Maximally Representative Allocations for Display Advertising

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YAHOO! RESEARCH

Joint work with

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Display Advertising

- Graphical ads on webpages
- $24 billion business (2008)
- Display advertising space can be bought
  - In advance from the publisher
  - In a spot market

- Guaranteed contracts: Publisher guarantees to deliver
  - Prespecifed number of impressions
  - Matching targeting constraints
  - Over specified time frame
  - Eg: 10 million impressions to California males in July 2009
- Spot market: Auction for each display ad opportunity
  - RightMedia Exchange: over 9 billion auctions every day

Marketplaces for Display

- Guaranteed contracts
  - Allows advertisers to hedge against future uncertainty of supply
  - Insures publishers against fluctuations in demand
- Spot market
  - Allows opportunity-specific bidding
  - Fine-grained targeting based on user tracking

Marketplaces for Display

- Guaranteed contracts
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- Spot market
  - Allows opportunity-specific bidding
  - Fine-grained targeting based on user tracking
- How should a publisher allocate her supply?

Obvious Solution 1: Guaranteed Contracts First

- Different impressions have different value
  - Two users with identical demographics
  - Different search behaviors reflecting purchase intent
  - Both equally satisfy guaranteed contract
  - Different prices on spot market!
- Revenue suboptimal to allocate first to guaranteed contracts

Obvious Solution 2 (Obvious Modification of 1)

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Obvious Solution 2 (Obvious Modification of 1)

- Cheapest impressions for guaranteed contracts
- Poor in terms of *fairness*
- Price is a signal of value
  - Impressions have common value component
  - One advertiser's information about user is relevant to all other advertisers
  - Price converges to true value in limit [Milgrom, Wilson]
- Cheapest impressions are also lowest quality impressions

The Publisher's Perspective

- Publisher faces trade-off between revenue and fairness
- Publisher also lacks access to all information to decide value of impression
  - User visits politics site
  - Amazon (as advertiser) sees user searched Amazon for ipod
  - Target sees user searched for coffee mug
  - Publisher only knows about politics site
- Spot market bids are revealed only after impression is placed on market

The Publisher As a Bidder

- Unknown spot market demand
  - Allocate to bidder on spot market if bid is "high enough"
  - Else to guaranteed contract
- Publisher acts as a bidder on behalf of guaranteed contracts
- Two simultaneous roles for publisher
  - Seller placing opportunity on spot market
  - Bidding agent "probing" spot market

The Publisher's Problem

- How to model trade-off between fairness and revenue?
  - What is a good allocation?
- Having chosen trade-off, how to bid on spot market?
  - What bidding strategy implements allocation?

Form of Solution

- Solution has two components
  - Allocation: Fraction of impressions at each price allocated to a contract
  - Bidding strategy: How to acquire allocation by bidding in an auction

- Quality measured by spot market price
- Perfectly representative allocation: Equal proportion of impressions at every price
- Revenue-fairness trade-off modeled using budget

Overview: Maximally Representative Allocations

- Quality measured by spot market price
- Perfectly representative allocation: Equal proportion of impressions at every price
- Revenue-fairness trade-off modeled using budget
- Maximally representative allocation: Minimize distance to perfectly representative allocation, subject to budget constraint

Overview

- Solving for maximally representative allocations
  - Single contract
  - Multiple contracts
- Implementing maximally representative allocations in an auction
  - Randomized bidding strategies

Model

- Single contract for d impressions
- Supply $s \geq d$
- *Bid landscape F*
  - Call highest bid on spot market 'price'
  - F is distribution of price
- s, F known to publisher
- Average target spend t

Allocation

- $\frac{a(p)}{s}$: Proportion of opportunities purchased at price p
  - $sf(p)dp$ impressions available at price p
  - $a(p)$ buys $\frac{a(p)}{s} \cdot sf(p)dp = a(p)f(p)dp$ impressions

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- Proposition: A right-continuous allocation $\frac{a(p)}{s}$ can be implemented (in expectation) by bidding in an auction if and only if $a(p_1) \geq a(p_2)$ for $p_1 \leq p_2$.
  - Draw bids from distribution $H(p) := 1 - \frac{a(p)}{s}$

Maximally Representative Allocation

- Given s, d, t, maximally representative allocation solves

$$
\begin{aligned}
\inf_{a(\cdot)} \quad & \int_p \mathbf{u}\left(\frac{a(p)}{s}, \frac{d}{s}\right) f(p)dp \\
\text{s.t.} \quad & \int_p a(p)f(p)dp = d \\
& \int_p pa(p)f(p)dp \leq td \\
& 0 \leq \frac{a(p)}{s} \leq 1.
\end{aligned}
$$

- Optimality conditions:

$$\mathbf{u}'\left(\frac{a(p)}{s}, \frac{d}{s}\right) = \lambda_1 - \lambda_2 p + \mu_1(p) - \mu_2(p),$$

  - $\mu_1(p), \mu_2(p) \geq 0$

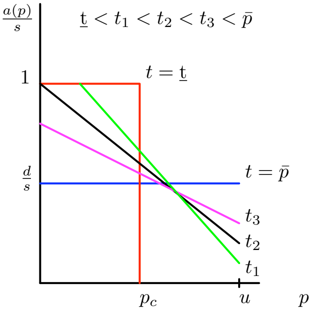
- Proposition: The maximally representative allocation for a single contract can be implemented by bidding in an auction for any *convex* distance measure **u**.

L_2 utility

- $u(x, y) = (x - y)^2$
- Proposition: The optimal allocation $a(p)$ is continuous in p.
- Optimal allocation has one of two forms:
  - $a(p)/s = z(p_{max} - p)$ for $p \leq p_{\max}$ (and 0 for $p \geq p_{\max}$)
  - $a(p)/s = 1$ for $p \leq p_{\min}$, and $a(p)/s = \frac{p_{\max} - p}{p_{\max} - p_{\min}}$ for $p \leq p_{\max}$ (and 0 thenceforth)

Effect of Varying Target Spend

- Vary t keeping d fixed
- At minimum feasible t, $p_{\text{min}} = p_{\text{max}} = F^{-1}(\frac{d}{s})$
- As t increases, (1) $\rightarrow$ (2) $\rightarrow$ d/s



Randomized Bidding Strategies

- Theorem: The optimal allocation for the L_2 distance measure can be implemented (in expectation) in an auction by the following random strategy: toss a coin to decide whether or not to bid, and if bidding, draw the bid from a uniform distribution.
  - Optimal allocation is $\frac{a(p)}{s} = \min\{1, z(p_{max} - p)\}$
  - Bid with probability $\min\{zp_{\max}, 1\}$
  - Draw bid UAR from $[\max\{p_{\max} - \frac{1}{z}, 0\}, p_{\max}]$

Multiple Contracts

- m advertisers with demands d_j
- Supply $s \geq \sum d_j$
- *Centralized* bidding strategy
  - Publisher submits one bid on behalf of all contracts
  - If bid wins, chooses winning contract
- *Decentralized* bidding strategy
  - One bid for each contract j

Decentralizable Allocations

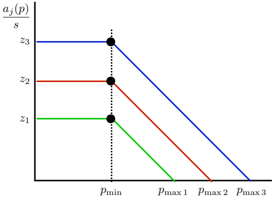
- Need to choose winner prior to seeing price
  - Automatically happens with decentralized strategies
  - Centralized strategy: $\frac{a_i(p)}{a_j(p)}$ independent of p
  - Such allocations are also decentralizable
- Theorem: A set of allocations $a_j(p)$ can be implemented in an auction via a decentralized strategy if and only if each $a_j(p)$ is non-increasing in p, and $\sum_j a_j(p)/s \leq 1$.
  - Bids compete: Not enough to draw bids from $1 - \frac{a_j(p)}{s}$!

- Optimal allocations $\frac{a_j(p)}{s}$ solve

$$
\begin{aligned}
\min \quad & \frac{s}{2}\sum_{j=1}^{m}\int_p \left(\frac{a_j(p)}{s} - \frac{d_j}{s}\right)^2 f(p)dp \\
\text{s.t.} \quad & \int_p a_j(p)f(p)dp = d_j \qquad \forall j \\
& \int_p p a_j(p)f(p)dp \leq t_j d \qquad \forall j \\
& a_j(p) \geq 0 \qquad \forall p, j \\
& \sum_{j=1}^{m} a_j(p) \leq s \qquad \forall p.
\end{aligned}
$$

Decentralizable Optimal Allocations

- Optimal allocation decentralizable if target spends are such that
  - Solutions decouple: Solve separately for each contract
  - $\frac{a_j(p)}{a_k(p)}$ is independent of p: Solve for common slope, $p_{\min}$, and contract specific values $p^j_{\max}$



Conclusion

- Summary
  - Moving guaranteed contracts into exchange environment
  - Randomized bidding trades off cost and quality
- Further directions
  - Unknown or stochastic supply
  - Strategic response to randomized bidding strategies