = \pi_2(t)[1-(1-\omega_N)D(t)] - (\gamma \lambda)(1-\omega_N)D(t)[1-D(t)] 
\]

capital-labor ratio) varied according to a Pareto distribution. See Houthakker (1955-56). Such a possibility is not obviously inconsistent with the assumptions that have been made about the novel technology, because while the capacity of the indivisible capital good embodying the innovation is assumed to be constant, the unit labor input requirements are subject to change over time, as are the associated production scales of the adopting firms at the extensive margin. But, that alone does not suffice to guarantee that the distribution of micro-level technical coefficients would have the Pareto form.

\textsuperscript{73} Inasmuch as the output capital ratio, \( v \) is taken to be constant for adopting firms, assuming they produce at the capacity of “the machine”, constancy of the real rate of interest (\( r \)) in the economy would set the share of output imputed to capital, and, under constant return to scale, this fixes \( \omega_N = [1- r v] \), whereas, for the industry as a whole the share of labor is decreasing as the diffusion of the relatively capital-using technology proceeds (eq. A.12) and can be found from \( \omega_N = 1 - [1-D(t)][r v] \). One cannot suppose that at the level of the individual firms the share of output going to capital was the elasticity of output with respect to capital input services, since with fixed coefficients that elasticity is not defined. But, the assumption made here applies to the aggregate production function for the innovating segment as a whole. One is then free to assume that at the micro and aggregate level of the ensemble of adopters in the consumer-goods sector, the average rate of return to (infinitely durable) capital is the real interest rate in the economy. This, however, does constrain the interpretation that can be placed on the source of heterogeneity in the population of adopters. For example, it would be difficult to consistently suppose that a threshold model based upon differences among firms in their expected output rate accounted for non-adoption by firms that were below the break-even scale of production for an innovation that was fixed-factor-using, and variable-factor-saving – as suggested in the discussion of section 3. Firms at the margin of adoption would be “breaking even”, and earning the opportunity cost rate of return on their investments, but as diffusion proceeded (with the threshold declining) a growing fraction of the adopters would be earning supernormal returns on their investment – as their output scale was above the breakeven point. Diffusion would drive a rise in the share of output that came to owners of capital within the consumer-goods sector’s “progressive” segment, and a fortiori in the sector as a whole – because the aggregate capital-output ratio there was rising as more and more firms adopted the capital-using innovation.
and the Hicks-neutrality case:

\[
\dot{A}(t) \mid Hicks - N = \dot{A}(t) \mid HaN + \theta(\gamma \lambda)(1 - \omega_N)[1 - D(t)],
\]

derived as eqs. A.19 and A.20, respectively.

Other things being equal, simulations of equation (A3.20) produce TFP growth rates that lie everywhere above those from equation (A.19), since the second of the right-hand terms in the first of these equations is positive.

One readily can find the first-order condition for the peak TFP growth rate,

\[
\frac{d}{dt} \left( \dot{A}(t) \mid HiN \right) = 0 \quad \text{(from eqs. A.9 and A.18, and)}
\]

The positive value of \( D(t) \) which satisfies that condition is found to be a function of the model’s four parameters \( \forall \), \( \lambda \), \( \omega_N \), and the normalizing constant, \( \delta \). Given \( D**(t) \), and the parameter \( \Phi \), which is a constant reflecting the initial position of the threshold variable in the \( z \)-distribution at \( t = 0 \), it is straightforward to solve for a general expression giving the date \( t** \) at which the peak growth rate of TFP occurs. The numerical simulations displayed in Figure 4, however, convey the essential points of the story rather more immediately.

[Figure 4 here]

The top panel in Figure 4 presents alternative diffusion paths generated by variant specifications regarding the rate of decline in the “break-even” threshold level \( z^* \), and the corresponding time profiles of the proportionate rate of diffusion, which is seen to undergone continuous retardation. The latter is more pronounced when the process is being driven by a comparatively fast decline in \( z^*(t) \).

Simulation results for the growth rate of average labor productivity and the multi-factor productivity residual appear on the left- and right hand side of the lower panel, respectively. These calculations have been made using three alternative specifications of the strength of the impact of learning from diffusion experience on the incremental improvement of the new technology’s relative efficiency. In the base case, condition \( 2 = 0 \) signifies the absence of any such learning effects. Under the assumption of a fast rate of decline in the threshold value \( z^*(t) \), the inflection point of the diffusion path occurs at \( D = 0.5 \), indicated by the dotted vertical line at the \( t=30 \) date. One can see that the peaks in the labor productivity growth rate are displaced to the right of that, the ‘delay’ being more pronounced the stronger are the endogenous learning effects.

The results show that the peak of the TFP growth rate is similarly displaced in time beyond the date of the inflection point of the diffusion path. This rightwards shift is more pronounced than that observed in the case of the growth rate of labor productivity. The latter reflects the strong contributions of increasing fixed input (capital) deepening during the phase when the absolute changes in the extent of diffusion become large.

The alternative cases presented by Figure 4 display the time profile of the multi-factor productivity residual under the assumption that there are positive Hicks-neutral efficiency improvements in the new technology that proceed pari-passus with the widening of
experience in the use of the new technology (i.e., with the extent of diffusion). Intuition is satisfied by observing that the greater is the elasticity of these learning effects on efficiency with respect to the extent of diffusion, the stronger the upward effect on the profile of the TFP growth rate.

Were the learned improvements in factor efficiency to be confined to enhancements in the efficiency of labor, as is the case under Harrod-neutrality, the general level of the TFP profile would be lower; also its peak would be reached still later in the diffusion process than the simulation results show the Hicks-neutrality specification. The intuition for this is quite direct: under Harrod neutrality improvements there is no source of capital efficiency improvements to offset the decline in the sector-wide average productivity of capital as diffusion proceeds.

Note that some special conditions are required in order for the foregoing assumption of incremental improvement in the relative efficiency of the new technique to be consistent with the (unchanged) specification of the diffusion path. The effects upon either the heterogeneity distribution, or the movement of the adoption threshold must change in an offsetting manner and so keep the threshold falling over time at a constant exponential rate, as the simulations posit. Alternatively, one may suppose that the distribution of critical heterogeneities in the population is displaced at a rate that offsets the declining pace of growth of labor- and capital-input efficiency due of experience-based learning. It is not implausible that the pace of upward shift in the expected output size distribution could be accelerating over time in such a fashion. Similarly, it is quite conceivable that the relative user-cost of the new capital in the industry might fall at a quickening pace, either because the economy-wide level was being forced upwards or because scale effects were lowering the real costs of producing the new fixed inputs. Of course, the constant rate of fall in the adoption threshold has been assumed here primarily for expositional convenience.

5. Conclusion: Going forward with empirical research on diffusion

As has been seen, the class of microeconomic models of new technology adoption that recognized the existence of underlying, critical heterogeneities in the population of potential adopters, and the relationship the latter’s distribution in the population and the distribution of the agents adoption decisions in time, offers a quite comprehensive framework for studying diffusion phenomena where decisions by the agents do not involve strategic considerations. The threshold model is not so much a theory of the diffusion of innovations as it is a paradigm within which one may articulate a variety of distinct theoretical models involving both the demand-side and the supply-side of the market for new technologies. Thus, it is capable of subsuming a quite different economic mechanisms, and its variant formulations can in principle account for diffusion processes that follow different lag structures that are very protracted, as well as those which are highly compressed in time.

By “connecting the dots” between the seminal contributions made in Zvi Griliches to the economics of technology diffusion, the study of distributed adjustment lags, and the growth rate of measured TFP, the preceding paper raises two sets of issues for empirical research that can be seen to cut across these three fields. The first issue is that of the “data constraint” on empirical efforts to identify particular micro-level mechanisms that generate time-series phenomena observed at the level of population aggregates. The exacting data
requirements -- consistent time-series data at the aggregate level and cross-section observations at the micro-level -- have been pointed out (in the conclusion of section 2) as a serious obstacle to econometric identification and measurement of specific sub-processes that may be contributing to slowing the speed of innovation’s diffusion into widespread acceptance.

Whether the incentives and material resources can be mobilized, in order to carry through an extensive program of data collection and data preparation that will be needed if empirical research in this field is to move forward in tandem with the elaboration of new analytical models is an important practical issue. Unfortunately, little has happened to alter the fundamental sources of the “data constraint” in this area, which, as Zvi Griliches pointed out in another connection, stem as much from the internal reward structure of the academic economics as they do from the policies of public and private research funding agencies.

But this realism does not counsel resignation and despair: there is much that can be done, and is worth doing that does not require the econometrically conclusive identification of the domains of empirical relevance of the many distinctive theoretical structures that have been and still could be proposed in this field. Even with the available data it seems it would be useful and not at all infeasible systematically to distinguish empirically between two broad classes of diffusion phenomena. On the one hand, would be those that do not involve significant irreversible investment expenditures, and are driven primarily by the relative rapid propagation of information -- the primary effect of which is to dispel uncertainties about the net benefits of adoption on the part of the agents. On the other hand, one expects to find diffusion phenomena whose dynamics involve temporally more prolonged transitions from restricted to widespread application of the new technology, and it may be conjectured that on careful examination of their individual circumstances, these cases will be found to reflect the induced modification of the economic and technical characteristic of the new technology in response to feedback induced by the process of diffusion, thereby permitting the new technological system to become attractive to an increasing proportion of a heterogeneous population of rational and already informed economic agents.

Logically, there can be a mixed category, where both information propagation and a moving equilibrium of technology adoption by informed agents both must be viewed as of more or less equal quantitative importance in setting the time constant for the overall diffusion process. It seems reasonable to suppose that most of the data that has been collected will be found to belong in the polar categories, and moreover, that what is conventionally measured as growth in total factor productivity at the aggregate industry and sectoral levels, and imputed to “innovation” will be attributable to the compositional effects within industries and sectors of the diffusion processes that belong in the category of he “low frequency” feedback dynamics.

Still other empirical enquiries would appear to be both feasible and useful in exploring links between diffusion processes and sub-field of the economics of technological change where Zvi Griliches’ work opened major research thoroughfares that others have been able to travel. The connection between changes in measured productivity at the level of the enterprise and innovative activities associated with R&D expenditures and patenting was a central preoccupation of Griliches’ research, about which the preceding pages of this paper
have remained silent.\footnote{74} Having here fulfilled the promise to exhibit the connections that may
and should be drawn between research on diffusion, distributed lags and TFP growth, perhaps
a further feasible task for the future is the empirical exploration of the nexus between
diffusion and R&D.

It seems entirely reasonable to suppose that there may be some identifiable feedback
from changes in the sales revenues received by suppliers of producer (or consumer) goods
that embody innovative production processes, and the internal financing of R&D investment
to further develop their products innovation -- improving the performance, or maintainability
of later vintages of the original innovation's design? Figure 5 summarizes the several
dynamic connections that were already represented in the simulation structure discussed in
Section 4, and ventures to indicate (conjecturally, with a question-mark) that a fourth set of
dynamic relationships bear closer investigation: these involve “post-innovation R&D
expenditures” that are both dependent upon the progress of an innovation’s diffusion, and in
turn affect the speed and eventual extent of its acceptance.

This opens a topic that has remained more or less untouched in the diffusion literature
and barely noticed in econometric research that has focused on the determinants of R&D
expenditures at the firm level and their impacts upon productivity growth. Here one can only
speculate briefly about three possible lines along which future research into the existence and
importance of such micro-level connections might unfold.

First, one can start by examining the possible diffusion-R&D nexus based upon the
relationship between gross sales revenues from the innovative product line, and the latter’s
effect the firm’s net earnings, which in turn might positively affect R&D budgeting decisions,
especially smaller enterprises where R&D was tightly constrained by retained earnings, or
lenders rationed credit to support R&D on the basis of new product sales growth. As second,
variant connection, perhaps more relevant where a larger firm at any point of time had a
number of product lines that differed in their degree of novelty, and the question of funding
further improvement in a newly introduced product (embodying an innovative production
technology) was primarily one allocating available retained earnings for R&D among the
claimant product lines. In this case, attention is directed to the possibility that there are life
cycles in R&D budgeting at the level of the individual product line: in the early phases of a
new product’s diffusion, rising sales volume – however small in absolute terms – may
strengthens the case for the firm to continue R&D expenditures to improve an emerging
("proven") addition to the product line, and so work to further expand the dimensions of the
market niche into which it is being accepted. But, by the same token, once an inflection point
in the diffusion path is passed, quite conceivably there will be some waning of the capital
budgeting committee’s enthusiasm for continuing to try to push the innovation into marginal
portions of the market, and the higher absolute level of sales revenues from the maturing
product will be increasingly diverted to supporting the improvement of younger product
lines.\footnote{75}

\footnote{74} See Griliches (1998), and notably in this context, Griliches and Mairesse (1984), Griliches, Hall and
Hausman (1986), Griliches and Regev (1995), for the legacy of this program of mico-level econometric
investigation is evident in the contributions of Hall (2009), Mairesse, Mohnen and Kremp (2009), Regev
(2009), and others that appear in Part IV of this Issue.

\footnote{75} Note that where funds for R&D are not constrained by sales of the new product itself the overlapping life-
cycles in product lines, as just described, would general feedbacks whose implications for the dynamics of the
threshold level for adoption are sharply distinguishable from those where product performance improvements
driven implementing by-product learning based upon the accumulated experience among either the suppliers or
A second distinct line of inquiry would lead toward better understanding of the mechanisms involved in user-producer feedback, generating suggestions for further improvement in performance based upon reports from field use through dealership and maintenance service networks for producer durable, or information returned to the firm by sales personnel. The literature in industrial organization economics and management, with few exceptions, has conceptualized learning-by-doing in production activities not as an object of management and the deliberate allocation of resources, but rather as the opportunistic implementation of incremental modifications in product design, fabrication methods, or modes of use that are essentially by-products of the operation of established production routines and therefore can be regarded as costless. Thus, they are held to be analytically distinct from R&D investments, which are treated as temporal precursors to the establishment of production. Yet, there is ample reason to think otherwise, and therefore to entertain the idea that some significant part of the expenditures reported as R&D may represent deliberate investments of design and engineering resources, and costly organizational integration and analysis of information flowing from interactions between sales and service personnel into managerial processes intended to optimize and capture the benefits of experience-driven “learning.”

Clearly there is evidence that this is the case in some industries where there is technical support for marketing of new complex products, and the control of quality and yield in fabrication is an important determinant of unit costs.

Thirdly, consider the role of the information revealed by sales reports about "structural holes" in the market -- areas where the innovation surprisingly fails to penetrate, which are identified by reports from the field and points to problematic features of the existing design? This is a variant of the "focusing devices" for technological change the adopters of the innovation. In particular it would tend to impart a more pronounced sigmoid pattern to the changes in the relative attractiveness of the specific innovative technique to the potential adopters. By contrast, in the model of Sect. 4, the (new) machine-building firms’ movements down their experience-based learning curves yields decreasing incremental gains in the price-performance ratio of the innovation-embodying product.

Hatch and Mowery (1998), in addition to providing a survey of the literature on learning by doing that supports the foregoing characterization, present an analysis of deliberate practices in the semiconductor fabrication industry that allocate resources to managing the learning process in order to increase yields while maintaining quality. See also, in this vein, Adler and Clark (1991), and Pisano (1997).

The literature on producer-user interactions, starting with Rosenberg (1982) Ch. 6 on “learning by using” – particularly in reference to optimization of aircraft performance, might be fruitfully harvested for additional evidence that translating the information captured during field use into productivity- or product performance-enhancing engineering modification entails both fixed and variable resource expenditures, some part of which surely wind up as expensed by the supplying firm but reported as “Development” under R&D for tax purposes.
argument -- a la Rosenberg (1972), but the feedback about dysfunctional features is market mediated, rather than technical and taking place within the innovating firm --which is mainly what Nate had in mind. Do we know anything about any of this? Shouldn't we? It would then be possible to take account of another feedback loop that links R&D into the system along with "diffusion", "lagged investment", and TFP growth.
Figure 1.

Figure 2.
Figure 3 (a)
Effect on diffusion of greater variance (smaller $\gamma$) of the underlying heterogeneity distribution.

Figure 3 (b)
Effect on aggregate labor productivity growth of greater variance (smaller $\gamma$) of the underlying heterogeneity distribution.

Simulation parameter values: $\lambda = \lambda_{\text{FAST}} = 0.3$; $\theta = 0.2$; $\omega_N = 0.5$; $\gamma = \{0.3, 0.45, 0.6\}$; $\frac{\pi_{\text{eq}}}{\pi(0)} = 0.4$; $\phi = 60$; where $D(t) = \left(1 + e^{-\phi t}\right)^{-1}$.

(Note: $\phi$ normalizes the initial value of $D(t)$ to 0.01.)
Figure 4: Effects of alternative values for $\lambda$ -- the rate of fall of the adoption 'threshold level $z'(t)$ -- on the diffusion path, and on the growth rates of labor productivity ($\Delta \ln \pi(t)$) and multifactor productivity ($\Delta \ln A(t)$). Given alternative diffusion-driven (Hicks-neutral) "learning effects" the relative productivity of the new technology.

Figure 4 Simulation parameter values: $\lambda_{\text{SLOW}} = 0.2$; $\lambda_{\text{FAST}} = 0.3$; $\theta = \{0, 0.2, 0.4\}$; $\omega_N = 0.5$; $\gamma = 0.5$; $\frac{\pi_N}{\pi_N(0)} = 0.4$; $\phi = 60$; where $D(t) = (1 + \phi e^{-\phi t})^{-1}$.

(Note: $\phi$ normalizes the initial value of $D(t)$ to 0.01.)
Figure 5: Summarizing the dynamic system:
Connecting diffusion, lagged distribution of investment, TFP growth – and perhaps lagged R&D expenditure too?
APPENDIX

A Heuristic Model Linking Aggregate Productivity Growth Rates to Micro-level Determinants of Technology Diffusion Dynamics

A.1 Definitions and assumptions

The following notation refers to an industry or sector producing a homogeneous good, $V$:

$\pi_j(t)$: is output per unit of labor input using the $j$-th technique at time $t$, where $j = o$ represents the “old” technique and $j = N$, the “new” technique; $\pi_N(t) \geq \pi_o(t)$ for all $t$.

$D(t)$: is the proportion of aggregate output produced using technique $N$, at time $t$;

$\pi(t)$: is aggregate labor productivity at time $t$.

Aggregate labor productivity $\pi(t)$ may be expressed as:

$$\pi(t) \equiv \pi_o(t)\left[1 - D(t)\left(1 - \frac{\pi_o(t)}{\pi_N(t)}\right)\right]^{-1}.$$  \hspace{1cm} (A.1)

There are two simplifying assumptions that restrict the dynamics of the technique-specific labor productivity rate:

**Assumption 1:** $\pi_o(t) = \pi_o$ for all $t$.

**Assumption 2:** $\pi_N(t) = \pi_N \{D(t)\}$, $\frac{\partial \pi_N}{\partial D} > 0$, $\frac{\partial^2 \pi_N}{\partial D^2} < 0$.

A.2 Determinants of the growth rate of aggregate labor productivity

The general expression for the dependence of the proportional growth rate of labor productivity on changes in the extent of the innovation’s diffusion is obtained by rewriting (A.1) as

$$\pi(t) = \left(\frac{\pi_0}{1 - [\beta(t)]D(t)}\right),$$  \hspace{1cm} (A.2)

where we define:

$$\beta(t) \equiv \left[1 - \frac{\pi_o}{\pi_N(t)}\right].$$

The proportionate growth rate of labor productivity, $\pi(t) \equiv \frac{\partial \{\ln \pi(t)\}}{\partial t}$, is then found by differentiation of (A.2):

$$\pi(t) = \left(\frac{d\pi}{dt}\right) = \left(\frac{\beta(t)}{1 - [\beta(t)]D(t)}\right)\frac{dD(t)}{dt} + \left[\left[1 - \beta(t)\right] \frac{d\pi(t)}{dt}\right] \frac{dD(t)}{dt},$$  \hspace{1cm} (A.3)

where the elasticity parameter in the endogenous innovation-improvement function, is denoted by
The first term on the RHS of (A.3) gives us the direct effect of diffusion on productivity growth, which is the total effect in the simplest case where neither the new nor the old technologies undergo any change in their respective unit labor input requirements, i.e. where \( \varepsilon(t) = 0 \) and \( \pi_N(t) = \pi_N \) for all \( t \). The second term on the RHS, obviously, gives the indirect effect. For future reference, the time-path of aggregate labor productivity will be denoted by \( \pi(t) = \pi_1(t) \) in the simplified case (\( \partial \pi_N = 0 \) ) where the diffusion of the innovation does not induce further reductions in the unit labor requirements for those firms that have adopted it.

It is now straightforward to show that aggregate labor productivity does not grow most rapidly when the extent of diffusion is rising at its fastest pace. This proposition holds generally, but it is most readily understood by examining the case where \( \varepsilon(t) = 0 \) and \( \pi_N(t) = \pi_N(o) \), so that \( \beta(t) = \beta > 0 \) for all \( t \). The general expression in equation A.3 simplifies to

\[
\pi_1(t) = \left( \frac{\beta}{1 - \beta D(t)} \right) \frac{dD(t)}{dt}, \quad \beta > 0 .
\]  

(A.3a)

Evidently, \( \pi_1 \) is not simply proportional to the change in the extent of diffusion \( (dD) \), and therefore it will not reach a maximum when \( dD/dt \) reaches its maximum. This is readily shown by differentiating \( \pi_1(t) \) with respect to time:

\[
\frac{d \pi_1(t)}{dt} = \left( \frac{\beta}{1 - \beta D(t)} \right) \left( \frac{d^2 D}{dt^2} \right) + \left( \frac{d \pi_1(t)}{dt} \right),
\]

(A.4)

from which it follows that at \( \max \left\{ \frac{dD}{dt}, \frac{d^2 D}{dt^2} \right\} \rightarrow 0, \) and \( \frac{d \pi_1(t)}{dt} \rightarrow \left( \frac{d \pi_1(t)}{dt} \right)^2 > 0 \).

For the typical case, \( \max (dD) \) occurs in the interval \((0, 1)\), implying that \( \left[ \pi_1 \mid \max (dD) \right] \) cannot be at a maximum. Since the term in brackets ( ) on the RHS of equation (A3a) is increasing monotonically in \( D(t) \), \( \max \left\{ \pi_1(t) \right\} \) must be reached later than \( \max (dD) \).

It is now straightforward to show that this result is more general when there a constant elasticity parameter that describes the dependence of the indirect “learning effects” upon changes in \( D(t) \), i.e., \( \varepsilon(t) = \varepsilon(o) \) for all. One may then define a new parameter,

\[
k = (1 - \varepsilon) + \varepsilon / \beta,
\]

and notice that the basic differential equation (A.3) for the sum of direct and indirect effects then may be written in the alternative form:
\[ \pi(t) = k \left( \frac{\beta}{1 - \beta D(t)} \right) \frac{dD(t)}{dt}, \quad \beta > 0, k > 0. \quad (A.5) \]

From this it follows immediately that the value of \( D(t) \) at which the whole expression for the labor productivity growth rate reaches its maximum will coincide with that obtained for the special case when only direct effects are present: \( \max \dot{\pi} = \max \dot{\pi}, \).

**A.4 Diffusion model specifications and the time-path of average labor productivity in the industry**

For eventual computational convenience, drawing upon the discussion in Section 2.3 of the text, we may introduce 3 specifying assumptions for the micro-level dynamics of the diffusion process:

**Assumption 3**: There is a stationary underlying distribution of the critical variate \( z \) in the population of potential adopters that is log-logistic in form, and the threshold value for agents to select the new technique is \( z^*(t) \) at time \( t \), which declines at the exponential rate \( \lambda \).

**Assumption 4**: The new technique is embodied in a fixed discrete input-bundle, only one unit of which is acquired by each adopting agent. Firms working with a unit of the innovative technology all have identical and constant flow output-flow capacity \( k_N \), whereas non-adopting firms have constant output-flow capacity \( k_O \).

From these specifications it follows that an index of the extent of diffusion at time \( t \), \( D(t) \), defined as proportion of the population that has adopted the innovation, will be a logistic function in the \( t \)-domain, with asymptotic saturation at \( D(\infty) = 1 \). This yields closed-form expressions for the level, the absolute and the proportional changes in \( D(t) \):

First, we have the form already familiar from the derivation (see section 2.4 of the text):

\[ D(t) = \left(1 + \{z^*(0)\} e^{-(\lambda/\gamma)t}\right)^{-1}, \quad \lambda > 0, \gamma > 0; \quad (A.6) \]

where \( \{z^*(0)\} = \Phi \) is a constant reflecting the initial position of the threshold variable in the \( z \)-distribution at \( t = 0 \).

Second, for all \( \gamma/\lambda > 0 \), we obtain

\[ \frac{dD(t)}{dt} = (\gamma/\lambda)[1 - D(t)]D(t); \quad (A.7a) \]

\[ \dot{D}(t) = (\gamma/\lambda)[1 - D(t)]. \quad (A.7b) \]

**Assumption 5**: There is an endogenous “improvement function” for the average labor productivity of workers using the new technology which is characterized by a constant (less-than unitary) elasticity of response to the increased extent of diffusion.

This specifying assumption is satisfied by:
\[ \pi_s(t) = \left[ \pi_s(o) \right] \left[ \frac{D(t)^\theta}{\kappa} \right], \quad 0 < \theta < 1, \] 

where \( \kappa \) is an arbitrary normalization constant.

The foregoing log-logistic diffusion specifications, in conjunction with equation (A.3), lead to the following simulation equations for the direct and indirect effects combined:

\[ \pi_{LL}(t) = \left( \frac{\beta(t)(1-\theta)+\theta}{1-\beta(t)\eta\lambda} \right) \left( D(t)[1-D(t)](\gamma\eta\lambda) \right), \] 

(A.9)

\[ \beta(t) = 1 - \alpha[D(t)]^{-\theta}, \]

where \( \alpha = \frac{\pi_0}{\pi_s(o)}(\kappa) \), and \( \pi_{LL}(t) \) denotes the aggregate growth rate of labor productivity for the underlying diffusion process based on the log-logistic heterogeneity specification.

In the special case in which there are no “learning effects”, i.e. \( \in(t) = 0 \), the simulation equation reduces to:

\[ \pi_{LL}(t) = \left( \frac{\beta}{1-\beta D(t)} \right) \left( D(t)[1-D(t)](\gamma\eta\lambda) \right). \] 

(A.10)

A.5 Diffusion dynamics and the growth rate of aggregate TFP

As the simulation model developed here maintains the underlying specifications that generate a logistic time path for the diffusion index, \( D(t) \), the \( LL \) subscript on variables denoting the productivity growth rates in the industry will be suppressed in the following exposition.

Accepting the conventional Abramovitz (1956)-Solow (1957) computation of the rate of growth of the total factor productivity (TFP) residual, the latter may be expressed as the factor share-weighted average of the average labor productivity and capital productivity growth rates:

\[ \dot{A} = \left[ \omega_L(t) \right] \pi(t) + [1-\omega_L(t)](v(t)). \] 

(A.11)

Given the expressions for \( \pi(t) \) from equation (A.9), it is a simple matter to derive corresponding expressions for \( \dot{\lambda} \), the proportional growth rate of TFP, once we have expressions for the time rates of change of output per unit of capital input, denoted \( (v) \), and for the share of labor in the aggregate output of the sector in question, \( \omega_L(t) \).

A.5(a) Labor’s share in aggregate output:
Since the new technology is embodied in a durable asset that must yield at least a positive rate of return, we introduce a further specification regarding the ensemble of production establishments in the segment of the industry that has adopted the process innovation.

**Assumption 6**: The segment of the final goods industry comprising production facilities that have adopted the innovation is characterized by an aggregate production function of the Cobb-Douglass form.

From that specification (the plausibility of which is discussed in Section 4.1 of the text, particularly in footnote 56) it follows that the elasticity of output with respect to labor input in the adopting segment of the industry will be constant over time. Thus, under conditions of competition in the product and factor markets, Assumption 6 implies that the share of labor in aggregate output of the adopting segment of the industry also will be a constant, \( 0 < \omega_N < 1 \).

As indicated by the remarks on **Assumption 1**, the share of labor in the old technology-sector of the economy is taken to be unity, so that the share of labor in the industry as a whole is:

\[
\omega_L(t) = 1 - D(t)[1 - \omega_N] .
\]

**A.5(b) The growth rate of capital productivity:**

The aggregate capital productivity growth rate depends upon the rate of change in the extent of diffusion, and the level and changes occurring in the productivity of capital used in the new technology segment of the industry (or sector). Denoting the latter by \( \nu_N(t) \), and recalling that the old technology uses only labor, the aggregate capital productivity in the industry is given by:

\[
\nu(t) = \left( \frac{D(t)}{\nu_N(t)} \right)^{-1} = \frac{\nu_N(t)}{D(t)} .
\]

From equation (A.13), by differentiation, and multiplication of both sides of the resulting expression by \( 1/\nu(t) \), the growth rate of aggregate capital productivity is simply:

\[
\nu(t) = \nu_N(t) - D(t) .
\]

Two alternative specifications are of interest in regard to \( \nu_N(t) \):

**Assumption 7a**: Improvements in the efficiency of the new technology due to endogenous, diffusion-dependent changes are Harrod-neutral (denoted \( HaN \)) – they raise \( \pi_N(t) \), but, leave \( \nu_N(t) = 0 \) for all \( t \).

Consequently, Harrod-neutrality in the innovation-using sector implies that

\[

\left[ \nu(t) \mid HaN \right] = -D(t) .
\]

Alternatively, one may entertain

**Assumption 7b**: Improvements in the efficiency of the new technology due to endogenous, diffusion-dependent changes are Hicks-neutral (denoted \( HiN \)), i.e. they result in \( \dot{\nu}_N(t) = \pi_N(t) \) for all \( t \).
Making use of eq. (A.8), Assumption 7b implies:

\[
\hat{\nu}(t) |_{HiN} = \theta \left( \hat{D}(t) \right) - \hat{D}(t) = -\left( 1 - \theta \right) \hat{D}(t)
\]  

(A.16)

For the industry as a whole, where measured aggregate TFP changes (interpreted as ‘efficiency growth’) result from the direct compositional effects of the innovation’s diffusion, and endogenous improvements the productivity of the adopters. Consequently, the variant specifications of the way that “learning” or other externalities of the innovation’s diffusion effects \( \pi_N(t), \nu_N(t) \), will yield different industry-wide growth rates for measured TFP.

A.6 Simulation equations for the aggregate TFP growth rate

Combining the results given by equations (A.11) and (A.15) or (A.16), alternatively, we obtain for the Harrod-neutrality and Hicks-neutrality cases, respectively:

\[
\begin{align*}
\dot{A}(t) |_{Harrod - N} &= \left( \pi_2(t) \right) \omega_L(t) - \left[ 1 - \omega_L(t) \right] D(t) \\
\text{and} \\
\dot{A}(t) |_{Hicks - N} &= \left( \pi_2(t) \right) \omega_L(t) - \left[ 1 - \omega_L(t) \right] (1 - \theta) D(t)
\end{align*}
\]  

(A.17)

Substituting for \( \omega_L(t) \) from equation (3.12), for \( D(t) \) from (3.7b), these expressions may be rewritten in the form:

\[
\begin{align*}
\dot{A}(t) |_{Harrod - N} &= \pi_2(t) \left[ 1 - (1 - \omega_N) D(t) \right] - (\gamma \lambda) (1 - \omega_N) D(t) [1 - D(t)], \\
\text{and} \\
\dot{A}(t) |_{Hicks - N} &= \left[ \dot{A}(t) |_{Harrod - N} \right] + \theta (\gamma \lambda) (1 - \omega_N) [1 - D(t)]
\end{align*}
\]  

(A.19)

From equations (3.18) and (3.9), one readily can find the first-order condition for the peak TFP growth rate, \( d \left( \dot{A}(t) |_{HiN} \right) = 0 \). The positive value of \( D(t)^* \) which satisfies that condition is a function of the four parameters \( \forall, 2, \delta, \omega_N \) and the normalizing constant, \( \theta \). Given \( D^*(t) \), and the parameter \( \Phi \) defined in equation (3.6), it is straightforward to solve for a general expression giving the date \( t^* \) at which the peak growth rate of TFP occurs.
References


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