Maximally Representative Allocations for Display Advertising

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YAHOO! RESEARCH

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Display Advertising

- Graphical ads on webpages
- $24$ billion business (2008)
- Display advertising space can be bought
  - In advance from the publisher
  - In a spot market
Guaranteed contracts: Publisher guarantees to deliver
- Prespecified number of impressions
- Matching targeting constraints
- Over specified time frame
- Eg: 10 million impressions to California males in July 2009

Spot market: Auction for each display ad opportunity
- RightMedia Exchange: over 9 billion auctions every day
Guaranteed contracts
- Allows advertisers to hedge against future uncertainty of supply
- Insures publishers against fluctuations in demand

Spot market
- Allows opportunity-specific bidding
- Fine-grained targeting based on user tracking
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How should a publisher allocate her supply?
Different impressions have different value
  Two users with identical demographics
  Different search behaviors reflecting purchase intent
  Both equally satisfy guaranteed contract
  Different prices on spot market!

Revenue suboptimal to allocate first to guaranteed contracts
Obvious Solution 2 (Obvious Modification of 1)

- Cheapest impressions for guaranteed contracts
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- Poor in terms of fairness
Obvious Solution 2 (Obvious Modification of 1)

- Cheapest impressions for guaranteed contracts
- Poor in terms of *fairness*
- Price is a signal of value
  - Impressions have common value component
  - One advertiser’s information about user is relevant to all other advertisers
  - Price converges to true value in limit [Milgrom, Wilson]
- Cheapest impressions are also lowest quality impressions
Publisher faces trade-off between revenue and fairness

Publisher also lacks access to all information to decide value of impression
  - User visits politics site
  - Amazon (as advertiser) sees user searched Amazon for ipod
  - Target sees user searched for coffee mug
  - Publisher only knows about politics site

Spot market bids are revealed only after impression is placed on market
The Publisher As a Bidder

- Unknown spot market demand
  - Allocate to bidder on spot market if bid is "high enough"
  - Else to guaranteed contract

- Publisher acts as a bidder on behalf of guaranteed contracts

- Two simultaneous roles for publisher
  - Seller placing opportunity on spot market
  - Bidding agent "probing" spot market
How to model trade-off between fairness and revenue?
  - What is a good allocation?

Having chosen trade-off, how to bid on spot market?
  - What bidding strategy implements allocation?
Form of Solution

- Solution has two components
  - Allocation: Fraction of impressions at each price allocated to a contract
  - Bidding strategy: How to acquire allocation by bidding in an auction
Overview: Maximally Representative Allocations

- Quality measured by spot market price
- Perfectly representative allocation: Equal proportion of impressions at every price
- Revenue-fairness trade-off modeled using budget
Overview: Maximally Representative Allocations

- Quality measured by spot market price
- Perfectly representative allocation: Equal proportion of impressions at every price
- Revenue-fairness trade-off modeled using budget
- Maximally representative allocation: Minimize distance to perfectly representative allocation, subject to budget constraint
Overview

- Solving for maximally representative allocations
  - Single contract
  - Multiple contracts
- Implementing maximally representative allocations in an auction
  - Randomized bidding strategies
Single contract for $d$ impressions

Supply $s \geq d$

Bid landscape $F$
- Call highest bid on spot market 'price'
- $F$ is distribution of price

$s, F$ known to publisher

Average target spend $t$
\[ \frac{a(p)}{s} \] is the proportion of opportunities purchased at price \( p \)

- \( sf(p)dp \) impressions available at price \( p \)
- \( a(p) \) buys \( \frac{a(p)}{s} \cdot sf(p)dp = a(p)f(p)dp \) impressions
\( \frac{a(p)}{s} \): Proportion of opportunities purchased at price \( p \)

- \( sf(p)dp \) impressions available at price \( p \)
- \( a(p) \) buys \( \frac{a(p)}{s} \cdot sf(p)dp = a(p)f(p)dp \) impressions

**Proposition:** A right-continuous allocation \( \frac{a(p)}{s} \) can be implemented (in expectation) by bidding in an auction if and only if \( a(p_1) \geq a(p_2) \) for \( p_1 \leq p_2 \).

- Draw bids from distribution \( H(p) := 1 - \frac{a(p)}{s} \)
Given $s, d, t$, maximally representative allocation solves

\[
\inf_{a(\cdot)} \int_{p} u \left( \frac{a(p)}{s}, \frac{d}{s} \right) f(p)dp \\
\text{s.t.} \quad \int_{p} a(p)f(p)dp = d \\
\int_{p} pa(p)f(p)dp \leq td \\
0 \leq \frac{a(p)}{s} \leq 1.
\]
Optimality Conditions

- Optimality conditions:

\[ u'(\frac{a(p)}{s}, \frac{d}{s}) = \lambda_1 - \lambda_2 p + \mu_1(p) - \mu_2(p), \]

- \( \mu_1(p), \mu_2(p) \geq 0 \)

- Proposition: The maximally representative allocation for a single contract can be implemented by bidding in an auction for any convex distance measure \( u \).
\( u(x, y) = (x - y)^2 \)

Proposition: The optimal allocation \( a(p) \) is continuous in \( p \).

Optimal allocation has one of two forms:

- \( a(p)/s = z(p_{\text{max}} - p) \) for \( p \leq p_{\text{max}} \) (and 0 for \( p \geq p_{\text{max}} \))

- \( a(p)/s = 1 \) for \( p \leq p_{\text{min}} \), and \( a(p)/s = \frac{p_{\text{max}} - p}{p_{\text{max}} - p_{\text{min}}} \) for \( p \leq p_{\text{max}} \) (and 0 thenceforth)
Effect of Varying Target Spend

- Vary \( t \) keeping \( d \) fixed
- At minimum feasible \( t \), \( p_{\text{min}} = p_{\text{max}} = F^{-1}(\frac{d}{s}) \)
- As \( t \) increases, \((1) \rightarrow (2) \rightarrow \frac{d}{s}\)

Figure 1: Effect of target spend on \( L_2 \) optimal allocation

The value of \( t \) provides a dial by which to move from the "cheap" allocation to the perfectly representative allocation. Figure 1 illustrates the effect of varying target spend on the optimal allocation.
Theorem: The optimal allocation for the $L_2$ distance measure can be implemented (in expectation) in an auction by the following random strategy: toss a coin to decide whether or not to bid, and if bidding, draw the bid from a uniform distribution.

- Optimal allocation is $\frac{a(p)}{s} = \min\{1, z(p_{\max} - p)\}$
- Bid with probability $\min\{zp_{\max}, 1\}$
- Draw bid UAR from $[\max\{p_{\max} - \frac{1}{z}, 0\}, p_{\max}]$
Multiple Contracts

- $m$ advertisers with demands $d_j$
- Supply $s \geq \sum d_j$

*Centralized* bidding strategy
  - Publisher submits one bid on behalf of all contracts
  - If bid wins, chooses winning contract

*Decentralized* bidding strategy
  - One bid for each contract $j$
Decentralizable Allocations

- Need to choose winner prior to seeing price
  - Automatically happens with decentralized strategies
  - Centralized strategy: $\frac{a_i(p)}{a_j(p)}$ independent of $p$
  - Such allocations are also decentralizable

- Theorem: A set of allocations $a_j(p)$ can be implemented in an auction via a decentralized strategy if and only if each $a_j(p)$ is non-increasing in $p$, and $\sum_j a_j(p)/s \leq 1$.
  - Bids compete: Not enough to draw bids from $1 - \frac{a_j(p)}{s}$!
Optimal allocations $\frac{a_j(p)}{s}$ solve

$$\min \frac{s}{2} \sum_{j=1}^m \int_p \left( \frac{a_j(p)}{s} - \frac{d_j}{s} \right)^2 f(p) dp$$

s.t. $$\int_p a_j(p) f(p) dp = d_j \quad \forall j$$
$$\int_p pa_j(p) f(p) dp \leq t_j d \quad \forall j$$
$$a_j(p) \geq 0 \quad \forall p, j$$
$$a_j(p) \geq 0 \quad \forall p, j$$
$$\sum_{j=1}^m a_j(p) \leq s \quad \forall p.$$
Optimal allocation decentralizable if target spends are such that
- Solutions decouple: Solve separately for each contract
- \( \frac{a_j(p)}{a_k(p)} \) is independent of \( p \): Solve for common slope, \( p_{\text{min}} \), and contract specific values \( p_{\text{max}}^j \)
Conclusion

Summary
- Moving guaranteed contracts into exchange environment
- Randomized bidding trades off cost and quality

Further directions
- Unknown or stochastic supply
- Strategic response to randomized bidding strategies