Stochastic Taxation and Asset Pricing in Dynamic General Equilibrium

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Abstract

Tax rates have fluctuated considerably since federal income taxes were introduced in the United States in 1913. This paper analyzes the effects of stochastic taxation on asset prices in a dynamic general equilibrium model. Stochastic taxation affects the after-tax returns of both risky and safe assets. Whenever taxes change, bond and equity prices adjust to clear the asset markets. These price adjustments affect assets with long durations, such as equities and long-term bonds, more than short-term assets. Under plausible conditions, investors require higher equity and term premia as compensation for the risk introduced by stochastic taxes.

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1 Introduction

One of the few certain forecasts about the tax system is that it will change. Since federal income taxes were introduced in 1913, the tax system of the United States has been reformed several times. In particular, marginal income tax rates have fluctuated considerably over this period, as depicted in Figure 1, which shows the federal marginal income tax rates for individuals in five different tax brackets.\(^1\) On top of marginal tax rate changes, other provisions of the tax code also changed, adding to the overall uncertainty of tax law.\(^2\) The recent policy discussion about tax reform makes it likely that the tax system will be modified in the near future. This paper examines the effects of stochastic taxation on asset returns.

From 1926 to 1999, the average annual real return of stocks in the United States was 9.99 percent. The corresponding real return of Treasury bills with a maturity of thirty days equaled only 0.77 percent. This implies an average equity premium of 9.22 percent (where the equity premium is defined as the difference between average returns of equities and short-term Treasury securities). The standard deviation of annual real stock returns during the same time period was 20.30 percent.\(^3\)

Conventional asset pricing models cannot generate a return distribution with these observed characteristics. The literature has focused on three related puzzles: Mehra and Prescott (1985) show that extremely high levels of risk aversion are necessary to explain the large equity premium (equity premium puzzle).\(^4\) Weil (1989) demonstrates that the risk-free rate increases dramatically at higher levels of risk aversion (risk-free rate puzzle). And Shiller (1981) argues that stock prices tend to be more volatile than the underlying uncertainty in the economy (excess volatility puzzle).\(^5\)

I investigate how time-varying tax policies affect asset prices and whether they introduce an additional risk factor in the economy, which increases equity and term premia. This paper generalizes the Lucas (1978) asset pricing model by introducing a flat consumption tax which follows a two-

\(^1\) A detailed description of the data is given in Appendix A.1.
\(^2\) Pechman (1985) traces the changes in the effective distribution of tax burdens over the period from 1966 to 1985. Tax burdens increased in the lower part of the income scale, declined sharply at the top, and remained roughly the same or rose slightly in between. He shows that effective taxes varied less than marginal tax rates.
\(^3\) The monthly Treasury securities returns, the stock returns, and the rate of inflation are taken from Ibbotson Associates (2000).
\(^4\) Hansen and Singleton (1983) and Hansen and Jagannathan (1991) provide an alternative illustration of the equity premium puzzle.
\(^5\) Kocherlakota (1996) and Campbell (1999) are recent surveys of this literature.
state Markov chain. Stochastic taxation differs qualitatively from other modifications of conventional asset pricing models in that it affects both safe and risky assets in a symmetric way. Consequently, this paper does not merely change the uncertainty associated with equities. This is the main difference from Rietz (1988), who shows that introducing unlikely but severe market crashes increases the equity premium. The crashes in Rietz’s model only affect the payoffs of equities but leave the payoffs of bonds unchanged.

The main result of the paper is that the effect of stochastic taxes on the risk premium increases with the duration of the assets. Whenever taxes change, asset prices adjust instantaneously to clear asset markets, and these price changes increase the variability of asset returns. The price adjustments are more severe for assets with long durations, such as equities and long-term bonds, than for assets with shorter durations. Individuals require higher expected returns for holding the assets with more severe price changes under identifiable and plausible conditions. Hence, equity securities tend to pay on average higher returns than short-term bonds.

Stochastic taxes have three effects on asset prices. First, they change the level of disposable income over time (income effect). Frequent tax changes increase the variability of consumption for a given production process. A higher variability of consumption significantly affects asset prices and leads to a higher equity premium, as previously shown in the asset pricing literature. Second, time-varying tax rates distort the relative price of consumption over time and affect the incentives to save and invest (substitution effect). Even if all the tax revenues are rebated to taxpayers and the consumption process remains completely unaffected by tax changes, stochastic taxes affect asset prices and the equity premium. This tax distortion has not been previously discussed in the equity premium literature. Third, taxes can influence the rate of growth of the economy and thereby affect asset prices (growth effect). Some tax regimes might be more conducive to economic growth than others.

A numerical example demonstrates that, for reasonable parameter values, stochastic taxation can by itself account for the return premium of equities over short-term bonds, without implying implausible returns of short-term bonds. The effects of stochastic taxation on the equity premium are significant even if all the tax revenues are rebated to the taxpayers.

Introducing heterogeneity does not change the main conclusions of this paper. In the heterogeneous model, the government decreases income inequality by imposing net taxes on the wealthy and paying net transfers to the poor. This policy affects the savings decisions and the portfolio choices of the individuals and changes asset prices and the equity premium in equilib-
rium. Tax rate changes benefit some individuals and harm others. People with diverging interests can use the available assets to insure each other against tax changes.

This paper is related to a vast literature in financial economics that addresses the equity premium, the risk-free rate, and the excess volatility puzzles and to a large literature in public economics that analyzes the effects of taxes on saving decisions and portfolio choice. The papers in the finance literature can be divided into four major groups. The first group of papers generalizes the preferences and the expectations of individuals. Epstein and Zin (1989) introduce ‘Generalized Expected Utility’ preferences that distinguish between risk aversion and the elasticity of intertemporal substitution. Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) show that adding habit formation to intertemporal preferences helps explain the asset puzzles. My model does not change the preferences and expectations of individuals and assumes that the utility function is time-separable and that expectations are rational.

The second group introduces incomplete markets. Mankiw (1986), Constantinides and Duffie (1996), Heaton and Lucas (1996), Storesletten, Telmer, and Yaron (1999), and Constantinides, Donaldson, and Mehra (2000) among others argue that individual income shocks cannot be diversified completely and that individual consumption growth is more variable than aggregate consumption growth. The first part of my paper uses a representative agent model, whereas the second part introduces heterogeneous agents and incomplete markets.

The third group adds frictions to the economy. Mankiw and Zeldes (1991) and Vissing-Jørgensen (1999) postulate that because of high trading and transaction costs only a relatively small proportion of individuals hold stocks, and that the consumption variability of stockholders is higher than that for non-stockholders. My paper adds frictions in the form of stochastic taxes, which affect the consumption process, distort consumption prices, and change the distribution of the growth rate of the economy.

The fourth group states that the data used to compute historical equity premia is biased. Rietz (1988) argues that the recent history in the U.S. excludes severe market crashes. He shows that such extreme events have a significant impact on asset pricing even if they occur with very low probability. Jorion and Goetzmann (1999) demonstrate that the U.S. equity premium is subject to survivorship bias, as the U.S. was the most successful capitalist system in the twentieth century. They show that most foreign equity markets had considerably lower appreciations than the U.S.

The papers in public economics analyze the effects of taxes on saving

A few papers discuss the effects of uncertain future taxes on the economy. Auerbach and Hines (1988) analyze the pattern of U.S. corporate investment incentives over the period between 1953 and 1986, incorporating the feature that investors are aware that next year’s tax code may not be the same as this year’s. Bizer and Judd (1989) present a dynamic general equilibrium model where taxpayers understand the uncertainty in tax policy when making their investment decisions. The impact of tax policy uncertainty on firm-level and aggregate investment is estimated by Hassett and Metcalf (1999), while Slemrod and Greimel (1999) investigate the relationship of the daily spread between taxable and municipal securities to a daily indicator of the likelihood of a tax reform that would eliminate the preference accorded to municipal securities. In a related paper (Sialm 2000), I show that tax reforms have an economically and statistically significant effect on asset returns. The impact of tax changes on asset prices is more pronounced for assets with long durations, such as stocks and long-term bonds, and less important for assets with shorter durations.

This current paper extends the literature in financial economics and the literature in public economics by analyzing the impact of frequent tax reforms on asset returns in a general equilibrium model. The effects of uncertain taxes on the distribution of asset returns and on the equity premium have not been previously studied.

The remainder of the paper is divided into five sections. The next section describes a simple representative agent model. Section 3 derives closed form solutions for asset returns in an environment with frequent tax reforms. Section 4 presents a numerical example where tax rate changes increase the equity premium, decrease the risk-free rate, and increase the variability of expected and actual asset returns under plausible parameter values. Section 5 introduces heterogeneous individuals who differ in their wealth levels. Here, the optimal portfolio choices and equilibrium asset returns are computed numerically, because this model cannot be solved analytically. Section 6 summarizes the main results and discusses possible extensions of this paper.
2 The Model

This paper generalizes the Lucas (1978) representative agent asset pricing model by introducing a flat consumption tax which follows a two-state Markov chain.\footnote{Other early works on consumption-based asset pricing models are LeRoy (1973) and Breeden (1979). Duffie (1996) and Ljungqvist and Sargent (2000) explain asset pricing theories. See Campbell, Lo, and MacKinlay (1997) for a summary of recent work on empirical implementations.} This subsection describes the technology, the tax system, the preferences of the representative agent, and the equilibrium conditions.

2.1 Technology

The output in the exchange economy is exogenous and perishable. Aggregate output $y > 0$ follows a geometric random walk with drift, where $z_t$ denotes its stochastic growth rate. Output growth $z_{t+1}$ has a mean of $\mu_t$ and a standard deviation of $\sigma_t$. The moments of the growth rate can be time-dependent. The distribution of the growth rate is known one period in advance.

$$y_{t+1} = y_t \exp(z_{t+1}), \quad z_{t+1} \sim N(\mu_t, \sigma_t^2).$$ \hspace{1cm} (1)

Two asset classes are traded in this economy: risk-free zero-coupon bonds with different maturities (B) and one risky equity security (S). A zero-coupon bond with remaining maturity $m$ pays a dividend of $d_t^{B,m} = 1$ if $m = 0$ and $d_t^{B,m} = 0$ otherwise. Each individual can issue or buy these bonds. There is no net aggregate supply of any bonds.

The equity security corresponds to the market portfolio and pays a dividend of $d_t^S = y_t$ at the beginning of each period $t$.\footnote{This paper follows the equity premium literature by assuming that equity is a claim on the aggregate resources of the economy. In reality, stocks are a levered claim on the resources and exhibit considerably more risk. The return premium of an asset increases with its leverage (Modigliani and Miller 1958). Thus, this assumption understates the actual premium of levered assets.} The prices of the two asset classes $p_t^S$ and $p_t^{B,m}$ are defined ‘ex-dividend’. The dividend and the price vectors of the assets at time $t$ are abbreviated with $d_t \in \mathbb{R}_t^1$ and $p_t \in \mathbb{R}_t^n$, where $n$ denotes the total number of assets traded in the economy ($n - 1$ bonds and 1 equity security). Assets can be traded without incurring any transaction costs and investors face no borrowing or short-selling constraints.
2.2 Tax System

The government imposes a flat consumption tax and all assets face the same effective tax rates.\textsuperscript{9} Future tax rates $\tau$ are stochastic and follow a two-state Markov-Chain with $0 \leq \tau_L \leq \tau_H < 1$. The transition probabilities $\phi_{ij}$ between the two states are defined as:

$$\phi_{ij} = \text{Prob}(\tau_{t+1} = \tau_j | \tau_t = \tau_i).$$

(2)

Time-varying tax rates may reflect unpredictable changes in the balance of power among different groups of taxpayers. This paper makes no attempt to explain the political process which generates frequent tax reforms.\textsuperscript{10} The tax revenues are exactly identical to the outlays of the government. The government uses a fixed proportion $\omega \in [0, 1]$ of the aggregate tax revenues $T_t$ to finance a public good $g_t = \omega T_t$ and rebates the remaining resources to the individuals as a lump-sum payment which can be used to purchase the aggregate consumption good without incurring any additional taxes.

The growth rate of the output and the dividends $z_{t+1} = \ln d_{t+1}^S - \ln d_t^S$ can depend on the current tax rate $\tau_t$ to allow for the possibility of distortionary taxes.\textsuperscript{11} The mean and the variance of the growth rate $z_{t+1}$ are $\mu_L$ and $\sigma_L^2$ if $\tau_t = \tau_L$ and $\mu_H$ and $\sigma_H^2$ if $\tau_t = \tau_H$.

2.3 Utility

The representative consumer purchases the $n$ available assets in quantities $x_t \in \mathbb{R}^n$ to maximize expected life-time utility. Utility is time-separable and

\textsuperscript{9}The current tax system in the United States is a progressive income tax where some sources of income are tax-exempt or tax-deferred. In addition, states usually impose sales taxes on consumption goods. A flat consumption tax simplifies the algebra in this paper significantly and enables the derivation of closed form solutions. Shoven and Sialm (2000) compute optimal portfolio choices for individuals holding assets taxed at different effective tax rates in tax-deferred and taxable savings accounts.

\textsuperscript{10}Bassetto (1999) shows that tax changes can be optimal in a model where the tax liabilities are unevenly spread in the population and where the government has a desire to redistribute wealth between different groups of individuals.

\textsuperscript{11}The growth rate of dividends $z_{t+1}$ is independent of next period’s tax rate $\tau_{t+1}$. This is a natural assumption because the tax rate $\tau_{t+1}$ (which is announced at the beginning of period $t+1$) cannot affect the dividend $d_{t+1}^S$ (which accumulated during the period $t$ and is paid to shareholders at the beginning of period $t+1$).

An alternative assumption would be that the government pre-announces the tax rates one year in advance. This complicates the analysis by replacing the two states of the Markov chain with four states, because then the current tax regime depends not only on the current tax rate but also on the pre-announced future tax rate. The equity premium in the base case is actually \textit{higher} if the tax rates are pre-announced.
the period utility is separable in the private consumption good $c$ and the public good $g$. The coefficient $\omega$ is a measure of the separability between the private consumption good and the outlays of the government. If $\omega = 0$ (no separability), then all the tax revenues are rebated to the taxpayers and the outlays of the government and the private consumption good are perfect substitutes. If $\omega = 1$ (full separability), then all the tax revenues are used to finance the public good. The asset pricing results with full separability are identical to the case where the government throws away the tax revenues.

The discount factor is denoted by $\beta \in (0,1)$. The period utilities of the two goods are denoted by $u(c)$ and $v(g)$, where $u'(c) > 0$ and $u''(c) \leq 0$.

The consumer’s problem is to maximize:

$$E_t \sum_{i=0}^{\infty} \beta^i (u(c_{t+i}) + v(g_{t+i})),$$  \hspace{1cm} (3)

where:

$$c_t = (1 - \tau_t)[(p_t + d_t)'x_{t-1} - p'_t x_t] + (1 - \omega)T_t,$$  \hspace{1cm} (4)

$$g_t = \omega T_t.$$  \hspace{1cm} (5)

Consumers are assumed to have a power-utility function with a coefficient of relative risk aversion $\alpha \in [0, \infty)$.

$$u(c_t) = \frac{c_t^{1-\alpha} - 1}{1 - \alpha}.$$  \hspace{1cm} (6)

Individuals with $\alpha = 0$ are risk-neutral and individuals with $\alpha = 1$ have logarithmic-preferences $u(c_t) = \ln(c_t)$. The risk aversion coefficient equals the reciprocal of the elasticity of intertemporal substitution.

The first-order conditions for the two asset classes are:

$$p_t^{B,m} u'(c_t)(1 - \tau_t) = \beta^m E_t (u'(c_{t+m})(1 - \tau_{t+m})), \hspace{1cm} (7)$$

$$p_t^{S} u'(c_t)(1 - \tau_t) = \beta E_t (u'(c_{t+1})(1 - \tau_{t+1})(p_{t+1}^S + d_{t+1}^S)). \hspace{1cm} (8)$$

An optimal solution to the agent’s maximization problem must also satisfy the following transversality conditions:

$$\lim_{i \to \infty} \beta^i E_t \left( u'(c_{t+i})(1 - \tau_{t+i})p_{t+i}^{B,m}x_{t+i}^{B,m} \right) = 0,$$  \hspace{1cm} (9)

$$\lim_{i \to \infty} \beta^i E_t \left( u'(c_{t+i})(1 - \tau_{t+i})p_{t+i}^{S}x_{t+i}^{S} \right) = 0. \hspace{1cm} (10)$$
2.4 Market Equilibrium

For market equilibrium, the quantities of each asset demanded must equal the exogenous supply. The zero-coupon bonds have zero aggregate supply and the aggregate supply of equity is normalized to one. In equilibrium, the tax revenues, the consumption of the representative agent, and the provision of the publicly provided good amount to:

\[
T_t = \tau_t d_t^S, \quad (11)
\]

\[
c_t = (1 - \tau_t)d_t^S + (1 - \omega)\tau_t d_t^S = (1 - \omega \tau_t) d_t^S, \quad (12)
\]

\[
g_t = \omega T_t = \omega \tau_t d_t^S. \quad (13)
\]

The (ex-post) marginal rate of intertemporal substitution of the private good equals in equilibrium:

\[
\frac{u'(c_{t+i})}{u'(c_t)} = \left( \frac{c_{t+i}}{c_t} \right)^{-\alpha} = \left( \frac{1 - \omega \tau_{t+i} d_{t+i}^S}{1 - \omega \tau_t d_t^S} \right)^{-\alpha}. \quad (14)
\]

The first-order conditions (7) and (8) give the relationship determining the price of the two assets. A zero-coupon bond with maturity \(m\) and unit face value should trade in equilibrium at the following price:

\[
p_{t}^{B,m} = \beta^m E_t \left( \frac{u'(c_{t+m})}{u'(c_t)} \frac{1 - \tau_{t+m}}{1 - \tau_t} \right). \quad (15)
\]

The price of the risky asset can be expressed in the following way if the transversality condition holds.

\[
p_t^S = \sum_{i=1}^{\infty} E_t \left( \beta^i \frac{u'(c_{t+i})}{u'(c_t)} \frac{1 - \tau_{t+i}}{1 - \tau_t} d_{t+i}^S \right), \quad (16)
\]

\[
= d_t^S \sum_{i=1}^{\infty} E_t \left( \beta^i \frac{1 - \tau_{t+i}}{1 - \tau_t} \left( \frac{1 - \omega \tau_{t+i}}{1 - \omega \tau_t} \right)^{-\alpha} \left( \frac{d_{t+i}^S}{d_t^S} \right)^{1-\alpha} \right),
\]

\[
= d_t^S \delta_t.
\]

The current tax regime determines the probability distribution of future tax rates and of future growth rates \(z_{t+i} = \ln d_{t+i}^S - \ln d_{t+i-1}^S\). The price-dividend ratio \(\delta_t\) and the price of the bond \(p_t^{B,m}\) do therefore not depend on the level of the equity dividends \(d_t^S\). They only depend on the current tax regime and the maturity \(m\) of the bonds.
Taxes affect asset prices over three mechanisms. First, tax rates influence the consumption levels if some portion of the tax revenues are used to fund the public good (i.e., $\omega > 0$). This affects the marginal rate of intertemporal substitution in equation (14). Tax changes result in a higher variability of consumption. Second, taxes distort the price of the private consumption good in different periods. Thus equations (15) and (16) include the ratio of the tax rates. Third, taxes influence the distribution of the growth rate of the economy.

A flat consumption tax has no effect on asset returns if the tax rate does not vary over time ($\tau_L = \tau_H$ or $\phi_{HH} = \phi_{LL} = 1$). The tax terms in the marginal rate of intertemporal substitution in equation (14) cancel if $\tau_t = \tau_{t+1}$. The tax factors in the pricing equations (15) and (16) also cancel. In this case, taxes do not influence the distribution of the growth rate of the economy either. A constant tax decreases consumption in all time periods by the same proportion and does not affect the marginal rate of intertemporal substitution if the utility function has a constant coefficient of relative risk aversion.

3 Results

This section discusses the main results of the representative agent model. I derive the prices of equity and bond securities and the equilibrium equity premium. Section 4 solves the model for plausible parameter values and shows that the effects of tax regime changes are economically significant.

3.1 Equity Valuation

The price-dividend ratio of equity is denoted by $\delta_t = \frac{d_t^S}{d_t^S}$. The first-order condition (8) can be expressed as:

$$\delta_t = \frac{p_t^S}{d_t^S} = \beta E_t \left[ u'(c_{t+1}) \frac{1 - \tau_{t+1} d_{t+1}^S}{1 - \tau_t d_t^S} (1 + \delta_{t+1}) \right],$$

$$= \beta E_t \left[ \left( \frac{d_{t+1}^S}{d_t^S} \right)^{1-\alpha} \left( \frac{1 - \tau_{t+1}}{1 - \tau_t} \left( \frac{1 - \omega \tau_{t+1}}{1 - \omega \tau_t} \right)^{-\alpha} \right) (1 + \delta_{t+1}) \right],$$

$$= \beta E_t \left[ \left( \frac{d_{t+1}^S}{d_t^S} \right)^{1-\alpha} E_t \left[ \frac{1 - \tau_{t+1}}{1 - \tau_t} \left( \frac{1 - \omega \tau_{t+1}}{1 - \omega \tau_t} \right)^{-\alpha} (1 + \delta_{t+1}) \right] \right].$$

The last equality follows from the independence of the dividend growth rate $z_{t+1}$ and next period’s tax rate $\tau_{t+1}$ and the price-dividend ratio $\delta_{t+1}$. 

9
As seen in Section 2.4, the price-dividend ratio does only depend on the current tax regime. The growth rate of the dividend depends by assumption only on the tax rate at time $t$ and not on uncertain future tax rates. The first factor is determined by the dividend-process and is denoted by

$$\gamma_t = \beta E_t(\frac{d_{t+1}^S}{d_t^S})^{1-\alpha} = \beta \exp\left((1-\alpha)\mu_t + 0.5(1-\alpha)^2\sigma_t^2\right).$$

This factor depends on the current tax rate ($\gamma_t = \gamma_L$ if $\tau_t = \tau_L$ and $\gamma_t = \gamma_H$ otherwise).

The second factor is determined by the tax-process and depends on the future price-dividend ratio and current and future tax rates. I use the following abbreviation:

$$\rho_{t,t+1} = \frac{1 - \tau_{t+1}}{1 - \tau_t} \left(\frac{1 - \omega\tau_{t+1}}{1 - \omega\tau_t}\right)^{-\alpha} > 0. \quad (18)$$

The tax pricing factor $\rho_{t,t+1}$ equals one if the tax rates do not change ($\rho_{HH} = \rho_{LL} = 1$). It is defined as $\rho_{HL}$ if the tax rates increase and as $\rho_{HL}$ if they decrease. Note that $\rho_{LH}$ is the reciprocal of $\rho_{HL}$. The expected value of $\rho_{t,t+1}$ equals $\rho_H = \phi_{HH} + \phi_{LH}\rho_{HL}$ in the high-tax state and $\rho_L = \phi_{LL} + \phi_{LH}\rho_{LH}$ in the low-tax state. The price-dividend ratio depends only on the current tax regime and is denoted by $\delta_t = \delta_H$ if $\tau_t = \tau_H$ and $\delta_t = \delta_L$ otherwise.

The price-dividend ratios in the two states are implicitly given by the following system of linear equations:

$$\delta_H = \gamma_H \left[\phi_{HH}(1 + \delta_H) + \phi_{LH}\rho_{HL}(1 + \delta_L)\right], \quad (19)$$

$$\delta_L = \gamma_L \left[\phi_{LL}(1 + \delta_L) + \phi_{LH}\rho_{LH}(1 + \delta_H)\right]. \quad (20)$$

Solving the system of linear equations for the two price-dividend ratios yields:

$$\delta_H = \frac{\gamma_H [\rho_H + \gamma_L(1 - \phi_{HH} - \phi_{LL})]}{1 - [\phi_{HH}\gamma_H + \phi_{LL}\gamma_L + \gamma_H\gamma_L(1 - \phi_{HH} - \phi_{LL})]}, \quad (21)$$

$$\delta_L = \frac{\gamma_L [\rho_L + \gamma_H(1 - \phi_{HH} - \phi_{LL})]}{1 - [\phi_{HH}\gamma_H + \phi_{LL}\gamma_L + \gamma_H\gamma_L(1 - \phi_{HH} - \phi_{LL})]]. \quad (22)$$

To ensure that the transversality condition (10) holds, I assume that $0 < \gamma_i < 1$ for $i \in \{L, H\}$. In this case, the price-dividend ratios in equations (21) and (22) are positive as shown in Appendix B.1.

The following proposition shows the relationship between the tax regime and the price-dividend ratio of equity securities in the general case (part i) and in the special case where the dividend growth rate is independent of the current tax rate (part ii).
Proposition 1  (i) The price-dividend ratio is higher in the high-tax regime if $\alpha$ is in set $A = \{ \alpha \in \mathbb{R}_+ : \gamma_H(\alpha) \rho_H(\alpha) \geq \gamma_L(\alpha) \rho_L(\alpha) \}$, and the price-dividend ratio is lower in the high-tax regime if $\alpha$ is in set $\bar{A} = \{ \alpha \in \mathbb{R}_+ : \gamma_H(\alpha) \rho_H(\alpha) < \gamma_L(\alpha) \rho_L(\alpha) \}$.

(ii) If the dividend process is independent of the tax process (i.e., $\mu_L = \mu_H$ and $\sigma_L = \sigma_H$), then the price-dividend ratio is higher in the high-tax regime if $\alpha < \tilde{\alpha}$, the price-dividend ratio is lower in the high-tax regime if $\alpha > \tilde{\alpha}$, and the price-dividend ratio is identical in the two regimes if $\alpha = \tilde{\alpha}$. The critical risk aversion $\tilde{\alpha}$ is given by:

$$\tilde{\alpha} = \frac{\ln(1 - \tau_L) - \ln(1 - \tau_H)}{\ln(1 - \omega \tau_L) - \ln(1 - \omega \tau_H)}.$$  \(23\)

Proof: All proofs can be found in Appendix B. \(\Box\)

If tax rates are not expected to change over time (i.e., $\tau_H = \tau_L$ or $\phi_{HH} = \phi_{LL} = 1$), then the price-dividend ratios are equal in the two states (i.e., $\delta_H = \delta_L = \gamma_H/(1 - \gamma_H) = \gamma_L/(1 - \gamma_L)$). This valuation corresponds to a conventional asset pricing model without tax changes.

The special case where the dividend process does not depend on the current tax regime (part ii) is discussed first. The critical risk aversion level where the price-dividend ratio is identical in the two tax regimes is denoted by $\tilde{\alpha}$. The coefficient $\tilde{\alpha}$ is larger than 1 and decreasing in $\omega$. $\tilde{\alpha}$ is defined for all $\omega \in (0, 1]$. If all the tax revenues are rebated to the tax payers as a lump-sum distribution ($\omega = 0$), then $\tilde{\alpha} = \infty$. If the tax revenues are used to finance a separable public good ($\omega = 1$), then $\tilde{\alpha} = 1$. The price-dividend ratio of equity securities is higher (lower) in the high-tax regime if risk aversion is lower (higher) than $\tilde{\alpha}$.

It is surprising that the price of equity can be higher in the high-tax regime. To better understand this result, it helps to analyze the different effects which determine asset prices. The valuation of stocks is driven by two effects with independent tax and dividend processes. First, the government takes away a higher proportion of the aggregate dividends in high-tax regimes (income effect). Individuals would like to compensate for this tax by consuming more and by decreasing their demand of risky assets. In equilibrium, the supply of assets cannot adjust and the price of equity has to decrease due to the income effect. Second, consumption is relatively more expensive in periods with high taxes since future taxes are expected to be equal to or lower than current taxes (substitution effect). Individuals want to consume less and invest more during these periods. In equilibrium, the price of the risky asset has to increase as a consequence of this substitution...
effect. The elasticity of intertemporal substitution determines which of the two opposite effects is more important. The second effect is stronger for individuals who are more willing to substitute consumption intertemporally (i.e., \( \alpha < \tilde{\alpha} \)), whereas the first effect is stronger for individuals with a low elasticity (i.e., \( \alpha > \tilde{\alpha} \)). The price-dividend ratios are identical in the two states if \( \alpha = \tilde{\alpha} \). In this case, the two effects exactly offset each other because the expenditure elasticity equals zero.

If all the tax revenues are used to finance the separable public good (i.e., \( \omega = 1 \)), then \( \tilde{\alpha} = 1 \) and equity valuations are higher in the high-tax regime only if individuals are less risk-averse than a log-utility individual. If all the tax revenues are rebated to the tax payers (i.e., \( \omega = 0 \)), then \( \tilde{\alpha} = \infty \) and equities are always valued higher in the high-tax regime. In this case, the aggregate consumption level does not depend on the tax rate and is equal to the before-tax dividend. Thus, the first effect of taxes on equity valuation is completely eliminated with full redistribution. However, the second effect is still important, because individuals have an incentive to consume less in periods where the tax on consumption is higher. Valuations in the high-tax regime tend to be higher than those in the low-tax regime at low levels of risk aversion and at low levels of separability.

Second, I look at the general case with dependent tax and dividend processes (part i of Proposition 1). A dependence between the two processes adds a third effect of taxes on equity valuation. The current tax regime affects the distribution of future output levels (growth effect). Equity prices are higher if the standard deviation of the dividend growth rate is higher and also if the average growth rate is higher (lower) if \( \alpha < 1 \) (\( \alpha > 1 \)). The dependence between the two processes has no effect on equity prices if \( \alpha = 1 \). It is difficult to characterize the set of risk aversion coefficients \( A \) for the general case, because the exponent of the pricing factor \( \gamma \) is a quadratic function of the risk aversion \( \alpha \). The set \( A \) can be empty and non-convex.

The expected gross return of equity at time \( t \) can be separated into the following two components because of the independence of the price-dividend ratio and the contemporaneous dividend:

\[
E_t(r_{t+1}) = E_t \left( \frac{p_{t+1}^S + d_{t+1}^S}{p_t^S} \right) = E_t \left( \frac{d_{t+1}^S}{d_t^S} \right) E_t \left( \frac{1 + \delta_{t+1}}{\delta_t} \right).
\]  

The expected returns of equity securities in the two tax regimes can be expressed in the following way, where \( \xi_t = E_t(d_{t+1}^S/d_t^S) = \exp(\mu_t + 0.5\sigma_t^2) \):

\[
E(r_H^S) = \xi_H \left( \phi_{HH} \frac{1 + \delta_H}{\delta_H} + \phi_{HL} \frac{1 + \delta_L}{\delta_H} \right),
\]

The expected return of equity securities in the two tax regimes can be expressed in the following way, where \( \xi_t = E_t(d_{t+1}^S/d_t^S) = \exp(\mu_t + 0.5\sigma_t^2) \):

\[
E(r_H^S) = \xi_H \left( \phi_{HH} \frac{1 + \delta_H}{\delta_H} + \phi_{HL} \frac{1 + \delta_L}{\delta_H} \right),
\]
\[ E(r^S_L) = \xi_L \left( \phi_{LL} \frac{1 + \delta_L}{\delta_L} + \phi_{LH} \frac{1 + \delta_H}{\delta_L} \right). \]  

Whenever the tax regime changes, the value of equity needs to adjust. The resulting jumps in the price of equity increase the return variability and generate a bimodal return distribution which has ‘fat tails’.

### 3.2 Bond Valuation

The prices of risk-free zero-coupon bonds can be derived from equation (7):

\[ p_{t,m} = \beta^m E_t \left[ \frac{u'(c_{t+m})}{u'(c_t)} \frac{1 - \tau_{t+m}}{1 - \tau_t} \right], \]

\[ = E_t \left[ \prod_{i=1}^{m} \left( \beta \frac{u'(c_{t+i})}{u'(c_{t+i-1})} \frac{1 - \tau_{t+i}}{1 - \tau_{t+i-1}} \right) \right], \]

\[ = \lambda_t E_t \left[ \rho_{t,t+1} \left( p_{t+1}^{B,m-1} + d_{t+1}^{B,m-1} \right) \right]. \]  

The third equality uses the following definition \( \lambda_t = \beta E_t (d_{t+1}^S/d_t^S)^{-\alpha} = \beta \exp(-\alpha \mu_t + 0.5 \alpha^2 \sigma_t^2) \). The separation into two components is possible because the price of the bond \( p_{t,m} \) does not depend on the level of the dividend \( d_t^S \).

The equilibrium prices of a zero-coupon bond with maturity \( m \) can be expressed recursively using the initial conditions \( p_{B,0}^H = p_{L,0}^H = 0 \) and \( d_{B,0}^H = d_{L,0}^H = 1 \). Note that \( d_{t,m}^B = 0 \) if \( m > 0 \).

\[ p_{H,m+1}^{B,m} = \lambda_H \left[ \phi_{HH}(p_{H,m}^{B,m} + d_{H,m}^{B,m}) + \phi_{HL} \rho_{HL}(p_{L,m}^{B,m} + d_{L,m}^{B,m}) \right], \]

\[ p_{L,m+1}^{B,m} = \lambda_L \left[ \phi_{LL}(p_{L,m}^{B,m} + d_{L,m}^{B,m}) + \phi_{LH} \rho_{HL}(p_{H,m}^{B,m} + d_{H,m}^{B,m}) \right]. \]

The next proposition proves in which tax regime the valuations of one-period zero-coupon bonds are higher.

**Proposition 2** (i) The price of a zero-coupon bond with maturity \( m = 1 \) is higher in the high-tax regime if \( \alpha \) is in set \( B = \{ \alpha \in \mathbb{R}_+ : \lambda_H(\alpha) \rho_H(\alpha) \geq \lambda_L(\alpha) \rho_L(\alpha) \} \), and the price is lower in the high-tax regime if \( \alpha \) is not in set \( B \).

(ii) If the dividend process is independent of the tax process (i.e., \( \mu_L = \mu_H \) and \( \sigma_L = \sigma_H \)), then the price of a zero-coupon bond with a maturity of one year is higher in the high-tax regime if \( \alpha < \bar{\alpha} \), the price is lower in the high-tax regime if \( \alpha > \bar{\alpha} \) and the prices are identical if \( \alpha = \bar{\alpha} \), where \( \bar{\alpha} \) is defined in equation (23).
If the dividend growth rate does not depend on the current tax rate (part ii), then the condition for bond prices is exactly identical to the condition for equity securities from Proposition 1. In this case, valuations of both assets are higher in the high-tax regime if $\alpha < \tilde{\alpha}$. The income and the substitution effects have the same intuition for bonds as for equity securities.

The growth effect, which is relevant if the dividend and the tax process are dependent, differs for equity and one-period zero-bonds. The growth rate affects the discount factor of bonds $\lambda$ but not the future payoffs of the bonds. For equity, both the discount factor $\gamma$ and the future dividends are affected. This explains why set $B$ for bonds differs in general from set $A$ for equity.

The yield to maturity of a zero-coupon bond is $r_{t}^{B,m} = (1/p_{t}^{B,m})^{1/m}$. The term structure of interest rates is the yield of zero-coupon bonds at different maturities.\(^{12}\) The gross return of a one-period risk-free bond amounts to:

$$r_{t}^{B,1} = \frac{1}{\lambda_{H}\rho_{H}},$$  \hspace{1cm} (30)

$$r_{t}^{B,1} = \frac{1}{\lambda_{L}\rho_{L}}.$$  \hspace{1cm} (31)

### 3.3 Equity Premium

The equity premium compares the expected return of equity to the return of risk-free one-period zero-coupon bonds. The following proposition states the conditions under which the equity premium increases in an environment with tax changes.

**Proposition 3** (i) The equity premium $\pi_i$ for $i \in \{L, H\}$ equals the sum of a premium due to dividend uncertainty $\pi_i^D$ and a premium due to tax changes $\pi_i^T$:

$$\pi_i = E_t(r_{t+1}^S) - r_{t}^{B,1} = \pi_i^D + \pi_i^T.$$  \hspace{1cm} (32)

The dividend premium $\pi_i^D$ is positive. The tax premium $\pi_i^T$ is positive if $\alpha$ is in set $C = \{\alpha \in \Re_+ : (\rho_L(\alpha)\gamma_L(\alpha) - \rho_H(\alpha)\gamma_H(\alpha))(1 - \rho_H(\alpha)) \geq 0\}$, and the premium is negative if $\alpha$ is not in set $C$.

(ii) If the dividend process is independent of the tax process (i.e., $\mu_L = \mu_H$ and $\sigma_L = \sigma_H$), then the tax premium $\pi_i^T$ is positive.

Part (i) states that the excess return of stocks over short-term bonds is due to two premia. The first premium $\pi_i^D$ equals the equity premium...

\(^{12}\)Cox, Ingersoll, and Ross (1985) study the term structure of interest rates in an intertemporal general equilibrium model.
in an environment without tax changes and is always positive. The second premium \( \pi_i^T \) is due to tax changes and equals zero if tax rates are constant over time (i.e., \( \tau_H = \tau_L \) or \( \phi_{HH} = \phi_{LL} = 1 \)) or if the coefficient of relative risk aversion equals \( \tilde{\alpha} \). The tax premium is negative if the risk aversion coefficient \( \alpha \) is not in set \( C \). It is even possible that the equity premium becomes negative if the distribution of the dividend growth rate is sufficiently different between the two tax regimes. The equity premium decreases if the tax changes are such that they reduce the aggregate uncertainty investors are exposed to. Part (ii) states that the tax premium is always positive if the distribution of dividends does not depend on the current tax rate. The tax premium reaches its minimum of zero at \( \alpha = \tilde{\alpha} \), because in this case asset returns do not depend on the tax regime. The tax premium increases as the risk aversion falls below, and increases above, the critical level \( \tilde{\alpha} \). The tax premium usually differs from zero even if individuals are risk-neutral (\( \alpha = 0 \)).

The three effects that drive the asset valuation give an intuition of the effect of tax changes on the equity premium. The first effect (income effect) increases the equity premium because tax changes increase the variability of consumption over time. It is well-known that an increase in consumption volatility increases the required risk-premia. The first effect disappears if all the tax revenues are rebated to the representative agent. In this case, the consumption process is not affected by tax changes. The second effect (substitution effect) remains important in the case with a full rebate. Varying tax rates affect the relative price of consumption over time. The third effect (growth effect) can increase or decrease the equity premium, depending on whether tax changes mitigate or worsen the productivity shocks.

Whenever taxes change, asset prices need to adjust to clear the asset markets. The necessary price changes are larger for assets with long durations and smaller for assets with short durations. Individuals require higher expected returns for holding long-duration assets, such as stocks and long-term bonds, compared to short-term bonds. The increase of the equity premium is due to an increase in the term premium.

4 Numerical Example

To determine whether stochastic taxation is economically significant and to see whether it can help to explain the high equity premium and the large variability of stock returns, the model is solved for plausible underlying parameter values. The first subsection justifies the numerical assumptions
used in this example. The second subsection discusses how tax changes affect asset returns, equity premia, and the standard deviations of asset returns. The third subsection performs sensitivity analyses to check the robustness of the numerical results.

4.1 Base Case Assumptions

The marginal income tax rates in Figure 1 are depicted for five tax brackets. I concentrate on the tax rates of relatively wealthy individuals, because those individuals hold a significant portion of the financial assets. Wealthy individuals are probably the marginal investors in the asset markets. The marginal income tax rates fluctuated much more for high-income than for medium- and low-income individuals. Taking average tax rates over the whole population underestimates the risk introduced by tax rate changes because most financial assets are controlled by high-income individuals.

The two tax rates are $\tau_L = 0.25$ and $\tau_H = 0.50$ and the transition probabilities are $\phi_{LL} = \phi_{HH} = 0.95$. This implies an average duration of a tax regime of twenty years. The long-run frequencies of the two regimes are the same because the transition matrix is symmetric ($\phi_{LH} = \phi_{HL}$). The average tax rate equals 37.5 percent and has a standard deviation of 12.5 percent.

The growth rate of the economy has a mean of two percent and a standard deviation of four percent. These moments correspond roughly to the real per-capita growth rate of GNP in the U.S. as reported in Appendix A.3. The distribution of the growth rate is assumed to be independent of the current tax regime. OLS regressions reported in Appendix A.3 show that both the growth rate and the square of the growth rate do not

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13 Appendix A.1 explains how the time series of the tax rates were constructed.
14 Those values are based on the estimated coefficients of a two-state Markov model of the U.S. tax system using the EM algorithm described in Hamilton (1990). The estimation results are given in Appendix A.2.
15 This assumption differs from those in some papers in the equity premium literature, which use the moments of aggregate consumption to calibrate the economy. There are two reasons for this modification. First, I am interested in analyzing the effects of tax changes on asset returns. Simply looking at the moments of after-tax consumption does not provide any insights into the effects of tax changes on asset returns. Second, tax rate changes are much more significant for high-income individuals, who own a large proportion of the financial assets, than for the average individual, who only owns a small proportion of financial assets. Using aggregated values of after-tax consumption ignores a large portion of the risk of changing tax rates, which is significant for individuals in relatively high marginal tax brackets. Vissing-Jørgensen (1999) discusses the biases of aggregating consumption levels over stockholders and non-stockholders.
depend significantly (at a five percent confidence level) on the marginal tax rates. Sensitivity analyses show that changes in these assumptions do not affect the results much.

The base case assumes a discount factor of $\beta = 0.975$. The analyses in this section will concentrate on risk aversion coefficients in the range between 0 and 5. Table 1 summarizes all the assumptions in the base case.

4.2 Asset Valuations

The following computations assume that the tax rates follow the Markov chain described in Section 2 and use the parameter values from Table 1. Figure 2 shows how the price-dividend ratio depends on the coefficient of relative risk aversion. The two extreme cases with full lump-sum rebates ($\omega = 0$) and with complete separability between the private and the public good ($\omega = 1$) are depicted in Panels A and B. The dashed curves show the price-dividend ratios in an environment without any tax rate changes. The ratios depend neither on the tax rate nor on the separability coefficient $\omega$. Therefore, the dashed curves are identical in Panels A and B. The price-dividend ratio without tax rate changes equals 18.19 at a risk aversion of $\alpha = 2.5$. The ratio increases considerably as individuals become less risk-averse.

The two solid curves depict the price-dividend ratios in the low-tax and the high-tax regime in an environment with tax rate changes. The valuation is always higher in the high-tax regime if all the revenues are rebated ($\omega = 0$). If all the tax revenues are used to finance the separable public good ($\omega = 1$), then the valuations are higher in the high-tax regime at low levels of risk aversion (i.e., $\alpha < \tilde{\alpha} = 1$) and higher in the low-tax regime at high levels of risk aversion. The valuations are exactly identical if individuals have logarithmic utility. In this case, tax regime changes have no effect on asset valuations, and the price-dividend ratio is exactly what it would be in an environment without tax changes.

If all the tax revenues are rebated as a lump-sum distribution to the representative agent (i.e., $\omega = 0$), then with a risk aversion of 2.5 the price-dividend ratio equals 21.29 in the high-tax state and 16.13 in the low-tax state. Tax changes result in very large variations in stock prices. Stock prices increase instantaneously by 31.99 percent ($= 21.29/16.13 - 1$) whenever tax rates are raised and fall by 24.24 percent ($= 16.13/21.29 - 1$) whenever they decrease. If the tax revenues are used to finance a separable public good (i.e., $\omega = 1$), then the price-dividend ratio equals 15.37 in the high-tax state and 23.37 in the low-tax state. Stock prices fall in this case by 34.23
percent whenever taxes increase and increase by 52.05 percent whenever taxes decrease. The model assumes that the timing of tax regime changes is not anticipated by the investors. In reality, market participants learn gradually about possible future tax reforms and those large price changes occur over longer time horizons as investors adjust their expectations about future tax changes.

Figure 3 depicts the expected returns of a one-period zero-coupon bond and equity in an environment with and without tax rate changes. If tax rates do not change over time, then the return of the short-term bond equals 7.28 percent at a risk aversion of \( \alpha = 2.5 \) and increases significantly at higher levels of risk aversion. This result is at odds with the average historical real return of short-term Treasury securities. Introducing tax changes decreases the expected return of short-term bonds. At a risk aversion of \( \alpha = 2.5 \), the mean return of the risk-free one-period bond equals 6.89 (6.38) percent if \( \omega = 0 \) (\( \omega = 1 \)).

Introducing tax regime changes increases the expected return of equity. The mean return of equity increases slightly from 7.71 percent to 7.86 (8.06) percent if \( \omega = 0 \) (\( \omega = 1 \)). The expected returns of the assets vary considerably between the different tax regimes. For example, the expected equity return is 5.66 (11.40) percent in the high-tax regime and 10.07 (4.72) percent in the low-tax regime if \( \omega = 0 \) (\( \omega = 1 \)) under the base case assumptions. This model of tax regime switches gives a plausible explanation for time-varying expected returns of assets as suggested by Campbell (1991). Expected returns tend to be higher in the high-tax regime if individuals are relatively risk averse and if a smaller share of the tax revenues is rebated to the taxpayers. The higher expected returns of equity and the lower returns of short-term bonds both increase the equity premium.

The dashed curves of Figure 4 show that the equity premium increases slowly with risk aversion in a conventional model without tax changes. For example, the equity premium equals only 0.43 percent at a coefficient of risk aversion of \( \alpha = 2.5 \). This contrasts with a mean historical premium in the U.S. of 9.22 percent relative to Treasury bills. Figure 4 demonstrates that the equity premium is larger in a model with tax rate changes (solid curves). Panel B shows that the equity premium is highly sensitive to changes in the coefficient of relative risk aversion if the government does not rebate the tax revenues to the taxpayers (i.e., \( \omega = 1 \)). The equity premium is lowest at a risk aversion of \( \alpha = 0.87 \). An average equity premium of 9.58 percent results at a coefficient of relative risk aversion of 5. The equity premium at this level of risk aversion without tax rate changes would have been only 0.89 percent.
Panel A of Figure 4 shows the equity premium if the government rebates all the tax revenues to the representative individual (i.e., \( \omega = 0 \)). The equity premium is higher in this case compared to the case where \( \omega = 1 \) if the risk aversion is smaller than \( \alpha = 2.0 \). It is still considerably higher than the equity premium without tax regime changes. However, the equity premium increases only slowly with risk aversion. In this paper, equity securities are unlevered, because their payoffs correspond to the total production of the economy. The equity premia of levered securities increase with the leverage. This additional factor helps to match the historical moments of equity returns.

Figure 5 shows the standard deviations of the stock returns with and without tax changes. The standard deviation of equity returns without tax changes (dashed line) equals 4.31 percent at \( \alpha = 2.5 \) and is much lower than the historical standard deviation of stocks of 20.30 percent. The standard deviations increase significantly after the introduction of tax regime changes. In the base case with a risk aversion of \( \alpha = 2.5 \), the standard deviation of equity returns equals 7.97 percent with full tax rebate and 11.23 percent with no tax rebate.\(^{16}\) With full rebate, the standard deviation decreases slightly with increasing levels of risk aversion. With no rebate, the standard deviation is ‘U-shaped’. The model of tax regime changes generates heteroskedastic equity returns. Tax regime changes increase the asset variability considerably and match actually observed values at plausible levels of risk aversion.

In an environment without tax changes, the term structure of interest rates is flat with a yield of 7.28 percent if \( \alpha = 2.5 \). With tax regime shifts, the term structure of interest rates of zero-coupon bonds is increasing and converges to the no-tax level at long maturities. As the investment horizon increases, tax regime changes become relatively less important and the difference between the yield in the two tax states decreases. Figure 6 depicts the term structure of interest rates with (solid curves) and without (dashed curves) tax rate changes. The average term premium amounts to 0.90 percent (=7.28-6.38) if \( \omega = 1 \) and 0.39 percent (=7.28-6.89) if \( \omega = 0 \).

The higher term premium accounts for a large portion of the equity premium. The effect of tax rate changes on assets depends primarily on the duration of the assets. Both equity and bonds have long durations and are highly sensitive to changes in tax rates.\(^{17}\)

\(^{16}\)The unconditional standard deviation is not simply the mean of the standard deviations in the two tax-states. It is strictly higher because the expected returns differ in the two states. The unconditional variance is defined as \( \text{Var}(r) = \mathbb{E}[\text{Var}(r|\tau)] + \text{Var}[\mathbb{E}(r|\tau)] \).

\(^{17}\)Abel (1999) divides the equity premium into a term premium and a risk premium.
4.3 Sensitivity Analyses

The assumptions are changed to check the robustness of the numerical example in the previous section. Figure 7 depicts the expected returns of short-term bonds and equity at different levels of persistence of the tax regimes with a symmetric transition matrix (i.e., $\phi_{HH} = \phi_{LL}$) and at a risk aversion of $\alpha = 2.5$. If tax rates are permanent ($\phi_{HH} = \phi_{LL} = 1$), then the equity premium corresponds to the case without tax rate changes. The price changes are large and infrequent at high persistence levels and small and frequent at low persistence levels. The equity premium is largest for intermediate persistence levels, when tax changes are common and price changes are relatively large. The level of the equity premium is lower if all the tax revenues are rebated to the tax payers.

Figure 8 shows the dependence of the returns of the two assets on the difference between the tax rates in the two states. The average tax rate is kept constant at its average level of 37.5 percent. The difference between the two tax rates in the base case is 25 percent. In this case, the average equity premium equals 1.68 percent with $\omega = 1$ and 0.98 percent with $\omega = 1$. As the difference between the tax rates in the two tax regimes increases, the mean return of equity securities increases and the mean return of fixed-income securities decreases. An increase in the tax rate differential results in larger relative price adjustments of equity. Investors require a higher premium to hold these more variable assets.

The numerical exercises performed previously assume that the distribution of the growth rates is identical in the two tax regimes. The results in Section 3.3 demonstrate that the premium due to tax changes can be either positive or negative if the growth rate of the economy depends on the tax rate. Figure 9 shows the average equity premia if the mean and the standard deviation of the growth rate differs between the two regimes. Changes in the differences of the mean growth rate affect the equity premium more than changes in the differences of the standard deviations. The equity premium decreases (increases) with $\mu_H - \mu_L$ if $\omega = 0$ ($\omega = 1$). Cyclical tax rates (i.e., tax rates that tend to be high when average economic growth is high) result in a lower equity premium with $\omega = 0$ and in a higher equity premium with $\omega = 1$. The equity premium is higher if the standard deviation of the growth rate differs sufficiently between the two regimes. In Figure 9, the premium due to tax rate changes is never negative. The differences between the moments of the growth rates in the two regimes have to be relatively large to generate a negative tax premium. If $\sigma_H = \sigma_L$, then the growth differential $\mu_H - \mu_L$ would need to be smaller than $-2.7$ percent for $\omega = 0$.
and the differential $\mu_H - \mu_L$ would need to be larger than 4.2 percent for $\omega = 1$ to generate a negative tax premium.

The previous results assume that tax rates follow a two-state Markov chain. The results do not change much if additional states are introduced. Alternative specifications of the Markov chain can either increase or decrease the equity premium relative to the base case with two states. For example, a three-state Markov chain with $\tau \in \{0.2219, 0.3750, 0.5281\}$ and with a probability of remaining in the current state of 0.9388 and equal probabilities to switch to any of the two other states has the same unconditional mean and variance of the tax rates and the same expected tax change in each period as the two-state Markov chain used previously. In this example, the equity premium decreases from 0.98 to 0.93 percent if $\omega = 0$ and it decreases from 1.68 to 1.57 percent if $\omega = 1$. The equity premium decreases slightly because very large tax changes become less frequent in a model of three states.

5 Heterogeneous Agent Economy

The model with heterogeneous agents builds on the representative agent model from Section 2. The agents differ in their wealth levels. The government decreases income inequality by imposing net taxes on the wealthy and paying net transfers to the poor. This redistribution policy affects the portfolio choices of the individuals and changes asset returns and the equity premium in equilibrium because asset markets are incomplete. Tax changes will generally benefit some individuals and harm others. People with diverging interests can use the available assets to insure each other against tax rate changes.\(^\text{18}\) This section discusses whether the effect of tax rate changes on asset returns will be less important with a more heterogeneous population. The optimal portfolio choices and equity premia cannot be derived analytically and I solve the model numerically. The first section formulates the model with heterogeneous individuals and the second section summarizes the main numerical results of this model.

5.1 The Model

There are two individuals in the economy with different initial wealth levels. The government taxes the consumption of the individuals at a uniform rate

\(^{18}\)Dumas (1989) formulates a two-person dynamic model with capital markets and shows that the two investors interact to share their risks.
and rebates all the tax revenues equally to the two individuals ($\omega = 0$). The wealthy individual will pay net taxes and the poor individual will receive net transfers from the government.

To simplify the numerical computations, the individuals can only invest in one zero-coupon bond with a maturity of one year. The number of bonds and stocks purchased is represented by $x_{t}^{B,i}$ and $x_{t}^{S,i}$. A one-period zero-coupon bond issued at time $t$ at the price of $p_{t}^{B} d_{t+1}^{B}$ pays a dividend of $d_{t+1}^{B} = d_{t}^{S}$ after one year. The principal value of the zero-coupon bond is set equal to the dividend of equity in the previous period $d_{t}^{S}$ to simplify the derivation of optimal policies which do not depend on the actual level of the current dividend.

The consumption of individual $i$ at time $t$ is:

$$c_{t}^{i} = (1 - \tau_{t})[(p_{t}^{S} + d_{t}^{S})x_{t-1}^{S,i} + d_{t}^{B} x_{t-1}^{B,i} - p_{t}^{S} x_{t}^{S,i} - p_{t}^{B} d_{t+1}^{B} x_{t}^{B,i}] + 0.5T_{t}.$$  \(33\)

In equilibrium, $\sum_{i} x_{t}^{S,i} = 1$ and $\sum_{i} x_{t}^{B,i} = 0$. The aggregate tax revenues $T_{t}$ equal:

$$T_{t} = \sum_{i} \tau_{t}(p_{t}^{S} + d_{t}^{S})x_{t-1}^{S,i} + d_{t}^{B} x_{t-1}^{B,i} - p_{t}^{S} x_{t}^{S,i} - p_{t}^{B} d_{t+1}^{B} x_{t}^{B,i}), \quad \text{(34)}$$

The relative wealth level of individual $i$ at time $t$ is defined as:

$$\kappa_{t}^{i} = \frac{(p_{t}^{S} + d_{t}^{S})x_{t-1}^{S,i} + d_{t}^{B} x_{t-1}^{B,i}}{\sum_{i} [(p_{t}^{S} + d_{t}^{S})x_{t-1}^{S,i} + d_{t}^{B} x_{t-1}^{B,i}]}, \quad \text{(35)}$$

$$= \frac{(p_{t}^{S} + d_{t}^{S})x_{t-1}^{S,i} + d_{t}^{B} x_{t-1}^{B,i}}{p_{t}^{S} + d_{t}^{S}},$$

$$= \frac{x_{t-1}^{S,i} + \frac{d_{t-1}^{B}}{d_{t}} \frac{1}{1 + \delta_{t}}}{\frac{d_{t}}{d_{t-1}}}. \quad \text{(36)}$$

The value function $W$ depends on the current output level $d_{t}^{S}$, the current tax rate $\tau_{t}$, and the relative wealth level $\kappa_{t}^{i}$. It can be expressed in recursive form:

$$W(d_{t}^{S}, \tau_{t}, \kappa_{t}^{i}) = \max_{x_{t}^{S,i}, x_{t}^{B,i}} E_{t} \sum_{j=0}^{\infty} \beta^{j} (u(c_{t+j}^{i})),$$
\[ \max_{x^{S,i},x^{B,i}} \left[ u(c_t^i) + \beta E_t \left( W(d^{S}_{t+1}, \tau_{t+1}, \kappa^i_{t+1}) \right) \right]. \]

The constant relative risk aversion utility function \( u(c) \) and the value function \( W(d, \tau, \kappa) \) are homogeneous of degree \( 1 - \alpha \) in \( c \) and \( d \), respectively. This property allows the elimination of the state variable \( d^S_t \):

\[ W(d^{S}_{t}, \tau_{t}, \kappa^i_{t}) = (d^{S}_{t})^{1-\alpha} W(1, \tau_{t}, \kappa^i_{t}) = (d^{S}_{t})^{1-\alpha} V(\tau_{t}, \kappa^i_{t}). \]  

The simplified value function \( V \) is defined as:

\[ V(\tau_{t}, \kappa^i_{t}) = \max_{x^{S,i},x^{B,i}} \left[ u\left( \frac{c^i_{t}}{d^S_{t}} \right) + \beta E_t \left( V(\tau_{t+1}, \kappa^i_{t+1}) \left( \frac{d^{S}_{t+1}}{d^S_{t}} \right)^{1-\alpha} \right) \right], \tag{38} \]

where:

\[ \frac{c^i_{t}}{d^S_{t}} = (1 - \tau_t) \left( (1 + \delta_t)x^{S,i}_{t-1} + \frac{d^{S-1}_{t}}{d^S_{t}} x^{B,i}_{t-1} - \delta_t x^{S,i}_{t} - p_t B^{B,i}_{t} \right) + 0.5 \tau_t, \]

\[ \kappa^i_{t} = x^{S,i}_{t-1} + x^{B,i}_{t-1} \frac{d^{S-1}_{t}}{d^S_{t}} \frac{1}{1 + \delta_t}. \]

The value function with two identical individuals equals \( V(\tau, 0.5) = 0.5^{1-\alpha} V^R(\tau) \), where \( V^R(\tau) \) is the value function of the representative individual at the tax rate \( \tau \). The aggregate portfolio choices and asset returns are identical in the case with two identical individuals and in a model with one representative individual.

### 5.2 Numerical Results

Equation (38) cannot be solved analytically. It must be solved numerically by a two-step procedure. In the first step, optimal savings and portfolio decisions and the corresponding value functions are computed at fixed asset prices. In the second step, asset prices are adjusted to clear the asset markets. These two steps are repeated until the excess demands for the two assets are sufficiently small. Appendix C describes the numerical methods in detail.

Figure 10 shows the consumption of an individual as a proportion of total output \( d^S \) in the two tax regimes for different initial relative wealth levels \( \kappa \). If the two individuals in the economy have identical wealth levels (\( \kappa = 0.5 \)), then each individual consumes exactly one-half of the total output. The consumption level does not depend on the tax regime, since all the tax revenues are by assumption rebated as lump-sum payments to the individuals.
The tax system diminishes inequality by increasing the consumption shares of poor individuals with $\kappa < 0.5$ and decreasing the consumption of wealthy individuals with $\kappa > 0.5$. Wealthy individuals enjoy a higher consumption in the low-tax regime and poor individuals are better off in the high-tax regime. If the wealthy individual owns 75 percent of initial wealth and the poor individual owns the remaining 25 percent, then the wealthy individual consumes 64.61 percent and the poor individual consumes 35.39 percent of total output in the high-tax regime.

Figure 11 depicts the net savings rates at different initial relative wealth levels. The net savings rate $ns$ is defined as the new investments in the two assets net of the initial wealth level.

$$ns_t^i = x_t^{S,i} + x_t^{B,i} \frac{p_t^B}{\delta_t} - \kappa_t^i$$

The net savings are zero if the two individuals are identical ($\kappa = 0.5$). The wealthy save in the low-tax regime and dissave in the high-tax regime. The savings patterns of the poor are exactly opposite. Figures 10 and 11 show that the poor consume and save more in the high-tax regime when they receive a relatively large transfer from the wealthy individuals. The individual savings rates are relatively small. Even a wealthy individual who owns all the assets ($\kappa = 1$) saves slightly less than 0.3 percent of the total output in the low-tax regime and dissaves slightly more than 0.4 percent in the high-tax regime. By assumption, aggregate savings are always zero as in the representative agent model.

Figure 12 depicts the portfolio choices of the investors. The average relative values of the two assets are depicted. The relative value of stocks equals $x_t^{S,i}$ and the relative value of the bonds equals $x_t^{B,i} \frac{p_t^B}{\delta_t}$. If $\kappa = 0.5$, then each of the two identical individuals holds fifty percent of the stocks and no bonds. The optimal equity proportion increases with the initial relative wealth level and the optimal bond proportion decreases with the initial wealth level. In a model with redistributive governmental policies, the wealthy are hurt and the poor benefit whenever the tax rates increase. The wealthy invest a relatively larger proportion of their portfolio in stocks than the poor. These portfolios partially hedge the risk due to tax rate changes because the equity prices increase as the tax rates increase. This financial gain offsets some of the losses due to higher taxes. The poor are willing to take losses in their financial portfolios when tax rates increase because they will receive larger transfers. These portfolio transactions improve the
risk-sharing between the wealthy and the poor.\footnote{The short-selling of bonds by the poor investors is only possible because the numerical simulations limit the variability of stock returns. Note that if the equity returns were extremely low, then the poor would only receive a small transfer from the government and this transfer might not be sufficient to pay back the loans.}

The introduction of heterogeneous agents does not affect asset prices significantly. Figure 13 depicts the effect of wealth distribution on the equity premium. On average, the equity premium decreases only slightly as the wealth distribution becomes more unequal. The introduction of a very simple form of heterogeneity has no significant effects on the main results in the representative agent model.

6 Conclusions

This paper generalizes the Lucas (1978) asset pricing model by introducing a flat consumption tax which follows a two-state Markov chain. This tax does not merely affect equity securities, it affects all assets symmetrically. Whenever taxes change, asset prices need to adjust instantaneously to clear asset markets. These price changes increase the variability of expected and actual asset returns. The price adjustments are more severe for assets with long durations, such as equity and long-term bonds, than for assets with shorter durations. Individuals require higher expected returns for holding the assets with more severe price changes under plausible and identifiable conditions.

Tax rate changes affect asset prices even if all the tax revenues are rebated to the representative individual and the consumption process remains completely unaffected by tax changes. This occurs because tax changes distort the price of consumption over time and affect savings and investment incentives.

A numerical example demonstrates that stochastic taxation by itself can account for the magnitude of the return premium of equities over short-term bonds, without implying implausible returns on short-term bonds.

Introducing a simple form of heterogeneity does not affect the main results of this paper. In the heterogeneous model, the government decreases income inequality by imposing net taxes on the wealthy and paying net transfers to the poor. This redistribution policy affects the portfolio choices and the savings decisions of the individuals, changing asset returns and the equity premium in equilibrium. Tax rate changes will benefit some individuals and harm others. People with diverging interests can use the
available assets to insure each other against tax rate changes.

This paper makes several simplifications which could be relaxed in future work. First, the model uses a simple exchange economy without real investment opportunities to illustrate the effects of tax rate changes. Endogenizing real investment choices will result in a more realistic model of the economy. Second, the heterogeneity in the second part of the paper is very simple and is only used to check the robustness of the results with a representative agent. Adding progressive taxes, limited stock market participation, and non-diversifiable income shocks would enrich the current model. Third, the current tax system in the United States is not a flat consumption tax system. It is a progressive income tax system where some income sources are exempt from taxes (e.g., tax-deferred accounts, municipal bonds). In particular, stocks and bonds face different effective tax rates. The effects of tax reforms will differ if the effective tax on stock returns is smaller than the tax on bond returns and if the variability of the tax rates of the two assets differs. The analysis under a more realistic tax system would be interesting. Fourth, the tax shocks in this paper are exogenous. Time-varying tax rates may reflect unpredictable changes in the balance of power among different groups of taxpayers. A political-economy model could explain the mechanism that generates frequent tax rate changes.
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A Data

A.1 Marginal Tax Rates

Taxable income was derived for five real income levels after deducting exemptions for a married couple filing jointly with two dependent children from the income levels. The proportion of total deductions relative to the adjusted gross income was assumed to equal the proportion of total deductions in the whole population for each year as reported by the Internal Revenue Service. The marginal income tax brackets and exemptions were determined using the Statistics of Income of the Internal Revenue Service (1954) for the years 1913-1943, Pechman (1987) for the years 1944-1987, and different issues of the Instructions to Form 1040 from the IRS for the remaining years between 1988-1999. The values of the Consumer Price Index from 1913-1957 were taken from Mitchell (1983) and for the other years from the U.S. Government Printing Office (2000). Total deductions as a proportion of adjusted gross income (AGI) were derived from different issues of the Statistics of Income of the IRS.

A.2 Regime Switching Model

I estimate the two-state Markov model of the tax system via the EM algorithm described in Hamilton (1990). The tax rate at time $t$ is assumed to be distributed normally, where the mean and the variance depend on the state of the Markov chain:

$$
\tau_t \sim N\left(\mu_t^i, (\sigma_t^i)^2\right), i = \{L, H\}. \quad (40)
$$

As in Section 2, the transition probabilities are denoted with $\phi_{HH}$ and $\phi_{LL}$. Table 2 summarizes the estimated coefficients of the regime-switching model and their standard errors. The regime switching model estimates also the variance of the tax rates around the mean tax rate in each regime. This additional source of variation is ignored in this paper. Adding an additional source of uncertainty to the tax system will most likely magnify the effects of tax rate changes.

A.3 Dependence Between Tax- and Output-Processes

The mean and standard deviation of the distribution of the growth rate of the economy is estimated from the real growth rate of U.S. GNP per-capita. The per capita real growth rate of real GNP uses the GNP as reported

To determine the effects of marginal tax rates on the mean and the standard deviation of the growth rate of the economy, I regressed the growth rate of the economy on the corresponding tax rate and the square of the growth rate on the tax rate. Table 3 summarizes the estimated coefficients of the following two regression equations:

\[
\begin{align*}
  z_t &= \alpha_0 + \alpha_1 \tau_t + \epsilon_t, \\
  z_t^2 &= \beta_0 + \beta_1 \tau_t + \eta_t.
\end{align*}
\]

The tax rate coefficients \(\alpha_1\) and \(\beta_1\) of the OLS equations in Table 3 are not significantly different from zero at a five percent confidence level. Taxes do not seem to have a statistically significant effect on the first two moments of the growth rate of the economy. It is interesting that the coefficient \(\alpha_1\) for the longer period is positive. This implies that the growth rate tends to be higher with higher tax rates. A tax rate increase of 10 percent for individuals with real income levels of $250,000 is associated with an increase in economic growth of 0.32 percent per year. The coefficient \(\beta_1\) is usually negative, which implies that the square of the growth rate is lower with higher tax rates. The base case of the numerical example in Section 4 assumes the the growth rate of the economy is independent of the tax regime since the relationship is not statistically significant.

B Proofs

B.1 Proof that Price-Dividend Ratios are Positive

This proof shows that the price-dividend ratios in equations (21) and (22) are positive if \(0 < \gamma_i < 1\) for \(i \in \{L, H\}\). The numerator of equation (21) \(N_H\) is positive if \(0 < \gamma_i < 1\) for \(i \in \{L, H\}\):

\[
N_H = \gamma_H [\rho_H + \gamma_L (1 - \phi_{HH} - \phi_{LL})]
\]

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\[
\gamma_H \left[ \phi_{HH} + \phi_{HL} \rho_{HL} + \gamma_L (1 - \phi_{HH} - \phi_{LL}) \right] = \gamma_H \left[ \phi_{HH} (1 - \gamma_L) + \phi_{HL} \rho_{HL} + \gamma_L (1 - \phi_{LL}) \right] > 0.
\]

The denominator \(D_H\) of equation (21) is positive if \(0 < \gamma_i < 1\), because:

\[
1 - D_H = \phi_{HH} \gamma_H + \phi_{LL} \gamma_L + \gamma_H \gamma_L (1 - \phi_{HH} - \phi_{LL})
\]

\[
= \gamma_H [\phi_{HH} + \phi_{HL} \rho_{HL}] + \gamma_L [\phi_{LL} + \phi_{LH} \gamma_H] - \gamma_H \gamma_L
\]

\[
\leq \gamma_H + \gamma_L - \gamma_H \gamma_L = \gamma_H + (1 - \gamma_H) \gamma_L < 1.
\]

Similar operations show that (22) is positive.

B.2 Proof of Proposition 1

(i) Equations (21) and (22) differ only in the first product of their numerators. The denominators in equations (21) and (22) are positive as proved in Appendix B.1. Therefore \(\delta_H \gtrless \delta_L\) if \(\gamma_H \rho_H \gtrless \gamma_L \rho_L\).

(ii) If \(\mu_L = \mu_H\) and \(\sigma_L = \sigma_H\), then \(\gamma_L = \gamma_H\). In this case only the first product in the numerator differs between equations (21) and (22). \(\rho_{HL} = \left[\frac{(1 - \tau_L)}{(1 - \tau_H)}\right] \left[\frac{(1 - \omega \tau_L)}{(1 - \omega \tau_H)}\right]^{-\alpha}\). \(\rho_{HL} = 1\) if \(\alpha = \tilde{\alpha}\) and \(\partial \rho_{HL} / \partial \alpha \leq 0\). Thus, \(\rho_{HL} \gtrless 1\) if \(\alpha \gtrless \tilde{\alpha}\). Because \(\rho_{LH} = 1 / \rho_{HL}\), \(\rho_{LH} \gtrless 1\) if \(\alpha \gtrless \tilde{\alpha}\). \(\rho_H = \phi_{HH} + \phi_{HL} \rho_{HL}\) and \(\rho_L = \phi_{LL} + \phi_{LH} \rho_{LH}\) are simply weighted averages of \(\rho_{HL}\) and \(\rho_{LH}\) with 1, respectively. Thus, \(\rho_H \gtrless 1 \gtrless \rho_L\) if \(\alpha \gtrless \tilde{\alpha}\). \(\delta_H \gtrless \delta_L\) if \(\rho_H \gtrless \rho_L\), because the denominators in equations (21) and (22) are positive as proved in Appendix B.1 and because by assumption \(0 < \gamma_L = \gamma_H\). Thus, \(\delta_H \gtrless \delta_L\) if \(\alpha \gtrless \tilde{\alpha}\). \(\square\)

B.3 Proof of Proposition 2

(i) Equations (28) and (29) show that the prices of the one-period zero-coupon bond equal \(p_{B,1}^H = \lambda_H \rho_H\) in the high-tax regime and \(p_{B,1}^L = \lambda_L \rho_L\) in the low-tax regime. Thus, \(p_{B,1}^H \gtrless p_{B,1}^L\) if \(\lambda_H \rho_H \gtrless \lambda_L \rho_L\).

(ii) Part (i) shows that the prices of the one-period zero-coupon bond equal \(p_{B,1}^H = \lambda_H \rho_H\) and \(p_{B,1}^L = \lambda_L \rho_L\). If \(\mu_L = \mu_H\) and \(\sigma_L = \sigma_H\), then \(\lambda_L = \lambda_H\). Since \(\lambda_H = \lambda_L > 0\), \(p_{B,1}^H \gtrless p_{B,1}^L\) if \(\rho_H \gtrless \rho_L\). As shown in the proof to Proposition 1, \(\rho_H \gtrless \rho_L\) if \(\alpha \gtrless \tilde{\alpha}\). \(\square\)
B.4 Proof of Proposition 3

The following proof holds in the high-tax state. The proof for the low-tax state is similar. (i) Equation (25) gives the return of equity in the high-tax state.

\[
E(r^S_H) = \xi_H \left( \frac{1 + \delta_H}{\delta_H} \phi_{HH} + \frac{1 + \delta_L}{\delta_H} \phi_{HL} \right), \tag{43}
\]

Plugging the return of the risk-free asset with a maturity of one year during the high-tax-state (30) into equation (43) and simplifying gives the following equation:

\[
E_t(r^S_H) = r^{B_1}_H \frac{\lambda_H \xi_H}{\gamma_H} (1 + v_H), \tag{44}
\]

where:

\[
v_H = \gamma_H \frac{\phi_{HH}(1 - \rho_H) \left( \frac{\gamma_H}{\gamma_L} \rho_L - \rho_H \right)}{\phi_{HH}(1 - \gamma_L) + \phi_{HL} \rho_H + \gamma_L \rho_{HL}}.\]

The one-period interest rate \(r^{B_1}_H = 1/(\lambda_H \rho_H) > 0\) is defined as the gross return and is therefore always strictly positive. The second factor \((\lambda_H \xi_H / \gamma_H = \exp(\alpha \sigma_H^2) \geq 1, \text{ because } \alpha \geq 0\) results from the uncertainty of dividend payments. The third factor \(1 + v_H\) results from tax rate changes. This factor equals 1 in an environment without tax rate changes (i.e., \(\phi_{HH} = \phi_{LL} = 1\) or \(\tau_H = \tau_L\), because \(\rho_H = \rho_L = 1\).

The premium due to dividend uncertainty \(\pi^D_H\) is positive:

\[
\pi^D_H = r^{B_1}_H \frac{\lambda_H \xi_H}{\gamma_H} - r^{B_1}_H = \frac{\exp(\alpha \sigma_H^2) - 1}{\lambda_H \rho_H} \geq 0.
\]

The premium due to tax uncertainty \(\pi^T_H\) is:

\[
\pi^T_H = r^{B_1}_H \frac{\lambda_H \xi_H}{\gamma_H} v_H = \frac{\exp(\alpha \sigma_H^2) v_H}{\lambda_H \rho_H}.
\]

The sign of \(\pi^T_H\) is identical to the sign of \(v_H\). The factor \(v_H\) is negative if \((\rho_L \gamma_L - \rho_H \gamma_H)(1 - \rho_H) < 0\). The tax premium \(\pi^T_H\) is negative if \(\rho_H \gamma_H < \rho_L \gamma_L\) whenever \(\alpha \leq \tilde{\alpha}\) and if \(\rho_H \gamma_H > \rho_L \gamma_L\) whenever \(\alpha > \tilde{\alpha}\), because \(\alpha \leq \tilde{\alpha}\) implies that \(\rho_H \geq 1\) and \(\alpha > \tilde{\alpha}\) implies that \(\rho_H < 1\).
Note that the condition $(\rho_L \gamma_L - \rho_H \gamma_H)(1 - \rho_H) < 0$ is equivalent to the condition $(\rho_H \gamma_H - \rho_L \gamma_L)(1 - \rho_L) < 0$, because if $\rho_H \leq 1$ then $\rho_L \leq 1$.

(ii) If $\mu_L = \mu_H$ and $\sigma_L = \sigma_H$, then $\gamma_L = \gamma_H$ and $v_H$ simplifies to:

$$v_H = \gamma_H \frac{\phi_{HH}(1 - \rho_H)(\rho_L - \rho_H)}{\phi_{HH}(1 - \gamma_L) + \phi_{HL}\rho_{HL} + \gamma_L\phi_{LH}}.$$ 

This term is always positive since the sign of $(1 - \rho_H)$ is identical to the sign of $(\rho_L - \rho_H)$, because $1$ lies between $\rho_L$ and $\rho_H$. □

## C Numerical Methods

Equation (38) cannot be solved analytically. I solve the investor’s problem using Matlab’s constrained maximization function in each of the two tax regimes for 25 equally-spaced nodes corresponding to different relative wealth levels $\kappa^i = \{-0.10, -0.05, 0.00, ..., 1.00, 1.05, 1.10\}$. The value-function for intermediate wealth levels is approximated using a cubic-spline interpolation. The log-normal distribution of the future dividends is approximated using a Gauss-Hermite Quadrature with 10 nodes.\footnote{See Judd (1998) for a more detailed description of this method.}

The starting values for the portfolio choices and for the value function at the 25 different relative wealth levels in each tax regime are taken from the model with a representative agent. The starting values for the stock proportion is $x_{S,i}^0 = \kappa^i$, for the bond proportion is $x_{B,i}^0 = 0$, and for the value function is $V_0(\kappa^i, \tau) = (\kappa^i)^{1-\alpha}V^R(\tau)$, where $V^R(\tau)$ is the value function for the representative agent. The equilibrium prices of the two assets in the model with a representative agent are used as the starting values for the prices. Note that the starting prices are identical for all relative wealth levels $\kappa^i$.

The optimal portfolio decisions and the equilibrium asset returns are derived in two steps. In the first step, the optimization problem (38) is solved for each node in the two tax regimes. The new value function is computed at the new optimal portfolio choices. This step is repeated until the changes in the value function and the optimal portfolio choices between two iterations are sufficiently small (i.e., the relative changes of the value function and of the optimal portfolio choices at all the nodes are less than $10^{-4}$). The excess demands of both assets are computed at each of the 25 nodes in the two tax regimes. If the excess demands are not sufficiently small (i.e., the relative excess demands at all the nodes are larger than $10^{-3}$), the
second step of the algorithm is started. In the second step, the prices of the
two assets are adjusted in each node using a Gauss-Jacobi iteration. With
these new prices, the optimal portfolio choices and the corresponding value
functions are computed using step one. The prices of the two assets are
adjusted until the excess demands of both assets are sufficiently small (i.e.,
the relative excess demands at all the nodes are less than $10^{-3}$).
Figure 1: Marginal Income Tax Rates at Different Real Income Levels
The marginal income tax rates over the period from 1913 to 1999 are depicted for families with real income levels of 50, 100, 250, and 500 thousand U.S. dollars (with 1999 consumer prices), and the marginal tax rate for the highest tax bracket.
Figure 2: Price-Dividend Ratios of Equity
The dashed curves show the price-dividend ratios in an environment without tax rate changes and the solid curves show the price-dividend ratios in the low-tax and the high-tax regime in an environment with tax rate changes. Panel A corresponds to the case where all the tax revenues are rebated to the tax payers (ω = 0) and Panel B depicts the case where the tax revenues are used to finance a separable public good (ω = 1).
Figure 3: Expected Returns of Short-Term Bonds and Equity
Panels A and B show the expected returns of a zero-coupon bond with a maturity of one year and Panels C and D show the expected returns of equity. Panels A and C correspond to the case where all the tax revenues are rebated to the tax payers ($\omega = 0$) and Panels B and D depict the case where the tax revenues are used to finance a separable public good ($\omega = 1$). The dashed curves show the expected returns in an environment without tax rate changes and the solid curves show the expected returns with tax rate changes.

One-Period Bond

**Panel A: $\omega = 0$**

**Panel B: $\omega = 1$**

Equity

**Panel C: $\omega = 0$**

**Panel D: $\omega = 1$**
Figure 4: Equity Premium
The dashed curves show the equity premia without tax rate changes and the solid curves show the premia with tax rate changes. Panel A corresponds to the case where all the tax revenues are rebated to the taxpayers ($\omega = 0$) and Panel B depicts the case where the tax revenues are used to finance a separable public good ($\omega = 1$).
Figure 5: Stock Variability
The dashed curves show the variability of equity returns without tax rate changes and the solid curves show the variability with tax rate changes. Panel A corresponds to the case where all the tax revenues are rebated to the tax payers ($\omega = 0$) and Panel B depicts the case where the tax revenues are used to finance a separable public good ($\omega = 1$).
Figure 6: Term Structure of Interest Rates
The dashed curves show the flat term structure in an environment without tax rate changes and the solid curves show that the term structure increases as the maturity of short-term bonds increases. Panel A corresponds to the case where all the tax revenues are rebated to the tax payers ($\omega = 0$) and Panel B depicts the case where the tax revenues are used to finance a separable public good ($\omega = 1$).
Figure 7: Asset Returns at Different Persistence Levels
The expected returns of one-year zero-coupon bonds and equity securities are depicted at different persistence levels. The equity premium is defined as the difference between the expected returns of the two securities. Panel A corresponds to the case where all the tax revenues are rebated to the taxpayers ($\omega = 0$) and Panel B depicts the case where the tax revenues are used to finance a separable public good ($\omega = 1$).
Figure 8: Asset Returns with Different Tax Rates
The expected returns of one-year zero-coupon bonds and equity securities are depicted at different levels of tax-differentials. The average tax level is kept constant at 37.5 percent. The equity premium is defined as the difference between the expected returns of the two securities. Panel A corresponds to the case where all the tax revenues are rebated to the tax payers ($\omega = 0$) and Panel B depicts the case where the tax revenues are used to finance a separable public good ($\omega = 1$).
Figure 9: Equity Premia with Dependence between Growth and Tax Regime

The equity premium depends on the difference in the mean growth rate of the dividends \( (\mu_H - \mu_L) \) and on the difference in the standard deviation of the growth rate \( (\sigma_H - \sigma_L) \). Panel A corresponds to the case where all the tax revenues are rebated as lump-sum distributions to the tax payers \( (\omega = 0) \) and Panel B depicts the case where the tax revenues are used to finance a separable public good \( (\omega = 1) \).
Figure 10: Relative Consumption Levels

The consumption share in the two tax regimes is depicted for individuals with different initial relative wealth levels. The wealthy consume relatively less in the high-tax regime and the poor consume relatively more in the high-tax regime.
Figure 11: Individual Savings Rates
This figure depicts the net savings rate for individuals at different wealth levels. The wealthy save during low-tax regimes and dissave during high-tax regimes. The savings choices of the poor are exactly opposite.
Figure 12: Portfolio Selection
The portfolio allocations are depicted for individuals with different wealth levels. The wealthy hold relatively more stocks than the poor.
Figure 13: Equity Premia
The equity premium in equilibrium changes only marginally as the heterogeneity in the economy increases.
Table 1: Numerical Assumptions
This table summarizes the base case parameter values used in the numerical example. $\alpha$ denotes the coefficient of relative risk aversion, $\beta$ the discount factor, $\tau_L$ and $\tau_H$ the tax rates in the low-tax and high-tax regimes, $\phi_{LL}$ and $\phi_{HH}$ the transition probabilities between the two regimes, and $\mu$ and $\sigma$ the mean and the standard deviation of the logarithm of the growth rate of dividends.

<table>
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<th>Coefficient</th>
<th>Value</th>
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<tr>
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<td>$\phi_{HH}$</td>
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<tr>
<td>$\sigma$</td>
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</table>
Table 2: Estimation of the Regime-Switching Model of Tax Rates

This table summarizes the estimates of the regime-switching model described in Hamilton (1990). $\mu^*_H$ and $\mu^*_L$ are the average tax rates in the high- and low-tax regimes, $\phi_{HH}$ and $\phi_{LL}$ are the transition probabilities of the Markov-chain, and $\sigma^*_H$ and $\sigma^*_L$ are the standard deviations of tax rates during the high- and the low-tax regimes. The standard errors of the regressions are summarized in brackets.

$$
\tau_t \sim N(\mu^*_i, (\sigma^*_i)^2), i = \{L, H\}.
$$

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<th>Tax-Bracket (in thousand)</th>
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<th>100</th>
<th>250</th>
<th>500</th>
<th>Max</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
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Table 3: The Effect of Tax Rates on the Growth Rate of the Economy

This table summarizes the OLS-regression results of the following two equations. \( z_t \) denotes the growth rate of per-capita GNP and \( \tau_t \) the marginal income tax rate. The standard errors of the regression coefficients are given in brackets. The significance levels are abbreviated with asterisks: ‘*’ and ‘**’ are significant at the 5, and 1 percent level

\[
\begin{align*}
    z_t &= \alpha_0 + \alpha_1 \tau_t + \epsilon_t, \\
    z_t^2 &= \beta_0 + \beta_1 \tau_t + \eta_t.
\end{align*}
\]

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<th>100</th>
<th>250</th>
<th>500</th>
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