Skill-specific rather than General Education:  
*A Reason for Slow European Growth?*

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Abstract

In this paper, we develop a model of technology adoption and economic growth in which households optimally obtain either a concept-based, “general” education or a skill-specific, “vocational” education. General education is more costly to obtain, but reduces the loss of a worker’s task-specific productivity whenever a new technology is incorporated into production. Firms weigh the cost of adopting and operating new technologies against increased revenues and optimally choose the level of adoption. Conditional on their education, households then choose between working in the technology-adopting sector and the non-adopting sector. We show that an economy whose policies favor vocational education will grow slower in equilibrium than one that favors general education. Moreover, the gap between their growth rates will increase with the growth rate of available technology.

Our theory suggests that while European education policies that favored specialized, vocational education might have worked well during the 60s and 70s when available technologies changed slowly, it may have contributed to slow growth and may have increased the growth gap relative to the US in the information age of the 80s and 90s when new technologies emerged at a more rapid pace.

Keywords: Education policy, Technology adoption, Balanced growth, Eurosclerosis.

JEL Classification: O40, O30, I21

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1 Introduction

European economic growth has been weak relative to that of the US since the 80s. For instance, both Germany and Italy grew faster than the US in per capita terms during the period between 1970 and 1980, while the situation reversed in the subsequent two decades. During the last two decades Europe has also, with a few notable exceptions, suffered from a “technology deficit” relative to the US. As measured by manufacturing productivity, the share of information technology equipment, or by its contribution to output growth, European technology has lagged behind. Furthermore, Europe has had a tradition of fostering specialized, skill-specific, “vocational” education at the upper-secondary and higher levels.

In this paper, motivated by the above-mentioned empirical facts, we formalize the hypothesis that public policies favoring vocational education over a concept-based, “general” education may contribute to slower technology adoption and economic growth, especially during times of rapid technological change. We posit that workers suffer a loss of task-specific productivity when a familiar production technology is replaced with a new one. This is more likely to happen to vocationally trained workers than to those with general education. Indeed, this dynamic advantage is the only benefit general education confers to workers in our framework; workers with identical current productivity levels supply the same instantaneous labor efficiency units irrespective of their type of education.

The notion that education helps in coping with technical change dates back at least to Nelson and Phelps (1966), who use reduced form specifications to postulate a higher return to education in an economy with more rapid technological change, and to Welch (1970), who provides supportive evidence for the dynamic advantage of education using wages of college graduates. The theoretical contribution of our paper is to embed this idea in an equilibrium growth model that jointly models the adoption decision of firms and the decision of individuals to acquire a particular type of education, and analytically characterize the effect of education policy on growth. On the empirical front, we provide an explanation consistent with the “Eurosclerosis” view that holds rigid government policies responsible for inhibiting economic adjustment, thereby causing low employment and slow growth.

1While Welch (1970) uses R&D intensity as a proxy for technical change, Bartel and Lichtenberg (1987, 1991) use age of equipment as a proxy for lack of change and find that the labor cost share and the wage rate decrease with equipment age. Gill (1988) finds that higher TFP industries employ a larger proportion of educated workers and that the wage profile for highly educated workers shifts out with increasing TFP growth and is also steeper. Benhabib and Spiegel (1994) find that the human capital stock affects the speed of technology adoption in a cross-country context, lending support to a specification in Nelson and Phelps (1966). Thus, the advantage of education in adapting to technical change has both theoretical precedence and empirical support in the literature.

2The economist Herbert Giersch is generally credited with coining the term “Eurosclerosis.”
focus is on continental Europe’s education policies.\footnote{We formally elaborate on a theme voiced by Lawrence and Schultze (1987), “The European economies...now experienced problems in graduating from a catch-up economy to one on the frontier of technology... Workers must have general training to adapt to new tasks, and European education, which has encouraged apprenticeships that provide specific skills, must adapt.”}

In our model, newly born individuals optimally and irreversibly choose between one of the two streams of education mentioned above, based on their intrinsic ability to absorb conceptual education, anticipated market conditions, and government subsidies for the two types of education. They weigh the dynamic advantage of general education against the higher cost of acquiring it and a lower initial level of task-specific productivity.\footnote{We assume, conservatively, that workers with skill-specific education, who are trained for a specific job, have an advantage in this regard.}

Firms have the choice of producing a single non-storable good either through technologies (production methods) \textit{used} in the previous period – which have become well-understood and readily usable in the present period at no cost – or by adopting, at a cost, new technologies up to the available frontier. This technology frontier evolves exogenously. The non-adopting firm, also referred to as the “low-tech” firm, can use the old technology without any adoption cost. The adopting firm, also referred to as the “high-tech” firm, has to pay a cost of adoption that depends on the distance between the new and the previously used level of technology in a convex fashion, as well as potentially higher wages to attract workers who face the risk of a loss in task-specific productivity caused by their move to an unfamiliar and more complex technology. Conditional on the education decision made at the beginning, during every period of the remainder of their lives individuals choose between working in these two types of firms.

We completely characterize the education and occupational choices of individuals and the adoption decision of firms, and show that the equilibrium growth rate is lower in an economy that allocates more of a given amount of resources toward vocational education. The equilibrium growth rate is equal under two different education policies if and only if the economy, under both policies, adopts new technologies at the same rate at which they become available.

The positive relationship between the fraction of the workforce with general education and growth may intuitively follow from our assumption on the dynamic advantage of such education, although demonstrating this requires a fully specified model such as the one we have developed. However, what is not immediately obvious is the effect of an increase in the rate of available technology; the true value of the model lies in showing that in such an event, countries with different education systems that had comparable growth initially could diverge. More specifically, the difference in the growth rate between an economy that focuses on vocational education and one that focuses on general education is shown to increase with
the exogenous rate of technology availability; the economy with better general education can more readily exploit the new technologies and might, in fact, be constrained only by their availability.

Our model suggests that while European education policies that favored specialized, vocational education may have worked well during the 60s and 70s when technologies were more stable, they may have contributed to slow growth and increased the European growth gap relative to the US during the information age of the 80s and 90s when new technologies emerged at a more rapid pace.\footnote{See, for instance, Greenwood and Yorukoglu (1997), who argue that there was an increase in the rate of technological change during the 70s. Hornstein and Krusell (1996) and Krusell, Ohanian, Rios-Rull, and Violante (2000) arrive at a similar conclusion.} The following observations made in the *European Competitiveness Report 2001* directly speak to this suggestion: “The growing consensus that the strong growth and productivity performance in the US is related to increased investment and diffusion of ICT goods and services has raised concerns that the weaker economic performance of EU Member States is caused by sluggishness in the adoption of these new technologies... in recent years skill shortages in important technology areas have been reported in several European countries... It appears that, unlike in previous years, when the long-term trend increase in the demand for skills was met by the supply of technology professionals from the educational system, the surge in demand for ICT-related skills in the 1990s found no corresponding supply forthcoming.” (ICT: Information and Communication Technologies)

We do not claim that the emphasis on skill-specific education alone is responsible for Europe's technology deficit or recent slow growth. Clearly, several other explanations such as generous unemployment insurance and inflexible labor laws would be required to complete the quantitative picture. Rather, our aim is to build a framework grounded in reasonable assumptions that focuses on the educational aspect, delivers key observed stylized facts, and lays a theoretical foundation for future empirical and quantitative work. We view the development of a tractable growth model, featuring heterogeneity in the type of education and a dynamic advantage of education, as a goal in its own right.

In addition to the above-mentioned early precursors in the literature, our paper is related to a few other, more recent, studies. Ljungqvist and Sargent (1998) model unemployment as an event that causes a loss in human capital in order to study the effect of European unemployment insurance schemes on the level of unemployment. Our focus is very different, but in a similar spirit we model stochastic losses in productivity arising from a change in production technologies. Violante (2002) posits a skill transferability function across jobs that depends on the technological distance between machine vintages in order to study the relationship between the rate of technological change and wage dispersion. The skill transition matrix we model is akin to his transferability function, though we index it by the
type of education. Neither of these papers is concerned with endogenizing the education decision and studying the effect of education policy on growth. Gould, Moav and Weinberg (2001) also focus on inequality caused by a depreciation of technology-specific skills, but this occurs randomly across sectors in their model; such depreciation is induced by a choice to work in the high-tech sector in our framework. Education is a choice variable for them, but there is no intentional adoption of new technology by firms.

Acemoglu (1998) develops a model in which an increase in skilled labor induces faster upgrading of skill-complementary technologies by firms; in our setup, an increase in the measure of workers with general education would have a similar effect. However, he does not distinguish between vocational and general education among skilled workers. Neither does education have a dynamic advantage in his model. Our research is complementary to Galor and Tsiddon (1997), who argue that during times of rapid technological progress the return to ability increases and that to specific human capital decreases, increasing mobility and the concentration of high-ability individuals in high-tech sectors, thereby fueling future growth. In this view, impediments to mobility in Europe could cause it to trail the US in economic performance. While our skill transferability matrix could be viewed as partly capturing such impediments to mobility, our primary focus is on educational policy differences between the US and Europe. Our work is also complementary to Galor and Moav (2000), who develop a model in which education does play a dynamic role, but focus on the effect of technological change on wage inequality. Unskilled labor is assumed to count for less in a composite labor input when growth is higher, and ability enables individuals to cope with technological progress. As in these two papers, we also assume an exogenous ability distribution, but given our focus on education policies, we endogenously model the acquisition of general education based on this ability. Prescott (1998) calls for theories of total factor productivity differences; we provide an education-based theory of such differences.

The rest of the paper is structured as follows. We summarize the stylized facts that motivate our study in Section 2. In Section 3 we present the economic environment and define a competitive equilibrium and a balanced growth path (BGP). The BGP is analyzed in Section 4. Section 5 contains our central theoretical results: an increase in subsidy for vocational education at the expense of general education will decrease growth, and the growth gap relative to an economy that focuses on general education will increase with the rate of arrival of new technologies. Section 6 concludes the paper. Proofs and derivations are presented in the appendix, unless otherwise noted.
2 Stylized Facts

In this section we present the stylized facts that provide empirical motivation for our study – slow European per capita growth and manufacturing productivity growth since the 80s, Europe’s “technology deficit”, and its bias toward vocational education.

Consider annualized per capita GDP growth for the US and two countries we will use throughout as representatives of Western Europe – Germany and Italy. In the 70s Germany (2.6%) and Italy (3.1%) grew faster than the US (2.1%). In the 80s, the US grew at the faster rate of 2.3%, compared to 2.0% and 2.2% for Germany and Italy. The US lead solidified in the 90s; it grew at 2.0%, while Germany and Italy managed only 1.0% and 1.2%. The difference in growth rates is even more pronounced during the 1995-98 period; the US grew at 3% while the two European countries managed only 1.3% each.\(^6\)

Since our theoretical framework focuses on technology adoption, productivity growth, technology usage, and technology production might be more relevant indicators of economic performance. Gust and Marquez (2000) study aggregate data and find that labor productivity in the major European countries did not accelerate in the latter half of the 1990s as it did in the US; TFP growth was also lower relative to that of the US. Our model is most likely to apply to the manufacturing sector; there, US labor productivity has outpaced that of Germany from as early as the mid-80s. The 1986-1990 and 1991-2000 figures for the US are 2.3% and 4.3%, and those for Germany are 2.0% and 3.8%. While Italy did better than the US in the initial period (3.8%), during the latter period its labor productivity growth of 2.2% was only half of US productivity growth.\(^7\)

The difference is much starker when one examines technology-driven industries – in the US, these industries recorded an average annual productivity increase of 8.3% in the 1990s, when compared to the 3.5% achieved in the same industries as the European Union.\(^8\)

There is abundant direct evidence that, with the exception of Sweden, Finland, and the Netherlands, Europe lags behind the US in technology usage.\(^9\) Schreyer (2000), presents data on the share of information technology (IT) equipment in total investment. In 1985, this share was 6.3% in the US, and 3.4% each for Germany and Italy. By 1996 the gap had

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\(^6\) Growth rates are from Scarpetta et. al. (2000).

\(^7\) See Table IV.1 in European Competitiveness Report 2001.

\(^8\) See page 66, Graph IV.5, and Table IV.6 in European Competitiveness Report 2001. Pharmaceuticals, Office machinery and computers, Motor vehicles, Aircraft and spacecraft, are a few of the industries classified as Technology-driven industries.

\(^9\) It is highly consistent with our theory that these three exceptions have higher figures (relative to the rest of Europe) for one or more of the following statistics: percentage of upper secondary students enrolled in general programs, net enrollment in university-level tertiary education, and percentage of public education expenditure devoted to subsidies for tertiary education. (See tables C3.2, C5.2a, and B3.2 in Education at a Glance: OECD Indicators, 1997.)
widened, with 13.4% for the US, and 6.1% for Germany and 4.2% for Italy. Schreyer also presents results from growth accounting studies which show the contribution of Information and Communication Technology (ICT) capital to output growth; these are presented in Table 1.10

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<tr>
<td>US</td>
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<td>0.34</td>
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<tr>
<td>Germany</td>
<td>0.12</td>
<td>0.17</td>
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<td>Italy</td>
<td>0.13</td>
<td>0.18</td>
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The contribution of ICT capital to output growth has been increasing for all countries, but the gap between the US and European countries has been increasing as well. Since our model is most suited to the technology-driven sectors, delivering a stylized version of this table is an important goal of our theoretical analysis.

For evidence that increased usage of such technology improves productivity, we turn to Stiroh (2001), who conducts econometric tests using industry-level data to show that IT-intensive industries experienced significantly larger labor productivity gains than other industries; he also finds a strong correlation between IT capital accumulation and labor productivity.11

There is evidence that ICT production is also correlated with TFP growth.12 In the US, the computer and semiconductor industries contributed more than 50% to nonfarm business TFP growth during 1974-90 despite their small share in output.13 Europe has lagged behind in ICT production as well (see Chart 3 in Gust and Marquez (2000)).

Though the actual magnitude of the productivity boom in the US during the 90s continues to be debated (see, for instance, Gordon (2000) for a skeptic’s view), the facts that Europe

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10 We present data from Table 4 in Schreyer (2000), which uses a ICT price index harmonized across countries. When these figures are calculated in percentage terms instead of absolute percentage points, similar patterns persist.

11 A positive correlation between the ICT share and TFP growth also emerges from Graph 6 in the European Commission (2000) report.

The widespread nature of productivity acceleration reported by Stiroh (2001) casts doubts on an alternate explanation for the US productivity advantage, that the US with its more liberal immigration policies attracts highly skilled specialists to the high-tech sectors, and American focus on general education has nothing to do with this advantage. This would not explain why industries as far flung as “Security and Commodity Brokers,” not typically populated by immigrants, experienced huge increases in productivity growth. The assumption that only workers with high levels of education immigrate to the US is not tenable, and many immigrants first acquire general education in US universities before they begin work. The immigration explanation also does not explain why there was a change in the European economic performance starting in the 80s when there was no corresponding change in European immigration laws.


13 See Table 4 in Oliner and Sichel (2000).
has lagged behind the US in the last two decades in technology usage and production, and experienced slower productivity growth, are unlikely to be overturned by recent evidence.

Since our hypothesis is that the diverging trends between the US and Europe can be explained by differences in the educational system, we now present evidence on European focus on vocational education.\textsuperscript{14} In 1991, 79.3% of upper secondary students in West Germany and 70.6% of Italian students were enrolled in vocational or apprenticeship programs. The EU average was 58%. In contrast, there is no separate stream of vocational education in the US at this level; however, the percentage of students who completed 30% or more of all credits in specific labor market preparation courses was just 6.8% in 1990.\textsuperscript{15} Since education at this level is typically fully funded by the government, this data suggests that the European governments spend a greater fraction of their resources on vocational training than the US.\textsuperscript{16} US vocational education is typically imparted in two-year community colleges; of those over the age of 18 enrolled in post-secondary education, only 13.8% were working toward a vocational Associate’s degree in 1991; this figure fell to 10.5% in 1994.\textsuperscript{17}

A related indicator is the net entry rate into universities, where general education is primarily imparted; it is 52% in the US but only 27% in Germany, 33% in France, and 26% in Austria.\textsuperscript{18} This lower European enrollment is reflected in attainment; while 25% of adults had completed university-level education and 8% had completed non-university tertiary education (primarily vocational) in the US in 1995, in Germany 13% had completed university education and 10% non-university education. The claim is sometimes made that the European high school graduates are better equipped than their American counterparts, and comparisons of college attainment alone might be misleading. If this claim were true, it would only deepen the puzzle of slow European adoption of technologies.

Allocation of educational resources differs between the US and continental Europe. The OECD Education Database indicates that the percentage of GDP devoted to primary and secondary education was about the same – 3.8% for the US and Germany in 1997 and 3.4% for Italy. However, the percentage devoted to tertiary education in the US (2.6%) far outstripped the percentages in Germany (1.1%) and Italy (0.8%). The expenditure per

\textsuperscript{14}The strict classification of education into general and vocational is at best an oversimplification and should be viewed as a metaphor for the rigidity and inflexibility of European upper secondary and post-secondary education. We also abstract from the diversity in education systems that exists even within continental Europe.

\textsuperscript{15}These figures are from Table 3.2 of Medrich et. al. (1994). Also see the European Commission’s 1998 report, \textit{Young People’s Training}. From OECD’s \textit{Education Policy Analysis} 1997, it also appears that the initial emphasis on vocational education in Europe is not overturned at later stages of education; in Germany only about 20% of university entrants do not have a general entry qualification.

\textsuperscript{16}The German apprenticeship program involves partial outlays by firms as well.

\textsuperscript{17}See Table 87 in Levesque et. al. (2000).

\textsuperscript{18}See Table C4.1 in \textit{Education at a Glance: OECD Indicators 1997}. 
student relative to GDP per capita on post-secondary non-tertiary (vocational) education was 49\% in Germany for 1997, amounting to 10,839 PPP dollars; little happens on this front in the US. The corresponding figures for tertiary education was 43\% in Germany but 59\% in the US. In PPP dollars, the tertiary education expenditure per pupil was 9,466 in Germany while in the US was 17,466 (it was 5,972 in Italy).

Therefore, it seems reasonable to conclude that more resources are allocated to vocational education in Europe relative to the US, where the emphasis has been on college (and thus mainly general) education. We use this observation to motivate revenue-neutral policy experiments in which European vocational education subsidy per student is higher than that of the US. The equilibrium enrollment and education attainment implied by the model presented below will then be qualitatively consistent with the data presented above.

3 The Environment

The economy is populated by a continuum of households and two representative firms, one in the sector that potentially adopts new technologies in every period, and one in the non-adopting sector.\(^{19}\) There is a single nonstorable consumption good in each time period and households supply labor to the firms. In this section we describe the maximization problems of both firms and of a typical household, and finally define equilibrium and a balanced growth path.

3.1 Firms and Technology Adoption

The firms in this economy are owned by infinitely lived entrepreneurs, who consume all profits from production each period, as they, like the workers, have no access to an intertemporal storage technology.

The firms in the low-tech sector that do not adopt new technologies in the current period face the production technology:

\[
Y_{n,t} = A_t \left( H_{n,t} \right)^\theta, 
\]

where \(Y_{n,t}\) is output of the nonadopting firm in period \(t\), \(H_{n,t}\) is the labor input used by that firm in period \(t\), \(A_t\) is the level of technology that is freely available in period \(t\) and \(\theta \in (0, 1)\) is an intensity parameter. The nonadopting firm takes \(A_t\) and the real wage rate (per effective unit of labor) \(W_{n,t}\) in the nonadopting sector as given.

\(^{19}\)We have abstracted from issues of industrial organization, such as free entry, to keep our model analytically tractable. It seems unlikely that we are missing any first-order issue on account of this simplification for the question we seek to address.
The firms in the high-tech sector of the economy that (potentially) adopt new technologies face the production technology

\[ Y_{a,t} = A'_t (H_{a,t})^\theta, \]

where \( Y_{a,t} \) is output of the adopting firm in period \( t \), \( H_{a,t} \) is the effective labor input used by that firm and \( A'_t \) is the level of technology used in period \( t \), which is a choice variable for the adopting firm. We let \( W_{a,t} \) denote the real wage rate (per effective unit of labor) in the adopting sector. The technology frontier grows exponentially, i.e.

\[ A_{f,t} = \lambda A_{f,t-1}, \tag{1} \]

where \( \lambda > 1 \) is the constant gross growth rate of the frontier technology. The technology choice of the adopting firm has to satisfy \( A'_t \leq A_{f,t} \). We assume that \( A_0 = A'_{t-1} = A_{f,t-1} = 1 \); i.e. the economy starts at the technology frontier.

We further assume that the nonadopter firm can use the highest technology that was used last period, and hence has become common practice; that is, \( A_t = A'_{t-1} \).\(^{20}\) It is important to note: 1) It is the technology actually adopted in the previous period rather than the frontier technology that was available last period that spills over costlessly to the next period, and 2) The adopting firms are small and do not take into account the influence that their adoption choice today has on the next period’s common practice.

The parameter \( \lambda \) is the potential growth rate of the economy. If the economy keeps pace with these periodic inventions by adopting them as fast as they occur, the actual growth rate will be the same as the potential one. An increase in \( \lambda \) is later used to model an increased pace of technological change.

To use a level of technology \( A'_t \in [A_t, A_{f,t}] \), the firm incurs a cost of adopting the new technology, which is increasing and convex in the distance between \( A_t \) and \( A'_t \). This cost captures the firm’s outlays for training workers, fixing “bugs” that will allow the full potential of the new technology to be realized today before they are ironed out next period by the technology provider, etc. We assume that the cost of adoption takes the form:

\[
C(A_t, A'_t) = \begin{cases} 
\frac{A'_t}{A_t} \left( \frac{A'_t}{A_t} - 1 \right)^2 & \text{if } A'_t > A_t \\
0 & \text{if } A'_t \leq A_t.
\end{cases}
\]

\(^{20}\) This can be viewed as a reduced form modeling of learning by doing in which productivity gains spill over across sectors. See, for instance, Young (1993) and the references therein.

We assume that there are no international spillovers of technology. If there are such spillovers, European technology would catch up to the US technology with whatever lag is assumed for the spillover. Data does not seem to support such automatic spillovers. Even with a mature technology such as personal computers, Microsoft data shows that in the US 90% of white-collar workers use them, whereas only 55% in Western Europe do so. The International Data Corporation reports that the US market for PCs grew 15% in 1996, while the Western European market grew only by 7.1%. (See the article, “Europe’s Technology Gap is getting Scary,” in the March 17, 1997, issue of Fortune.)
Since our analysis will focus on balanced growth paths (BGP) in which the growth rate 
\( x = \frac{A_t'}{A_t} \leq \lambda \) is constant, whenever there is no ambiguity we drop time subscripts from \( A_t, A_t' \) and \( A_{f,t} \). If \( A \) is viewed in units of machines, the cost function implies that along a BGP the cost of retooling each machine with the new technology, \( \frac{C}{A} = \frac{x}{x-1}^2 \) is constant.\(^{21}\)

### 3.2 Households

There is a measure one of potentially infinitely lived agents. Each agent has a constant probability of \( \alpha \) of dying at the end of each period, identical across agents (see Yaari (1965)). We invoke a law of large numbers and assume that \( \alpha \) is also the deterministic fraction of the population that dies in any given period. At the beginning of each period, a measure \( \alpha \) of new agents is born and ready for education at the upper secondary level to replace those who died at the end of the previous period.

#### 3.2.1 Education Choices

Each agent is born with an ability \( a \in [0, 1] \) for higher education, which is distributed uniformly across the population; i.e. according to the cumulative distribution function \( F(a) = a \). At the beginning of her life each agent has to choose between general education \( g \) and vocational education \( v \). There is a utility cost of general education, \( e(a) \), which is strictly decreasing in \( a \). This captures the greater difficulty in learning conceptual material, cost of longer duration of education, lower subsidies relative to vocational training, etc. Once the education choice \( i \in \{g, v\} \) has been made, it is irreversible until agents die.

#### 3.2.2 Skill Accumulation

At any given time, there are two types of workers – those with general (typically college) education (\( g \)) and those with vocational education (\( v \)). At the start of every period each worker has to decide whether to work in the adopting or the nonadopting sector. Each worker has a skill level for her current occupation of \( H \in \mathcal{H} = \{1, h\} \) which follows a Markov chain, with transition probabilities that depend on the sector of the economy the agent chooses to work in. We assume that \( h > 1 \). We refer to a worker with \( H = 1 \) as a low-skill worker (indicated by subscript \( l \)) and one with \( H = h \) as a high-skill (indicated by subscript \( h \)) worker.

If a worker with skill level \( H \) works in the nonadopting sector her skill level is perfectly persistent, i.e. she will have skill level \( H' = H \) tomorrow. This assumption captures the

\(^{21}\) The modeling choice of not making the cost of adoption depend directly on the measure of the labor force with general education appears to be a conservative one. Our results are likely to be strengthened if “skilled” labor is required to adopt technologies.
lower risk of skill depreciation (or appreciation, for that matter) faced by workers employed in the sector in which an older technology is used.

If the worker chooses to work in the technology-adopting sector, her skill evolves according to the transition probability:

$$T_i(H, H') = \frac{1}{h} \begin{pmatrix} T_{il} & 1 - T_{il} \\ 1 - T_{ih} & T_{ih} \end{pmatrix},$$

for $i \in \{g, v\}$. For example, $T_{il}$ denotes the probability that a low-skill worker of education type $i$ who chooses to work in the adopting sector remains at the same skill level.\(^{22}\) We again invoke the law of large numbers and assume that $T_{ih}$ is the deterministic fraction of the population that, conditional on having skill level $H$ and working in the adopting sector, will have skill level $H' = H$ tomorrow. Since this economy features only idiosyncratic uncertainty, but no aggregate uncertainty, all aggregate variables follow nonstochastic time paths.

We adopt the following timing convention. An agent enters the current period with initial skill level $H$; then she decides in which sector to work and then the skill level $H'$ relevant for the current period’s job is realized (it either remains the same, if the nonadopting sector is chosen, or evolves according to $T_i$). In other words, a workers’ skill level is scrambled around before she works and is paid the appropriate wage. The worker then enters next period with initial skill level $H'$.

We assume that workers who obtain vocational education start their working career with skills $H = h$, whereas workers with general education start with skill levels $H = 1$. This assumption captures the job-readiness that vocational education imparts for the technology currently in use; indeed, if that technology does not change, vocationally educated workers would forever have high productivity as they have been trained specifically for it.\(^{23}\)

Our main assumption is that workers with general education find it easier to adopt to new technologies than workers with vocational education; this is the only margin along which general education is superior to vocational education.\(^{24}\) Formally:

\(^{22}\) The transition probability could be made a function of the growth rate of the frontier technology; the probability of retaining or acquiring a higher skill level for an occupation would presumably depend negatively on the speed of technological progress, and more strongly so for vocationally trained agents. However, as in the adoption cost specification, ignoring this dependence appears to be a conservative move.

\(^{23}\) This makes lemma 6 below a bit easier to prove. However, the overriding reason for the assumption is empirical plausibility; it is also a conservative assumption for our purposes.

\(^{24}\) Technical and ancillary assumptions are included in the text and not highlighted the way Assumption 1 is.
Assumption 1:

\[ T_{gl} < T_{vl}, \quad T_{gh} > T_{vh}, \]
\[ T_{gl} > 1 - T_{gh}, \quad T_{vl} > 1 - T_{vh}, \]
and \( T_{gl} < 1 - T_{vh}. \)

The first two inequalities capture the above-mentioned dynamic advantage of general education. The next two inequalities ensure that a high-skill agent of a given education type has a higher probability of being high-skilled when working in the adopting sector than a low-skill agent of the same type. The last inequality ensures that, upon choosing the high-tech sector, even an initially low-skilled agent with general education has a higher probability of becoming high-skilled than an initially high-skilled agent with vocational education. Let \( E_{iH} \) denote the skill level that agent of education type \( i \) with current skill level \( H \) can expect to have after deciding to work in the high-tech firm.

\[ E_{il} = T_{il} + (1 - T_{il})h; \quad E_{ih} = (1 - T_{ih}) + T_{ih}h. \] (2)

Given \( h > 1 \) and the previous assumptions we obtain the strict ordering: \( E_{vl} < E_{vh} < E_{gl} < E_{gh}. \)

### 3.2.3 Endowments and Preferences

A newborn household of type \( a \) has preferences over infinite stochastic streams of consumption \( c = \{c_t\}_{t=0}^{\infty} \). The only endowment the household has is one unit of time in each period that can be supplied in the labor market. In order to simplify our analytical treatment of the economy, we assume that workers do not have access to an intertemporal storage technology or any assets that enables them to smooth consumption over time. Note however that agents, by choosing to always work in the nonadopting sector, guarantee themselves a perfectly smooth consumption profile over time (in a BGP). Therefore an agent, depending on her education, current skill level \( H \), and industry choice \( j \in \{n, a\} \), consumes

\[ c_t = W_{n,t}H = W_{n,t}H', \] if she works in the nonadopting sector, and:

\[ c_t = W_{a,t}H' = \begin{cases} W_{a,t}H & \text{with probability } T_{iH} \\ W_{a,t}H-1 & \text{with probability } 1 - T_{iH}, \end{cases} \]

---

25 In order to focus on the potential obsolescence of job-specific skills wrought by new technology, we have ignored on-the-job learning in our specification. In our model, moving to the high-tech sector allows workers who have drawn a bad “match” (i.e. have a low job-specific skill level), a chance to draw a better match (higher job-specific skill level). On-the-job-learning would provide an alternate way of becoming more skilled. Incorporating such learning in our model would be an interesting extension, but one that is likely to make analytical characterization of the equilibrium infeasible.

26 Although there is no aggregate uncertainty in this model, the idiosyncratic uncertainty requires us to spell out the underlying stochastic structure of the skill level process. If \( H' \) denotes the history of skill levels of an agent \( a \in [0, 1] \), consumption would have to be written as \( c_t(H') \). Since we will cast our economy in recursive language in the next section, we leave the dependence of \( c_t \) on \( H' \) implicit here.
if she works in the adopting sector, where $H_{-1} = H - \{H\}$. Households maximize:

$$U(c) = (1 - \beta(1 - \alpha)) \left\{ Et \sum_{j=0}^{\infty} [\beta(1 - \alpha)]^j \log(c_{t+j}) - I_g \log(A_t \exp(e(a))) \right\},$$

by choosing the sector to work in each period and the type of education, where $I_g = 1$ if the household chooses to obtain general education and 0 otherwise.\(^{27}\) The expectation $E_t$ is taken with respect to the underlying stochastic process governing skill level transitions that occur whenever the agent works in the adopting sector.

### 3.3 Recursive Competitive Equilibrium

The aggregate state of this economy is given by the current level of technology $A$, the technology frontier $A_f$, and the cross-sectional distribution of workers over their education levels $i \in \{g, v\}$ and their job-specific skill levels $H \in \{g, h\}$. Let $\mu_{iH}$ denote the fraction of the work force of type $(i, H)$. The aggregate state is thus $s = (A, A_f, \mu)$.\(^{28}\) From our assumptions it is also clear that there exists a cutoff level $a^*(s) \in [0, 1]$ such that all agents with ability $a(s) \geq a^*(s)$ will choose to obtain general education ($I_g = 1$), whereas all agents with $a(s) < a^*(s)$ will choose to obtain vocational education ($I_g = 0$).

#### 3.3.1 Workers’ Problem in Recursive Formulation

A worker with a particular type of education is fully identified by $(i, H)$. Let $V_{iH}(s)$ denote the expected continuation utility of an agent of type $(i, H)$ if the aggregate state is $s$, excluding the potential disutility from obtaining a general education in the first period of an agent’s life. After having made the education decision, the only economic decision a worker has to make each period is whether to work in the nonadopting or the adopting sector. Given wages $W_n(s), W_a(s)$, the value functions satisfy,

$$V_{iH}(s) = \max_{a, n} \{T_{id} [(1 - \beta(1 - \alpha)) \log(W_a(s)) + \beta(1 - \alpha) V_{id}(s')] + (1 - T_{id}) [(1 - \beta(1 - \alpha)) \log(W_a(s)h) + \beta(1 - \alpha) V_{id}(s')] \},$$

$$(1 - \beta(1 - \alpha)) \log(W_n(s)) + \beta(1 - \alpha) V_{id}(s')) \} \tag{3}$$

\(^{27}\)The normalization of the utility cost of education by $\log(A_t)$ is required to ensure that the relative cost of education for newborn agents does not diminish over time when the economy grows. Though we have not explicitly modeled it, if the cost of general education involves a greater time commitment, which results in lost wages and quality-adjusted leisure, such a normalization would be appropriate.

\(^{28}\)Note that an agent’s ability level $a$ will only affect her education decision in the first period of her life, but not subsequent consumption levels and choices in which sector to work conditional on the educational choice. Therefore, we do not have to further index allocations by ability level $a$. 

13
and

\[
V_{ih}(s) = \max_{a,n} \{T_{ih} \left[ (1 - \beta(1 - \alpha)) \log(W_a(s)h) + \beta(1 - \alpha)V_{ih}(s') \right] + (1 - T_{ih}) \left[ (1 - \beta(1 - \alpha)) \log(W_a(s)) + \beta(1 - \alpha)V_{il}(s') \right],
\]

\[
(1 - \beta(1 - \alpha)) \log(W_n(s)h) + \beta(1 - \alpha)V_{ih}(s')).
\]  

(4)

Let \(I_{ih}(s) = 1\) if agent \((i, H)\) chooses to work in the adopting sector and 0 if she chooses to work in the non-adopting sector. If a worker is indifferent between both sectors, \(I_{ih}(s) \in [0, 1]\) and we interpret \(I_{ih}(s)\) as the fraction of agents of type \((i, H)\) working in the adopting sector.

Aggregate effective labor supply in both sectors implied by workers’ choices are then given as:

\[
H_a^n(s) = \sum_{i=\nu,A} \sum_{H=1,h} \mu_{iH} (1 - I_{ih}(s)) H
\]

\[
H_a^a(s) = \sum_{i=\nu,A} \sum_{H=1,h} \mu_{iH} I_{ih}(s) E_{iH},
\]

where \(E_{iH}\) is defined in (2).

### 3.3.2 Equilibrium

We are now in a position to define a recursive competitive equilibrium.

**Definition 1** A recursive competitive equilibrium consists of value functions \(V_{iH}(s)\) and policy functions \(I_{iH}(s)\) for the household, implied aggregate labor supply functions \((H_a^a(s), H_a^n(s))\), a cutoff level \(a^*(s)\), labor demand functions for firms \((H_a^a(s), H_a^n(s))\), and a technology adoption function \(A'(s)\) for the adopting firms, wage functions \((W_n(s), W_a(s))\) and an aggregate law of motion \(\Phi\) mapping today’s aggregate state \(s\) into tomorrow’s aggregate state \(s'\) such that:

1. Given \((W_n(s), W_a(s))\) and \(\Phi\), the \(V_{iH}\) solve Bellman equations (3) and (4), and \(I_{iH}\) are the associated policy functions.

2. Given \((W_n(s), W_a(s))\),

\[
H_a^d(s) \in \arg\max_{H \geq 0} A'(H)^\theta - W_a(s)H
\]

\[
(H_a^d(s), A'(s)) \in \arg\max_{H \geq 0, A' \leq A_f} A'(H)^\theta - W_a(s)H - C(A, A').
\]

3. \(H_n^d(s) = H_n^a(s)\); and \(H_a^d(s) = H_a^a(s)\).

4. The cutoff \(a^*(s)\) satisfies: \(V_{gl}(s) - \log(A \exp(e(a^*(s)))) = V_{eh}(s)\).\(^{29}\)

\(^{29}\)We later make assumptions to guarantee an interior \(a^*\).
5. The aggregate law of motion $\Phi$ is induced by the Markov transition functions $T_i$, the policy functions $A'(s)$ and $I_{iH}(s)$ and the cutoff $a^*(s)$ (as described below).

Given a state $s = (A, A', \mu)$ today, the state $s' = \Phi(s) = (A', A'_f, \mu')$ tomorrow is determined as follows. The frontier evolves exogenously, $A'_f = \lambda A_f$. Given the endogenously determined technology adoption function $A'(s)$, we have $A' = A'(s)$. This leaves the next period’s distribution over types, $\mu'$, to be determined. Let $\eta_v(s)$ denote the fraction of newborn agents deciding to get vocational education and $\eta_g(s)$ denote the fraction of newborn agents deciding to obtain general education; given the threshold ability mentioned above, and a uniform ability distribution, these fractions are $\eta_v(s) = a^*(s)$ and $\eta_g(s) = 1 - a^*(s)$ respectively. We will first present the Markov transition function relating skills today and skills tomorrow for the $(1 - \alpha)$ surviving old workers (education remains fixed after birth, which accounts for the zeros off the diagonal blocks):

\[
\begin{array}{cccccc}
(g, l) & (g, h) & (v, l) & (v, h) \\
g, l & 1 - I_{gl}(s) + I_{gl}(s) T_{gl} & I_{gl}(s) (1 - T_{gl}) & 0 & 0 \\
g, h & I_{gh}(s) (1 - T_{gh}) & 1 - I_{gh}(s) + I_{gh}(s) T_{gh} & 0 & 0 \\
v, l & 0 & 0 & 1 - I_{vl}(s) + I_{vl}(s) T_{vl} & I_{vl}(s) (1 - T_{vl}) \\
v, h & 0 & 0 & I_{vh}(s) (1 - T_{vh}) & 1 - I_{vh}(s) + I_{vh}(s) T_{vh}.
\end{array}
\]

By assumption, the $\alpha \eta_v(s)$ newborn workers who obtain vocational education start with high skills, whereas the $\alpha \eta_g(s)$ newborn workers who obtain general education start with low skills. The distribution over skill and education types tomorrow is therefore given as:

\[
\begin{align*}
\mu'_{gl}(s) &= \alpha \eta_g(s) + (1 - \alpha) \left[ \{1 - I_{gl}(s) + I_{gl}(s) T_{gl}\} \mu_{gl} + I_{gh}(s) (1 - T_{gh}) \mu_{gh} \right] \\
\mu'_{gh}(s) &= (1 - \alpha) \left[ I_{gl}(s) (1 - T_{gl}) \mu_{gl} + \{1 - I_{gh}(s) + I_{gh}(s) T_{gh}\} \mu_{gh} \right] \\
\mu'_{vl}(s) &= (1 - \alpha) \left[ \{1 - I_{vl}(s) + I_{vl}(s) T_{vl}\} \mu_{vl} + I_{vh}(s) (1 - T_{vh}) \mu_{vh} \right] \\
\mu'_{vh}(s) &= \alpha \eta_v(s) + (1 - \alpha) \left[ I_{vl}(s) (1 - T_{vl}) \mu_{vl} + \{1 - I_{vh}(s) + I_{vh}(s) T_{vh}\} \mu_{vh} \right].
\end{align*}
\] (5)

3.3.3 A Balanced Growth Path

A balanced growth path is defined to be a recursive competitive equilibrium for which all elements of the equilibrium, normalized by the current level of technology in an appropriate fashion, are constant. Since growth in this economy is driven exclusively by the adoption of new technologies, the growth rate along a balanced growth path is given by $x \equiv A'$. We normalize wage per skill unit as $w_n = \frac{W_n}{A'}$ and $w_a = \frac{W_a}{A'}$. The normalized firm maximization problems become:

\[
\begin{align*}
\Pi_n &= \max_{H \geq 0} H^\theta - w_n H \\
\Pi_a &= \max_{H \geq 0, 1 \leq x \leq x} x H^\theta - w_a H - \frac{1}{2} (x - 1)^2,
\end{align*}
\] (6) (7)
where $\bar{x} = \frac{A_L}{A}$ is the maximal growth rate of technology that the adopting firm can choose. That is, the adopting firm’s problem on the BGP now involves a choice of the growth rate of technology rather than its level. As for the workers’ problem, we normalize consumption (wages) by the level of technology $A$ to obtain the following modified Bellman equations:

$$v_{il} = \max_{a, n} \{ T_{il} [(1 - \beta(1-\alpha)) \log(w_n) + \beta(1-\alpha)v_{il}] + (1 - T_{il}) [(1 - \beta(1-\alpha)) \log(w_nh) + \beta(1-\alpha)v_{ih}], (1 - \beta(1-\alpha)) \log(w_n) + \beta(1-\alpha)v_{il}] \}$$

and

$$v_{ih} = \max_{a, n} \{ T_{ih} [(1 - \beta(1-\alpha)) \log(w_nh) + \beta(1-\alpha)v_{ih}] + (1 - T_{ih}) [(1 - \beta(1-\alpha)) \log(w_n) + \beta(1-\alpha)v_{il}], (1 - \beta(1-\alpha)) \log(w_nh) + \beta(1-\alpha)v_{ih}] \}.$$  

Note that value functions in a BGP reduce to a quadruple of numbers $(v_{gl}, v_{gh}, v_{vl}, v_{vh})$, the BGP cutoff level $a^*$ then satisfies the following indifference condition that governs educational choice:

$$v_{gl} - e(a^*) = v_{vh}. \tag{10}$$

Finally, in a BGP the cross-sectional distribution over education and skill level $\mu$ is constant over time, i.e. $\mu = \mu' = \bar{\mu}$. Equations (5) then imply the following $\bar{\mu}$:

$$\begin{align*}
\bar{\mu}_{gl} &= \frac{1 - (1-\alpha) [1 - I_{gh}(1 - T_{gh})]}{1 - (1-\alpha) [1 - I_{gh}(1 - T_{gh})] - I_{gl}(1 - T_{gl})} \eta_g \\
\bar{\mu}_{gh} &= \frac{1 - (1-\alpha) [1 - I_{gh}(1 - T_{gh})] - I_{gl}(1 - T_{gl})}{(1-\alpha)I_{gl}(1 - T_{gl})} \eta_g \\
\bar{\mu}_{vl} &= \frac{1 - (1-\alpha) [1 - I_{vl}(1 - T_{vl})]}{1 - (1-\alpha) [1 - I_{vl}(1 - T_{vl})] - I_{vl}(1 - T_{vl})} \eta_v \\
\bar{\mu}_{vh} &= \frac{1 - (1-\alpha) [1 - I_{vl}(1 - T_{vl})] - I_{vl}(1 - T_{vl})}{1 - (1-\alpha) [1 - I_{vl}(1 - T_{vl})]} \eta_v.
\end{align*} \tag{11}$$

Here, $\eta_g = 1 - a^*$ is the fraction of newborn agents obtaining general education and $\eta_v = a^*$ is the fraction of agents obtaining vocational education. It can be seen from (11) that $\bar{\mu}_{gl} + \bar{\mu}_{gh} = \eta_g$ and $\bar{\mu}_{vl} + \bar{\mu}_{vh} = \eta_v = \left(1 - \eta_g \right)$. We therefore have the following definition.

**Definition 2** A balanced growth path consists of values $(v_{gl}, v_{gh}, v_{vl}, v_{vh})$, associated policies $(I_{gl}, I_{gh}, I_{vl}, I_{vh})$, labor supplies $(H^a_n, H^a_h)$, labor demands $(H^d_n, H^d_h)$ and a growth rate of technology $x$, wages $(w_n, w_a)$, a cutoff ability level $a^*$ and an invariant distribution $\bar{\mu} = (\bar{\mu}_{gl}, \bar{\mu}_{gh}, \bar{\mu}_{vl}, \bar{\mu}_{vh})$ such that:

---

\(^{30}\)One can now see that the education cost in the original recursive formulation needs to be multiplied by the technology factor $(1 - \beta(1-\alpha)) \log(A)$ in order to keep the normalized education cost constant along a BGP. The normalized value function is related to the original value functions according to: $v_{iH} = V_{iH} - (1 - \beta(1-\alpha)) \log(A)$. 

16
1. Given \((w_n, w_a)\) the values \((v_{gl}, v_{gh}, v_{vl}, v_{vh})\) satisfy Bellman equations (8) and (9), and \((I_{gl}, I_{gh}, I_{vl}, I_{vh})\) are the associated policies.

2. Given \((w_n, w_a)\), \(H^d_n\) solves (6) and \((x, H^d_a)\) solve (7).

3. \(H^d_n = H^s_n\); and \(H^d_a = H^s_a\).

4. The cutoff \(a^*\) satisfies (10).

5. The distribution \(\tilde{\mu} = (\tilde{\mu}_{gl}, \tilde{\mu}_{gh}, \tilde{\mu}_{vl}, \tilde{\mu}_{vh})\) satisfies (11).

We confine ourselves to the analysis of BGP equilibria; in particular we are interested in how different educational policies affect the attainment of general education, and hence the growth rate of the economy along the BGP, as the speed of technological advancement \(\lambda\) increases.

4 Analysis of the BGP

We first outline the steps in our strategy for characterizing the BGP; the applicable subsections and figures are given within parentheses:

- Solve firms’ problems to obtain the relative labor demand function \(\frac{H^d_n}{H^d_a} \left(\frac{w_a}{w_n}\right)\) and “growth demand” schedule \(x(w_a)\) from the problem for adopting firms (section 4.1, figure 1).

- Solve the workers’ Bellman equations for individual labor supply decisions. Aggregate labor supply to obtain \(\frac{H^s_n}{H^s_a} \left(\frac{w_a}{w_n}\right)\) (sections 4.2, 4.3, figure 2).

- Characterize labor market equilibrium and solve for the “growth supply” schedule \(x^s(w_a)\), conditional on \(\eta_g\) (section 4.4, figure 1).

- Characterize \(x^s(\eta_g)\); that is solve for the growth rate (intersection of \(x(w_a)\) and \(x^s(w_a)\)), conditional on \(\eta_g\) (section 4.5, figure 1).

- Characterize the education decision and equilibrium \(\eta^*_g\). The “unconditional” \(x^s\) and other relevant prices and quantities can then be backed out from the previous steps (section 4.6, figure 3).

We now consider each of the above steps in detail.
4.1 Firms, Labor Demand and Technology Adoption

For a given wage in the nonadopting sector $w_n$ the labor demand of the firm in that sector is given by, $H^d_n(w_n) = \left( \frac{\theta}{w_n} \right)^{\frac{1}{\sigma'}}$, and profits are obtained as, $\Pi(w_n) = \left( \frac{\theta}{w_n} \right)^{\frac{\sigma}{\sigma'}} (1 - \theta) > 0$. For the adopting sector we first solve for the conditional labor demand as a function of the wage $w_a$, and the growth rate, $x$, as:

$$H^d_a(w_a; x) = \left( \frac{\theta}{w_a} \right)^{\frac{1}{\sigma'}} ,$$  

(12)

where $x = \frac{\lambda'}{\lambda}$ is the growth rate of technical progress chosen by the firm. As a function of $x$, the relative labor demand is thus given by:

$$\frac{H^d_a}{H^d_n} = \left( \frac{xw_n}{w_a} \right)^{\frac{1}{\sigma'}} .$$

(13)

In order to guarantee concavity of the objective function in the critical range and ensure the first order condition is sufficient, we assume $\theta < \frac{1}{2}$. Using the conditional labor demand function in the objective function we can rewrite the maximization problem of the adopting firm as:

$$\max_{1 \leq x \leq \bar{x}} x^{\frac{1}{\sigma'}} \left( \frac{\theta}{w_a} \right)^{\frac{\sigma}{\sigma'}} (1 - \theta) - \frac{1}{2}(x - 1)^2 .$$

This maximization problem has a unique solution $x^* \in (1, \bar{x}]$ which may be at the corner $\bar{x}$.\textsuperscript{31} In the appendix, where most of the proofs and derivations are presented, we show:

**Lemma 1** A necessary and sufficient condition for a balanced growth path with growth rate $x = \lambda$ is:

$$\lambda \overset{\sigma}{\theta} \left( \frac{\theta}{w_a} \right)^{\frac{1}{\sigma'}} \geq \lambda - 1 .$$

(14)

Define $\bar{w}_a = (\lambda \theta) / (\lambda - 1)^{\frac{\sigma}{\sigma'}}$. The previous lemma states that only if and only if $w_a \leq \bar{w}_a$ can there exist a BGP with growth rate $\lambda$. The following corollary is immediate.

**Corollary 1** A necessary and sufficient condition for a balanced growth path with growth rate $x < \lambda$ is that $w_a > \bar{w}_a$.

The last two propositions show how the growth rate of the economy along a BGP depends on the equilibrium wage in the adopting sector. In particular, for $w_a \leq \bar{w}_a$ the growth rate is constant at $\lambda$ and the economy always uses the frontier technology in the adopting sector. For $w_a > \bar{w}_a$ the growth rate of the economy is declining in $w_a$; to summarize:

$$x(w_a) \left\{ \begin{array}{ll} \frac{\lambda}{1 - \theta} \left( \frac{\theta}{w_a} \right)^{\frac{1}{\sigma'}} = x^* - 1 & \text{if } w_a \leq \bar{w}_a \\
\text{solves } (x^*)^{\frac{\sigma}{\sigma'}} \left( \frac{\theta}{w_a} \right)^{\frac{\sigma}{\sigma'}} = x^* & \text{if } w_a > \bar{w}_a \end{array} \right. \right.$$  

\textsuperscript{31}Given our earlier assumption that the economy starts out at the frontier, on the BGP, $\bar{x} = \lambda$, or the constraint is not binding.
The graph of the function $x(w_a)$ is shown in Figure 1. The other component, the “supply” curve, $x^*(w_a)$, is derived in subsequent sections.

Finally, we can compute profits of the adopting firm (and hence consumption of its owner), as:

$$
\Pi(w_a) = \begin{cases} 
\lambda \frac{\theta}{w_a} \left( \frac{\theta}{w_a} \right)^{\theta} (1 - \theta) - \frac{1}{2}(\lambda - 1)^2 & \text{if } w_a \leq \bar{w}_a \\
(x(w_a) - 1) \left[ (1 - \theta)x(w_a) - \frac{1}{2}(x(w_a) - 1) \right] & \text{if } w_a > \bar{w}_a.
\end{cases}
$$

The profit of the adopting firm is decreasing in the wage necessary to induce workers to choose the high-tech sector. It is intuitive that when the operating cost of the high-tech firm is high, the firm would choose to spend less in technology adoption. In other words, as a function of $w_a$, the “demand” for adoption and thus growth $x$, is decreasing. The available technology limits the firm’s adoption when $w_a$ is low; this explains the flat portion of $x(w_a)$ in $[0, \bar{w}_a]$.

As $\lambda$ increases, $\bar{w}_a$, and thus the wage interval over which maximal growth occurs, decreases. Since the cost of adoption increases with the size of the technological jump, it is again intuitive that the firm is willing to adopt technologies at the increased rate of availability only for lower operating costs (i.e. lower wages).

![Figure 1: Determination of $x(\eta_a)$](image)

### 4.2 The Household Labor Supply Decision

In this section we study the sectoral choice of workers, for given wages $(w_n, w_a)$. By working in the nonadopting sector the household is assured of the same skill level tomorrow as today.
By working in the adopting sector it faces the risk of transiting to a potentially lower skill level but also potentially higher wages. The optimal choice involves a tradeoff between these effects.

At this point we make the following assumption on the endogenous wages: $w_a > w_n$. We shall later provide conditions involving only parameters of the economy to ensure that this assumption is satisfied in equilibrium.\footnote{As we will show, this “guess and verify” strategy guarantees that there is a unique equilibrium with $w_a > w_n$ but does not preclude the existence of one with $w_a \leq w_n$. The preclusion of the latter equilibrium is not automatic in our setup; low-skilled agents might choose the adoption sector even if it offers lower wages for the option of drawing a higher job-specific productivity. We focus on the former equilibrium as it seems to be the more plausible case.} It immediately follows that $v_{ih} > v_{il}$ and the optimal decision for agents with currently low job-specific skill is to move to the adopting sector (they have nothing to lose in terms of skill, but earn higher wages and possibly find that they have higher skill for the new job); that is, $I_{il} = 1$ for $i = g, v$.

It might initially seem counterfactual that the workers with lowest occupational skills are the surest to participate in the adoption process. However, under several interpretations of skill, this is not so. In our setup, it is the type of education that is human capital in the conventional sense, and workers with higher human capital (those with general education) do have an overall advantage in adoption. However, $H$ is to be interpreted as productivity idiosyncratic to a particular job, task, or an industry. As potentially skill-obsoleting innovation takes place, the above result can be rationalized as workers who have nothing to lose making riskier choices; a move to the high-tech sector is their way of finding a better “match.”

For agents with currently high skill level the situation is different; they have to weigh the of risk losing their high skill in the process of adoption against a higher wage. The relative wage gain has to be sufficiently high to induce agents to bear this risk. Given the optimal choice for the low-skilled agents, their value is given by setting the choice in (8) to adoption, which upon simplification yields:

$$v_{il} = \left[ \log(w_a h) - T_{il} \log(h) \right] + \frac{\beta(1 - \alpha)}{(1 - \beta(1 - \alpha))} [(1 - T_{il})(v_{ih} - v_{il})].$$

(15)

If $T_{il} = 1$, the low-skilled worker will surely begin the next period as a low-skill person; the second term vanishes and his only gain in entering the high-tech sector is the higher wage rate he earns in the current period. If $T_{il} = 0$, he will surely be a high-skill worker next period; in addition to higher wages this period, he also benefits from the added value of starting next period as a high-skill agent. For all intermediate values of $T_{il}$, his value is a combination of these two effects.

From (9), we can show that if a worker who currently has a high level of skill chooses to
work in the nonadopting sector, we have \( I_{ih} = 0 \) and:

\[
v_{ih}|_{I_{ih}=0} = \log(w_nh).
\]  

(16)

If, on the other hand, the agent chooses to work in the adopting sector, \( I_{ih} = 1 \), and:

\[
v_{ih}|_{I_{ih}=1} = \left[ \log(w_nh) + \log\left( \frac{w_a}{w_nh} \right) + T_{ih} \log(h) \right] - \frac{\beta(1-\alpha)}{(1-\beta(1-\alpha))} [(1 - T_{ih})(v_{ih} - v_{il})].
\]

(17)

Subtracting (15) from (17) yields:

\[
v_{ih}|_{I_{ih}=1} - v_{il} = \frac{(1 - \beta(1 - \alpha)) (T_{ih} - (1 - T_{il})) \log(h)}{1 - \beta(1 - \alpha) (T_{ih} - (1 - T_{il}))} \equiv c(T_{ih}, T_{il}) \log(h),
\]

(18)

with \( c(T_{ih}, T_{il}) \in (0, 1) \). An increase in \( T_{ih} \), which will increase the chance that a high-skill worker does not suffer a loss of capital when he moves, or a decrease in \( T_{il} \), which will decrease the risk of staying a low-skilled if he does suffer this loss, will both increase \( c(T_{ih}, T_{il}) \) and hence the value to being more skilled.

Substituting (18) into (17) and comparing to (16), one can show that it is optimal for an agent to work in the adopting sector \( (I_{ih} = 1) \) if only if:

\[
\frac{w_a}{w_n} \geq \Gamma_i(T_{ih}, T_{il}, h),
\]

(19)

where:

\[
\Gamma_i(T_{ih}, T_{il}, h) \equiv \exp\left[ \frac{(1 - T_{ih})}{1 - \beta(1 - \alpha) (T_{ih} - (1 - T_{il}))} \log(h) \right] > 1.
\]

\( \Gamma_i \) can be viewed as a measure of the risk involved in moving to the high-tech sector. The high-tech wage premium has to be larger than this measure to warrant entry into the high-tech sector. High productivity agents are more likely to move into the adopting sector when \( T_{ih} \) is high and \( T_{il} \) is low, as these decrease her chance of ending up with a low productivity level in the new job. They are also more likely to move when \( h \), the amount of skill that could be potentially lost, is low. For a wage premium of exactly \( \frac{w_a}{w_n} = \Gamma_i(T_{ih}, T_{il}, h) \) agents with education \( i \in \{g, v\} \) are indifferent between the two sectors.

It is important to note that \( \frac{w_a}{w_n} \) cannot be readily identified with the college premium that is typically reported in the empirical literature. Moreover, high and low skilled agents can coexist within the high-tech sector and actual wages \( (w_aH) \) depend not only on the rental rate \( (w_a) \), but also the agents’ relative skill levels \( (H) \).

Given our assumptions, we can prove the following lemma.

**Lemma 2** Along the BGP, \( I_{ih} = 1 \) implies \( I_{gh} = 1 \).
Proof. Given assumption 1, we have \( \Gamma_g < \Gamma_v \) and thus the result follows.

This lemma shows that moving into the adopting sector is a less risky choice for agents with general education. Therefore, if it is optimal for agents with vocational education to work in the adopting sector, it is necessarily optimal for those with general education.

Based on the value of the high-tech wage premium, \( \frac{w_{ht}}{w_{nt}} \), relative to the risk measures \( \Gamma_i \), we obtain the following optimal labor supply decisions by households and associated value functions (we will refer to these case numbers in subsequent sections):

**Case 1 (Only low-skill agents are in the high-tech sector):** If \( 1 < \frac{w_{ht}}{w_{nt}} < \Gamma_g \), then \( I_{gl} = I_{vl} = 1 \) and \( I_{gh} = I_{vh} = 0 \). The value functions are given by:

\[
\begin{align*}
v_{vl} &= \frac{(1 - \beta(1 - \alpha)) \log(w_a) + (1 - T_{il}) \log(h)] + \beta(1 - \alpha)(1 - T_{il}) \log(w_nh)}{(1 - \beta(1 - \alpha)T_{il})} \\
v_{ih} &= \log(w_nh).
\end{align*}
\]

**Case 2 (Low-skill agents and some high-skill agents with general education are in the high-tech sector):** If \( 1 < \frac{w_{ht}}{w_{nt}} < \Gamma_g \), then \( I_{gl} = I_{vl} = 1 \) and \( I_{gh} \in [0, 1] \) and \( I_{vh} = 0 \) and the value functions are as in case 1.

**Case 3 (Low-skill agents and all high-skill agents with general education are in the high-tech sector):** If \( 1 < \frac{w_{ht}}{w_{nt}} < \Gamma_g \), then \( I_{gl} = I_{vl} = 1 \) and \( I_{gh} = 1 \) and \( I_{vh} = 0 \). The value functions are given by:

\[
\begin{align*}
v_{gl} &= \frac{(1 - \beta(1 - \alpha)) \log(w_a) + (1 - T_{gl}) \log(h)] + \beta(1 - \alpha)(1 - T_{gl}) \log(w_nh)}{(1 - \beta(1 - \alpha)T_{gl})} \\
v_{vl} &= \frac{(1 - \beta(1 - \alpha)) \log(w_nh) - T_{il} \log(h)] + \beta(1 - \alpha)(1 - T_{il}) \log(w_nh)}{(1 - \beta(1 - \alpha)T_{il})} \\
v_{gh} &= \frac{[\log(w_a) + T_{gh} \log(h)] - \beta(1 - \alpha)}{(1 - \beta(1 - \alpha))}c(T_{gh}, T_{gl}) \log(h) \\
v_{vh} &= \log(w_nh).
\end{align*}
\]

**Case 4 (Low-skill agents, high-skill agents with general education, and some high-skill agents with vocational education are in the high-tech sector):** If \( 1 < \frac{w_{ht}}{w_{nt}} = \Gamma_g \), then \( I_{gl} = I_{vl} = 1 \) and \( I_{gh} = 1 \) and \( I_{vh} \in [0, 1] \). The value functions are as in case 3.

**Case 5 (All agents in the high-tech sector):** If \( 1 < \Gamma_g < \Gamma_v < \frac{w_{ht}}{w_{nt}} \), then \( I_{gl} = I_{vl} = I_{gh} = I_{vh} = 1 \). The value functions are given by:

\[
\begin{align*}
v_{gl} &= \frac{(1 - \beta(1 - \alpha)) \log(w_a) - T_{il} \log(h)] + \beta(1 - \alpha)(1 - T_{il}) \log(w_nh)}{(1 - \beta(1 - \alpha)T_{il})} \\
v_{vl} &= \frac{[\log(w_a) + T_{ih} \log(h)] - \beta(1 - \alpha)}{(1 - \beta(1 - \alpha))}c(T_{ih}, T_{il}) \log(h)
\end{align*}
\]
Note that agents with vocational education are born with high job-specific skills, in any BGP where \( \frac{w_a}{w_n} < \Gamma_v \), \( \bar{\mu}_{vl} = 0 \), and the supply of labor into the adoption sector is driven purely by the behavior of agents with general education. Whenever \( I_{ih} \in (0,1) \), the high-skilled agents are indifferent between choosing the adopting and the nonadopting sector. We will interpret a value such as \( I_{ih} = \frac{1}{2} \) as each agent of that type choosing a lottery over which sector to work in, with probabilities \( \left( \frac{1}{2}, \frac{1}{2} \right) \). We then invoke the law of large numbers to conclude that \( \frac{1}{2} \) is also the deterministic fraction of type \((v, g)\) that ends up working in each sector.\(^{33}\)

4.3 Aggregate Labor Supply

In the previous section, we solved for the optimal labor supply decisions as function of the wage premium \( \frac{w_a}{w_n} \). Now we derive the aggregate labor supplies in a BGP equilibrium, using the stationary distribution over agents as derived in (11).\(^{34}\) The aggregate labor supply for both sectors is:

\[
H^*_n = \bar{\mu}_{gh}(1 - I_{gh})h + \bar{\mu}_{vh}(1 - I_{vh})h
\]

\[
H^*_a = \bar{\mu}_{gl}E_{gl} + \bar{\mu}_{vl}E_{vl} + \bar{\mu}_{gh}I_{gh}E_{gh} + \bar{\mu}_{vh}I_{vh}E_{vh}.
\]

Using the stationary distribution and (2), we derive the relative labor supply as well the labor supply for each sector for the different ranges of the high-tech wage premiums:

**Case 1:** If \( 1 < \frac{w_a}{w_n} < \Gamma_v \) then \( I_{gl} = I_{vl} = 1 \) and \( I_{gh} = I_{vh} = 0 \) and thus:

\[
H^*_n = \left( \frac{1 - (1 - \alpha)T_{gl} - \alpha\eta_g}{1 - (1 - \alpha)T_{gl}} \right) h
\]

\[
H^*_a = \frac{\alpha h (1 - T_{gl}(1 - \frac{1}{\eta_g}))}{1 - (1 - \alpha)T_{gl}} \eta_g
\]

\[
\frac{H^*_a}{H^*_n} \left( \frac{w_a}{w_n} \right) = \frac{\alpha h (1 - T_{gl}(1 - \frac{1}{\eta_g})) \eta_g}{1 - (1 - \alpha)T_{gl} - \alpha\eta_g}
\]

\[
= \Xi^1(\eta_g) \leq \frac{\eta_g}{1 - \eta_g},
\]

where the last inequality is strict for all \( \eta_g \in (0,1) \). Note that \( \Xi^1(\eta_g) \) is strictly increasing in \( \eta_g \).

\(^{33}\)We don’t need realizations of the lottery to be independent across agents. The law of large numbers can then be justified by Feldman and Gilles (1985). Alternatively one may simply interpret \( I_{ih} = \frac{1}{2} \) to mean that the first half of the \( \bar{\mu}_{vg} \) agents chooses the nonadopting sector and the second half chooses the adopting sector, which is optimal for both groups.

\(^{34}\)Given the optimal labor supply decisions, we set \( I_{il} = 1 \) for \( i \in \{g, v\} \), in the stationary distribution.
Case 3: If $1 < \Gamma_g < \frac{w_a}{w_n} < \Gamma_v$ then $I_{g,t} = I_{v,t} = I_{gh} = 1$ and $I_{vh} = 0$:

$$
\begin{align*}
H_n^s &= (1 - \eta_g)h \\
H_a^s &= \left[\frac{(1 - (1 - \alpha)T_{gh})E_{gl} + (1 - \alpha)(1 - T_{gl})E_{gh}}{[1 - (1 - (1 - \alpha)T_{gh}) + (1 - \alpha)(1 - T_{gl})]}\right] \eta_g \\
\frac{H_a^s}{H_n^s} \left( \frac{w_a}{w_n} \right) &= \left[\frac{(1 - (1 - \alpha)T_{gh}) (1 - T_{gl}(1 - \frac{1}{K})) + (1 - \alpha)(1 - T_{gl}) (1 - (1 - T_{gh})(1 - \frac{1}{K}))}{[1 - (1 - (1 - \alpha)T_{gh}) + (1 - \alpha)(1 - T_{gl})]}\right] \eta_g \\
&= \Xi^2(\eta_g) \leq \frac{\eta_g}{(1 - \eta_g)}
\end{align*}
$$

where the last inequality is strict for $\eta_g \in (0,1)$; furthermore one can show that $\Xi^2(\eta_g) > \Xi^1(\eta_g)$ for $\eta_g \in (0,1)$ and that $\Xi^2(\eta_g)$ is strictly increasing in $\eta_g$.

Case 5: Finally, if $1 < \Gamma_g < \Gamma_v < \frac{w_a}{w_n}$ then $I_{g,t} = I_{v,t} = I_{g,h} = I_{v,h} = 1$ and thus:

$$
\begin{align*}
H_n^s &= 0 \\
H_a^s &= \mu_{gl}E_{gl} + \bar{\mu}_{el}E_{el} + \bar{\mu}_{gh}E_{gh} + \bar{\mu}_{vh}E_{vh} \\
\frac{H_a^s}{H_n^s} \left( \frac{w_a}{w_n} \right) &\to \infty,
\end{align*}
$$

where the $\bar{\mu}$'s are evaluated with all the $I$'s set to 1.

The labor supply for case 2 is a convex combination of cases 1 and 3; likewise the supply for case 4 is a convex combination of cases 3 and 5. Thus, the effective labor supplied to the sector that adopts new technologies, $H_a^s$, is weakly increasing in the wage premium and the supply of labor to the nonadapting sector, $H_n^s$, is weakly decreasing in the wage premium. A larger measure of agents choosing the high-tech firm as the relative wage increases, and assumption 1 that guarantees $E_{ih} > E_{gl}$, are responsible for this outcome. The relative labor supply for the adopting sector is therefore (weakly) increasing in the wage premium and is given by the following step function:35

$$
\frac{H_a^s}{H_n^s} \left( \frac{w_a}{w_n} \right) \begin{cases}
= \Xi^1 & \text{if } 1 < \frac{w_a}{w_n} < \Gamma_g \\
\in [\Xi^1, \Xi^2] & \text{if } \frac{w_a}{w_n} = \Gamma_g \\
= \Xi^2 & \text{if } 1 < \Gamma_g \leq \frac{w_a}{w_n} < \Gamma_v \\
\in [\Xi^2, \infty) & \text{if } \frac{w_a}{w_n} = \Gamma_v \\
\to \infty & \text{if } 1 < \Gamma_g \leq \Gamma_v < \frac{w_a}{w_n}.
\end{cases}
$$

The relative labor supply schedule is shown in figure 2.

4.4 Labor Market Equilibrium

In this section we derive the relation between the growth rate $x$ and the resulting wage of the adopting sector $w_a$ from the labor market equilibrium. We will use this schedule, together with the technology adoption decision of the firm, $x(w_a)$ discussed in section 4.1, and depicted in Figure 1, to determine the growth rate, $x^*$, and the equilibrium wage in the

35Though we often omit it for simplicity, it should be borne in mind that $\Xi^1$ and $\Xi^2$ are functions of $\eta_g$. 

24
adopting sector, \( w_a^* \), for a given (and to be endogenously determined) fraction \( \eta_g \) of agents with general education.

The \( x^a(w_a) \) curve traces out the equilibrium relation between \( x \) and \( w_a \) in the labor market. (Since it is the condition between \( x \) and \( w_a \) that is consistent with the supply of labor into the adoption, \( H_a \), we can view this as a “supply” schedule.)

### 4.4.1 Wage Premium in Equilibrium

Recall that the relative demand for labor is given from equation (13) as: 
\[
\left( x \frac{w_a}{w_n} \right)^{\frac{1}{1-\sigma}}. 
\]
Relative labor demand is a continuous, strictly decreasing function with 
\[
H^d_n \left( \frac{w_a}{w_n} \right) \big|_{x=1} = H^d_n \left( \frac{w_a}{w_n} \right) \rightarrow \infty 
\]
and 
\[
\lim_{w_n \to \infty} H^d_n \left( \frac{w_a}{w_n} \right) = 0. 
\]
The labor supply correspondence satisfies 
\[
\lim_{w_n \to \infty} H^s_n \left( \frac{w_a}{w_n} \right) = 1 
\]
and is increasing and continuous on (1, \( \infty \)).

We now assume: 
\[
T_{gl} < \frac{1-2\alpha}{1-\alpha}. 
\]
This sufficient condition, stated in terms of model parameters alone, guarantees there exists a unique relative wage \( \frac{w_a}{w_n} > 1 \) that clears the labor market.\(^{36} \)
At a relative wage of \( \frac{w_a}{w_n} = 1 \) we need the relative labor supply to be low enough to cause excess demand in order to drive the premium to a value greater than one in equilibrium. For \( \frac{w_a}{w_n} < \Gamma_g \), the relative labor supply is independent of the transition matrix for the workers with vocational education, since they are born with high productivity, never choose to work in the high-tech sector, and therefore remain high-skilled forever. Only low-skilled workers with general education work in the high-tech sector and there will be more of them with a higher \( T_{gl} \); \( H^s_n \) thus increases with \( T_{gl} \), \( H^s_n \) decreases with \( T_{gl} \), and the relative labor supply increases with \( T_{gl} \). Therefore, in order to assure that at \( \frac{w_a}{w_n} = 1 \) relative labor supply is sufficiently small, we need \( T_{gl} \) to be sufficiently small.

**Lemma 3** For any BGP equilibrium growth rate \( x \in [1, \lambda] \) there exists an associated unique labor market equilibrium satisfying: 
\[
\frac{w_a}{w_n} > 1 \text{ and } H^a_n \in (\Xi^1, \infty). 
\]

Figure 2 illustrates the labor market equilibrium characterized by the above lemma. In this example, case 2 of the labor market equilibrium applies \( \left( \frac{w_a}{w_n} = \Gamma_g \right) \). Since \( \Xi^1 \) and \( \Xi^2 \) increase with \( \eta_g \), lower cases of the labor market equilibrium are more likely with higher \( \eta_g \); when \( x \) increases, the demand curve shifts outward, and a higher case is likely to result in equilibrium.

\(^{36} \)A weaker condition is given in the proof to Lemma 3 in the appendix.
4.4.2 Characterization of Labor Market Equilibrium

Given that relative labor supply is a step “function” there are several possible cases for the equilibrium wage premium and corresponding labor allocations. We first argue that the wage premium in equilibrium can’t be too high.

Lemma 4 For any BGP growth rate, \( x \in [1, \lambda] \), the equilibrium in the labor market satisfies \( \frac{w_a}{w_n} \leq \Gamma_v \).

For \( \frac{w_a}{w_n} > \Gamma_v \), \( w_n \) becomes arbitrarily large and an equilibrium is not possible. Therefore \( \frac{w_a}{w_n} \in (1, \Gamma_v] \) for any possible growth rate of the BGP \( x \in [1, \lambda] \); we need to be concerned only with cases 1 through 4 in the labor market. Since the labor supply decision depends on the relative wage, it is the relative labor market condition depicted above that pins down the relevant case. In this subsection, given the case that obtains in equilibrium, we move from relative supplies to absolute ones: we derive, for given growth rate \( x \), the associated absolute wage \( w_a \) in the labor market. Inverting this relation enables us to construct the entire \( x^s(w_a) \)-schedule.

Which of the four cases occurs depends on the growth rate \( x \) on which the labor market equilibrium is conditioned; as mentioned earlier the applicable case increases with \( x \). If the equilibrium occurs in case 1 or case 3, the relative supply is fixed at \( \Xi^1 \) or \( \Xi^2 \), and the relative labor demand pins down the relative wage. The labor demanded by the adopting sector, given by (12), is equated to the labor supplied into that sector, given either by (21) or (23), to derive \( x^s \) as a function of \( w_a \) and \( \eta_g \). The relative labor demand (13) can be equated to the fixed relative supply to back out \( w_n \) as a function of the \( w_a \) determined above. If the
equilibrium occurs in case 2 or case 4, the relative wage is fixed at $\Gamma_g$ or $\Gamma_v$. In these cases the $gh$- and $vh$-type agents (respectively) would be indifferent to entering the adopting sector, and the fraction of agents that enter to maintain the above-mentioned fixed relative wage as an equilibrium has to be determined using the absolute and relative labor market conditions simultaneously.

In the appendix, we perform the above steps to derive the $x^*(w_a)$ schedule conditional on $\eta_g$ as a continuous function given by:

$$x^*(w_a) = \begin{cases} w_aK_1(\eta_g) & \text{if } 1 \leq x < x_1 \\ K_2(w_a, \eta_g) & \text{if } x_1 \leq x \leq x_2 \\ w_aK_3(\eta_g) & \text{if } x_2 < x < x_3 \\ K_4(w_a, \eta_g) & \text{if } x \geq x_3 \end{cases}$$

with the critical points for $(x, w_a)$ given by:

$$x_1 = [\xi^1]^{1-\theta} \Gamma_g, \quad w_{a1} = \frac{\theta \Gamma_g}{h^{1-\theta}} \left[ \frac{1 - (1 - \alpha)T_{g1}}{\alpha(1 - T_{g1}(1 - \frac{1}{\delta}))\eta_g} \right]^{1-\theta},$$

$$x_2 = [\xi^2]^{1-\theta} \Gamma_g, \quad w_{a2} = \frac{\theta \Gamma_g}{h^{1-\theta}} \left[ \frac{1}{1 - \eta_g} \right]^{1-\theta},$$

$$x_3 = [\xi^3]^{1-\theta} \Gamma_g, \quad w_{a3} = \frac{\theta \Gamma_g}{h^{1-\theta}} \left[ \frac{1}{1 - \eta_g} \right]^{1-\theta},$$

where $1 < x_1 < x_2 < x_3$ and $w_{a0} < w_{a1} < w_{a2} < w_{a3}$. All the $K$’s are increasing in $\eta_g$; in addition, $K_2$ and $K_3$ are increasing in $w_a$. The $x^*(w_a; \eta_g)$ schedule is thus increasing and shown in figure 1.

### 4.5 Determination of the Growth Rate $x$

#### 4.5.1 Existence, Uniqueness and Characterization for given $\eta_g$

We now use the $x(w_a)$ and $x^*(w_a)$ schedules to characterize the equilibrium growth rate $x$ conditional on $\eta_g$. We prove:

**Lemma 5** For each $\eta_g \in (0, 1)$ there exists a unique BGP growth rate $x^*(\eta_g) \in (1, \lambda)$

The unique growth rate, conditional on $\eta_g$, is graphically depicted by the intersection of the $x(w_a)$ and $x^*(w_a)$ curves, as in figure 1. We can also give a sufficient condition for the growth rate $x$ (for a given $\eta_g$) to be less than the maximal possible growth rate $\lambda$.

**Corollary 2** Suppose $w_{a0} = \frac{\theta}{h^{1-\theta}} \left[ \frac{1 - (1 - \alpha)T_{g1}}{\alpha(1 - T_{g1}(1 - \frac{1}{\delta}))\eta_g} \right]^{1-\theta} \geq \bar{w}_a = (\lambda \theta) / (\lambda - 1)^{1-\theta}$. Then the BGP growth rate satisfies $x^*(\eta_g) < \lambda$, for any given $\eta_g \in [0, 1]$.

This simply follows from the derivation of the $x^*$ presented above; if the critical wage for case 1 ($w_{a0}$) is larger than the critical wage below which maximal growth obtains ($\bar{w}_a$), the intersection of the $x$ and $x^*$ schedules will occur in the downward sloping portion of $x(w_a)$,
and the growth rate will be less than the maximal value. This is more likely to occur for a lower (endogenous) fraction of workers with general education, \( \eta_g \), and a lower value for the effective labor supplied by the skilled, \( h \), and a lower value for the probability a low-skilled agent with general education stays low-skilled, \( T_{gl} \). The dependence on \( \eta_g \) and \( h \) is intuitive. The dependence on \( T_{gl} \) stems from the earlier observation that the relative labor supply in the adoption sector is increasing in this parameter since in this region only the \( gl \)-types choose the high-tech sector; therefore a lower value is consistent with a higher wage in the adoption sector.

### 4.5.2 Effects of Education Composition on Growth Rate \( x \)

We now examine how the \( x(w_a) \) and \( x^s(w_a) \) schedules change with \( \eta_g \) in order to investigate how the equilibrium growth rate depends on the fraction of workers with general education. First we note that \( \eta_g \) does not affect the adoption decision of firms, for a given \( w_a \), and hence does not affect the \( x(w_a) \) curve. By demonstrating that the \( x^s(w_a) \) curve shifts up (to the left) with an increase in \( \eta_g \), we establish the following lemma:

**Lemma 6** The growth rate \( x^* \) is (weakly) increasing in \( \eta_g \).

The increase in \( x^s(w_a) \) with \( \eta_g \) is not automatic since for some ranges of the wage premium, workers with both types of education choose the adopting sector; therefore an increase in the fraction of workers with general education, which necessarily decreases the fraction of those with vocational education, makes the dependence of \( H_a \), and thus \( x \), on \( \eta_g \) ambiguous. However, since the vocational education agents enter with high job-specific skills and do not choose the high-tech sector until \( \frac{w_v}{w_a} > \Gamma_v \) (case 4), the behavior of \( x^s(w_a) \) is primarily driven by those with general education; the more abundant they are in the population, the greater is the supply of labor into the adopting sector. This increased supply makes it cheaper for the high-tech firm to adopt new technologies, leading to a higher equilibrium growth rate.

### 4.6 Determination of the Education Decision

In this section we discuss how the equilibrium fraction of agents with general education, \( \eta_g \), is determined. Recall that \( \eta_g \) is related to the threshold ability level \( a^* \), above which all agents obtain general education and below which all agents obtain vocational education, by \( a^* (\eta_g) = 1 - \eta_g \). Write (10) as follows in order to obtain the equilibrium threshold ability as a solution to the following equation:

\[
\nu_{gl} (\eta_g) - \nu_{vh} (\eta_g) = e \left( a^* (\eta_g) \right).
\] (24)
This captures the fixed point problem induced by the education decision – newborn agents anticipate a certain fraction of the workforce with general education, \( \eta_g \), which determines the high-tech wage premium, and thus the value to getting general education; their decision to obtain one or the other type of education has to be consistent with the posited \( \eta_g \).

We now make the following functional form assumption for the cost of obtaining general education.

**Assumption 2:** The function \( e : [0, 1] \rightarrow \mathbb{R}_+ \) is given by: \( e(a) = \frac{1}{a} - 1 \).

With this assumption we see that for the ablest agents the cost of obtaining general education is 0, whereas for the least able agents this cost becomes prohibitively high. Then the right hand side of equation (24), as a function of \( \eta_g \), becomes 

\[
 e\left(\frac{\eta_g}{1-\eta_g}\right). 
\]

This is a strictly increasing function of \( \eta_g \), equal to zero when \( \eta_g = 0 \) and approaching infinity as \( \eta_g \to 1 \).

Our analysis thus far allows us to obtain an explicit expression for the left hand side of equation (24), as a function of \( \eta_g \):

\[
 v_{gl} (\eta_g) - v_{vh} (\eta_g) = \Psi_1 \log \left( \frac{w_a}{w_n} \right) - \log(h). \tag{25}
\]

- For cases 1 and 2, that is, for \( \frac{w_n}{w_a} \in (1, \Gamma_g) \) we can derive:

\[
 v_{gl} (\eta_g) - v_{vh} (\eta_g) = \Psi_1 \log \left( \frac{w_a}{w_n} \right) - \log(h). \tag{25}
\]

- For cases 3 and 4, that is, for \( \frac{w_n}{w_a} \in (\Gamma_g, \Gamma_v) \) we can derive:

\[
 v_{gl} (\eta_g) - v_{vh} (\eta_g) = \log \left( \frac{w_a}{w_n} \right) - \{1 - \Psi_2 \} \log(h). \tag{26}
\]

In these expressions:

\[
 \Psi_1 = \frac{1 - \beta(1 - \alpha)}{1 - \beta(1 - \alpha)T_{gl}}, \\
 \Psi_2 = \frac{(1 - T_{gl})(1 - \beta(1 - \alpha)T_{gl}T_{gh})}{[1 - \beta(1 - \alpha)(T_{gh} - (1 - T_{gl}))][1 - \beta(1 - \alpha)T_{gl}]}.
\]

which depend only on the model parameters.\(^{37}\)

For case 2, \( \frac{w_n}{w_a} = \Gamma_g \), and for case 4 we have \( \frac{w_n}{w_a} = \Gamma_v \), so that a change in \( \eta_g \) does not alter the utility differential, as long as the labor market equilibrium remains in the same

\(^{37}\) Note that, since \( \Psi_1 < 1 \), and the term within braces in (26) is smaller than 1 and the equilibrium relative wages \( \frac{w_a}{w_n} \) increase from case 1 to case 4, \( v_{gl} - v_{vh} \) increases as we move from case 1 to case 4.
case. For cases 1 and 3, a change in $\eta_g$ changes the wage premium on the labor market and thus the utility differential. Using expressions for the relative wage, one can write the above utility differentials in terms of $\eta_g$ and the model parameters alone. In the appendix, we derive expressions for $\left(\frac{w_a}{w_n}\right)(\eta_g)$ and prove the lemma:

**Lemma 7** There exist $0 \leq \eta_{g1} \leq \eta_{g2} \leq \eta_{g3} \leq 1$, such that for $\eta_g \in [0, \eta_{g1}]$ case 4 of the labor market equilibrium obtains, for $\eta_g \in [\eta_{g1}, \eta_{g2}]$ case 3 obtains, for $\eta_g \in [\eta_{g2}, \eta_{g3}]$ case 2 obtains, and for $[\eta_{g3}, 1]$ case 1 obtains. Moreover, there exists $\bar{\eta}_g^3 \in [\eta_{g1}, \eta_{g2}]$, such that for $\eta_g \in [\eta_{g1}, \bar{\eta}_g^3]$, $x < \lambda$ (case 3b), and for $\eta_g \in [\bar{\eta}_g^3, \eta_{g2}]$, $x = \lambda$ (case 3a). Likewise, there exists $\bar{\eta}_g^1 \in [\eta_{g3}, 1]$, such that for $\eta_g \in [\eta_{g3}, \bar{\eta}_g^1]$, $x < \lambda$ (case 1b), and for $\eta_g \in [\bar{\eta}_g^1, 1]$, $x = \lambda$ (case 1a).

We then derive explicit expressions for the utility differential as:

$$
(v_{gl} - v_{ch})(\eta_g) = \begin{cases} 
\log(\Gamma_c) - \{1 - \Psi_2\} \log(h) & \text{for } \eta_g \in [0, \eta_{g1}] \quad \text{(Case 4)} \\
\log \left( C_1 \left( C_2 (\eta_g)^{2\theta-1} (1 - \eta_g)^{1-\theta} + \left( \frac{1}{\eta_g} - 1 \right)^{1-\theta} \right) \right) - \{1 - \Psi_2\} \log(h) & \text{for } \eta_g \in [\eta_{g1}, \bar{\eta}_g^3] \quad \text{(Case 3b)} \\
\log \left( \frac{\lambda}{|\varrho (\eta_g)|^{1-\theta}} \right) - \{1 - \Psi_2\} \log(h) & \text{for } \eta_g \in [\bar{\eta}_g^3, \eta_{g2}] \quad \text{(Case 3a)} \\
\Psi_1 \log(\Gamma_g) - \log(h) & \text{for } \eta_g \in [\eta_{g2}, \eta_{g3}] \quad \text{(Case 2)} \\
\Psi_1 \log \left( B_1 (B_2 (\eta_g)^{\theta-1} + B_3 (\eta_g)^{2\theta-1}) \right) - \log(h) & \text{for } \eta_g \in [\eta_{g3}, \bar{\eta}_g^1] \quad \text{(Case 1b)} \\
\Psi_1 \log \left( \frac{\lambda}{|\varrho (\eta_g)|^{1-\theta}} \right) - \log(h) & \text{for } \eta_g \in [\eta_{g3}, 1] \quad \text{(Case 1a)}. 
\end{cases}
$$

(27)

The definitions of the constants $\Psi_1$ and $\Psi_2$ are given above; the expressions for the positive constants $C_1$, $C_2$, $B_1$, $B_2$, and $B_3$ are given in the appendix. The utility differential is weakly decreasing in $\eta_g$; as the fraction of workers with general education increases, the labor supplied to the adoption sector increases, and the high-tech wage premium in the labor market decreases, which in turn decreases the utility differential. Note that the negative relationship between $\frac{w_a}{w_n}$ and $\eta_g$ implies, in a reversal of the order of occurrence in the labor market, we move from case 4 to case 1 as $\eta_g$ increases.

The derivation in the appendix also shows the intuitive result that an increase in $\lambda$ shifts the utility-differential curve (weakly) upward; it leaves utility differentials constant for all cases but cases 1a. and 3a, for which utility differentials strictly increase. Furthermore, such an increase makes the higher utility differential cases more likely to result by (weakly) increasing the cutoff levels $\{\eta_{g1}, \eta_{g2}, \eta_{g3}\}$ and $\{\bar{\eta}_g^1, \bar{\eta}_g^3\}$.
We now assume that $h < \Gamma_v$. This assumption ensures $v_{gl} > v_{sh}$ at $\eta_g = 0$ and ensures an interior equilibrium; if the initial job-specific skill level of the vocationally educated, $h$, is too high, even the ablest individual would not prefer general education and the accompanying low initial skill level. Using the above expressions for the utility differential and the assumed $e(a)$ we then prove the following lemma.

**Lemma 8** Suppose the assumptions made above are satisfied. Then there exists a unique equilibrium $\eta^*_g \in (0, 1)$.

The equilibrium $\eta^*_g$ is illustrated in figure 3. In this example, case 2 of the labor market equilibrium results.

5 Comparing US and European Policies

In section 2, we presented evidence on European educational policies that favor vocational education over general education, while in the US the situation is the reverse. In this section, we study the effects of this policy difference on the growth rates of and the growth gap between the two regions, as implied by our model.

5.1 Policy Differences

We will denote by $G$ the normalized amount of government expenditure available for subsidizing both types of education. (In the original problem, since this will be multiplied by $A$, government subsidies will be growing at the rate of technology.) Let $s_v$ denote the per student subsidy given to a vocational education student and let $s_g$ denote the per student...
subsidy given to a general education student. Then the government resource constraint, given a uniform ability distribution and an ability threshold $a^*$ is:

$$a^*s_v + (1 - a^*)s_g = G,$$

with $s_v, s_g > 0$. We will consider a revenue neutral experiment; that is, assume that $G$ is the same for the US and Europe and that $(s_v)_\text{Europe} > (s_v)_{US}$ is given.

Assume, for simplicity, that in the first period of their lives, agents have linear preferences in the cost of education net of taxes and subsidies. The linearity is not as crucial as separability. The effective cost of general education – the right hand side of (24) – becomes:

$$e(a^*(\eta_g)) + s_v - s_g = \frac{\eta_g}{1 - \eta_g} - \frac{G - s_v}{\eta_g} = C(\eta_g).$$

Note that $C(\eta_g = 0) = -\infty$ and $C(\eta_g = 1) = \infty$ and that $C(\eta_g)$ is strictly increasing and continuous on $(0, 1)$. Thus existence of equilibrium is guaranteed. In figure 3, a lower $s_v$ will shift the cost difference curve, $C(\eta_g)$, to the right.\(^{38}\)

5.2 Growth Rates and Growth Gaps with Different Policies

Denote by $\Delta(\lambda)$ the gap between the potential growth rate of the economy and the actual rate, when the frontier evolves exogenously at the rate $\lambda$. Formally, the growth gap is $\Delta(\lambda) \equiv \lambda - x^*(\eta_g(\lambda))$. Given our assumptions on policy differences, we have $C^{US}(\eta_g) < C^{EUR}(\eta_g)$ for $\eta_g \in (0, 1)$. We show:

**Lemma 9** Suppose the assumptions made above are satisfied. We have:

1. $\eta_g^{US} > \eta_g^{EUR}$
2. Either $x^{US} = x^{EUR} = \lambda$ or $\lambda \geq x^{US} > x^{EUR}$.
3. Either $\Delta^{US}(\lambda) = \Delta^{EUR}(\lambda) = 0$ or $\Delta^{EUR}(\lambda) > \Delta^{US}(\lambda) \geq 0$.

\(^{38}\)An alternate scheme which is qualitatively equivalent to the one proposed in the main text is the following. Suppose that all agents who obtain vocational education pay a lump-sum tax $\tau$ that finances a subsidy of $s$ for general education. Budget balance of the government requires that: $s = \frac{\tau(1 - \eta_g)}{\eta_g}$. With $C$ now defined as:

$$(e(1 - \eta_g) - s) - \tau = \frac{\eta_g}{1 - \eta_g} - \frac{\tau}{\eta_g} \equiv C(\eta_g),$$

the analysis in the text with differential subsidies goes through. Higher tax rates resemble education policies that are geared towards subsidizing general education. Higher $\tau$’s shift the cost difference curve, $C(\eta_g)$, to the right. We assume that $\tau^{US} > \tau^{EUR}$; note that these are not total taxes in the two regions, but only the levies to finance general (tertiary) education.
The first statement asserts that the fraction of workers with general education is higher in the US. The second statement asserts that either the US and Europe both grow at the potential rate \( \lambda \), or the US grows at a strictly higher rate. If both grow at the maximal rate, the growth gap of both countries with respect to the potential is zero; otherwise the growth gap of Europe is strictly higher. The difference between the US and Europe is illustrated in figure 4.

### 5.3 Effect of an Increase in Speed of Innovation

We now analyze whether an increase in the rate of technological progress widens the growth gap between the US and Europe. We are thus interested in comparative statics with respect to \( \lambda \). Making use of the observation that the utility differential is weakly increasing in \( \lambda \), we prove:

**Proposition 1** Equilibrium general education attainment increases with \( \lambda \): \( \frac{d\eta_g}{d\lambda} \geq 0 \).

The higher \( \lambda \) increases the demand for labor in the adopting sector and thus the high-tech wage premium \( \frac{w_a}{w_n} \); this increases the incentive to acquire general education. Now we consider a marginal increase in the speed of technological progress \( \lambda \). The above-mentioned increase in demand is also responsible for the following result.

**Proposition 2** \( \frac{d\Delta(\lambda)}{d\lambda} \geq 0 \). Almost surely (i.e. for the measure one of parameter combinations for which the equilibrium \( \eta_g \) is not equal to one of the thresholds), \( \frac{d\Delta(\lambda)}{d\lambda} \in \{0, 1\} \).

\[39\] This is reminiscent of Ljungqvist and Sargent’s (1998) experiment of increasing economic turbulence to study its effect on European unemployment.
That is, the gap in the growth of an economy relative to the potential growth rate $\lambda$, is itself (weakly) increasing in $\lambda$. This leads us to the central proposition of the paper.

**Proposition 3** \( \frac{d\Delta^{EUR}(\lambda)}{d\lambda} \geq \frac{d\Delta^{US}(\lambda)}{d\lambda} \) with strict inequality if \( x^{US} = \lambda > x^{EUR} \).

\[ x^{(w_a)}: \text{US} \\
\lambda' \\
x' \\
\lambda \\
X(w_a) \\
w_a \]

**Figure 5: Effect of an Increase in $\lambda$**

Though the proposition is phrased in terms of growth gap of each region relative to the (new) potential growth rate, its implication for the gap in growth rates between the two regions is obvious. If the rate of change of available technologies, $\lambda$, increases, two economies that were earlier growing at the maximum possible rate could find themselves growing at different rates, with the economy that focuses on general education, and thus has a low $w_a$, having the clear advantage. The equilibrium wage of the vocationally focused economy may be higher than the new threshold level that makes maximal adoption profitable. As mentioned in the introduction, several economists feel that the rate of technological change indeed increased in the mid-seventies, reaching its peak in the nineties. Our model suggests that the US, with a much higher fraction of its workforce possessing general education was able to adopt these available technologies at a faster rate than Europe could. Even if both regions adopt technology at faster rate, our model predicts that there may be a gap in their rate of adoption, consistent with the data in table 1.

This effect is illustrated in figure 5 (an expansion of figure 1), which shows the effect of an increase in $\lambda$ to $\lambda'$.\(^{40}\) The US continues to be constrained only by the availability of technologies, while Europe potentially falls behind.

\(^{40}\)For simplicity, we show the $x^{(w_a)}$ curve as unchanged. As noted in the proof to proposition 2, $\eta_p$ could increase as a result of an increase in $\lambda$ which has a potential to exacerbate the effect illustrated. This also shows that the exogenous limit to growth, though realistic, is not the only force behind the above results.
6 Conclusion

We have developed a growth model featuring a dynamic advantage of general over vocational education, technology adoption by firms, and educational and occupational decisions by households, to argue that two economies that grow at potential when the rate of technological progress is low, could diverge when this rate increases. Our analysis thus provides one possible explanation for the growth gap Europe, which focuses on skill-specific education, has suffered since the eighties relative to the US, which focuses more on conceptual education. It must be emphasized that the use of balanced growth analysis is mostly an analytically convenient way to study the issue of slow European technology adoption. One could instead construct a steady state model and cast European catch-up or falling behind purely as a transitional issue, relying on numerical instead of analytical characterization. If educational reforms are instituted, such as the much discussed reforms to make German universities more competitive, then the growth gaps we have analyzed are going to be necessarily transitional. Indeed, one needs to be cautious about literally mapping variations in general education policy to permanent growth rate differences; an analysis using a panel data set with shorter run growth rates and policy variables applicable for those shorter periods might be more appropriate.

While casual evidence suggests that manufacturing productivity growth is strongly correlated with the share of the workforce with tertiary education (European Competitiveness Report 2001, Table IV.2), rigorous attempts to extend our analysis along quantitative dimensions are warranted. One possibility would be to calibrate the model presented in this paper to quantify the predicted gap in growth between the US and Europe. Another is to conduct a cross-country, cross-industry, econometric study to assess whether acceleration in adoption rates has been particularly higher in the US relative to Europe in those industries that have seen greater increases in available technologies. The model points to the high-tech premium, \( \frac{w_g}{w_h} \), as a crucial equilibrium variable; the mapping of this into empirically reported premia (such as the college premium) deserves further attention. Using years of education as a proxy, cross-country growth studies have found only a weak effect of human capital in explaining growth. Our study points out that the type of education obtained, rather than the number of years of education per se, could have a crucial bearing on the rate of economic growth.\(^{42}\) These are topics of ongoing research.

\(^{41}\) See for instance Hyde Flippo’s, “Can the German University be Saved?” an online supplement (http://www.german-way.com), to The German Way (Passport Books), which reports a steep increase in the percentage of high school students earning the academic diploma that leads to college study in the city-state of Hamburg, and points to the emergence of private universities such as Universität Witten/Herdecke.\(^{42}\) Also see Murphy, Schleifer, and Vishny (1991) in this regard.
A Appendix

A.1 Proof of Lemma 1:

Proof. The first order necessary and sufficient condition for the firm is

\[(x^*)^{\ast} \rightarrow \left( \frac{\theta}{w_a} \right)^{\ast} \geq x^* - 1\]

\[= \text{ if } x^* < \bar{x}\]

Suppose there exists BGP with \(x = \lambda\). Then

\[\bar{x} = \frac{A_f}{A} = \frac{A_f^{-1}}{A} \ast \frac{A_f}{A_f^{-1}} = 1 \ast \lambda = \lambda\]

where \(A_f^{-1} = 1\) because in a BGP with growth rate \(\lambda\) the actual level of technology must equal the potential level at each point of time (remember that we assumed that \(A_0 = A_{f-1}\)). In order for the firm to optimally choose \(x^* = \lambda\) a necessary and sufficient condition is hence (14).

A.2 Proof of Corollary 1:

Proof. Suppose there exists BGP with \(x < \lambda\).

First we show that \(x^* < \bar{x} = \frac{A_f}{A}\). Suppose not; i.e. suppose \(x^* = \bar{x}\). But then

\[x^* = \frac{A_f^{-1}}{A} \ast \frac{A_f}{A_f^{-1}} \geq \lambda\]

since \(A \leq A_f^{-1}\). For a BGP we need \(\lambda \leq x^* = x < \lambda\), a contradiction. Hence \(x^* < \bar{x}\). But then the optimal choice \(x^*\) satisfies

\[(x^*)^{\ast} \rightarrow \left( \frac{\theta}{w_a} \right)^{\ast} = x^* - 1\]

(28)

But the unique solution \(x^*\) to this equation satisfies \(x^* < \lambda\) if and only if

\[(\lambda)^{\ast} \left( \frac{\theta}{w_a} \right)^{\ast} < \lambda - 1\]

or \(w_a > \bar{w}_a\). ■

A.3 Proof of Lemma 3:

Proof. It is sufficient to show that \(\lim_{w_a \rightarrow 1} \frac{H_f^\eta (w_a)}{H_f^\eta (w_a)} = 1 < \frac{H_f^\eta (1)}{H_f^\eta (1)} = x^{\ast} \rightarrow x^{\ast} \) for all \(x \geq 1\) and all \(\eta \in [0,1]\). Thus, it is sufficient to show that

\[\Xi^1 < 1\]
for all \( \eta_g \in [0,1] \). From the formula for \( \Xi^1 \) we see that \( \Xi^1 \) is strictly increasing in \( \eta_g \). Thus it is sufficient to show that \( \Xi^1(\eta_g = 1) < 1 \). We can see that

\[
\Xi^1(\eta_g = 1) = \frac{\alpha (1 - T_{gl} (1 - \frac{1}{\eta}))}{1 - (1 - \alpha)T_{gl} - \alpha} < 1
\]

if and only if

\[
\frac{T_{gl}}{1 - T_{gl}} < \frac{1 - 2\alpha}{\alpha} h.
\]

Given \( h > 1 \), a sufficient condition for this to be true is, \( \frac{T_{gl}}{1 - T_{gl}} < \frac{1 - 2\alpha}{\alpha} \), which is guaranteed by the assumption made in the text about \( T_{gl} \). The rationale for this assumption can be more formally given here. At a relative wage of \( \frac{w_{an}}{w_n} = 1 \) we need the relative labor supply to be low enough to cause excess demand in order for the premium to be greater than one in equilibrium. For \( \frac{w_{an}}{w_n} < \Gamma_g \) the relative labor supply is independent of \( T_{el} \) since \( \mu_{vl} = 0 \) (all \( v \) types are born with high productivity and remain there forever). Now \( \mu_{gh} \) decreases with \( T_{gl} \) and thus \( \mu_{gl} \) increases with it. Therefore \( H_n^* = (\mu_{gh} + \mu_{vh}) h \) decreases with \( T_{gl} \). If \( H_n^* \) increases with \( T_{gl} \) we will without ambiguity have the implication that the relative supply is increasing in \( T_{gl} \) and to keep the relative supply low, \( T_{gl} \) will have to be low. We have \( H_n^* = \mu_{gl} E_{gl} = \mu_{gl} (T_{gl} + (1 - T_{gl}) h) \) in this region. The first factor is increasing in \( T_{gl} \) while the second is decreasing. However, if \( \alpha h < 1 \) one can show that \( H_n^* \) is indeed increasing in \( T_{gl} \), as is the relative supply. Therefore, in order to assure that at \( \frac{w_{an}}{w_n} = 1 \) relative labor supply is sufficiently small requires that \( T_{gl} \) is sufficiently small. 

### A.4 Proof of Lemma 4:

**Proof.** Suppose not. For a wage premium of \( \frac{w_{an}}{w_n} > \Gamma_v \), the optimal sector choices of high-skilled workers are \( I_{gh} = I_{vh} = 1 \), with relative labor supply \( H_n^* \left( \frac{w_{an}}{w_n} \right) \rightarrow \infty \). Given (13), equilibrium is possible in the labor market only if \( x \rightarrow \infty \) and / or \( \frac{w_{an}}{w_n} \rightarrow 0 \). Since \( x \) is bounded above by \( \lambda \) and \( \frac{w_{an}}{w_n} > \Gamma_v > 0 \) for this case, neither condition can hold. 

### A.5 Derivation of \( x^*(w_a) \):

**Case 1:** Equilibrium wages fall in the region \( \frac{w_{an}}{w_n} \in (1, \Gamma_g) \). This occurs if the growth rate of the economy \( x \in [1, x_1) \), where \( x_1 = \left[ \Xi^1 \right]^{1-\theta} \Gamma_g \). In this region relative labor supply, which determines equilibrium quantities, is given by \( H_n^* = \Xi^1 \). We use the labor supplied to the adopting sector from (21), and demand from (12), and derive the \( x^* \) schedule as:

\[
x^* = w_n \kappa_1(\eta_g),
\]

where,

\[
\kappa_1(\eta_g) \equiv \frac{1}{\theta} \left[ \frac{\alpha (1 - T_{gl} (1 - \frac{1}{\eta})) h \eta_g}{1 - (1 - \alpha) T_{gl}} \right]^{1-\theta}.
\]
Hence for $x \in [1, x_1)$ the $x^*(w_a)$-schedule is linearly increasing in $w_a$, and strictly increasing in $\eta_g$.

Note that case 1 occurs only for values of $\eta_g$ large enough to ensure $x_1 > 1$. We denote the endpoints of the relevant adoption sector wages for this case as $w_{a0}$, $w_{a1}$.

**Case 2:** Equilibrium wages satisfy $\frac{w_a}{w_n} = \Gamma_g$. The equilibrium relative labor allocation, from the labor supply equation, satisfies $\frac{H_n}{H_a} \left( \frac{w_a}{w_n} \right) \in [\Xi^1, \Xi^2]$. From the relative labor demand schedule, one can see that this case of the labor market equilibrium occurs for growth rates satisfying $x \in [x_1, x_2]$, where $x_2 \equiv \Xi^2 \cdot (1 - \theta) \cdot \Gamma_g$. In this region, $\frac{H_n}{H_a} = \left( \frac{x}{x_g} \right)^{\frac{1}{\theta + 1}}$.

In order to determine absolute equilibrium labor input in both sectors we have to determine the fraction $f = I_{gh}$ of the high-skilled agents with general education (who are indifferent between both sectors at equilibrium wages) that have to choose the adopting sector in order to rationalize the equilibrium relative wage. Equilibrium in both labor markets require

$$
\frac{\partial \Gamma_g}{\partial w_a} = H_n = H^n_a = \bar{\mu}, \frac{h}{h} + (1 - f) \bar{\mu} h, h
$$

$$
\frac{\partial w}{\partial x} = H_a = H^g_a = \bar{\mu} E_{gl} + \bar{\mu} = E_{vl} + f \bar{\mu} E_{gh}.
$$

Note that $\bar{\mu} = 0$ and $\bar{\mu} = 1 - \eta_g$. Using the fact that $\frac{H_n}{H_a} = \left( \frac{x}{x_g} \right)^{\frac{1}{\theta + 1}}$ and substituting in the formulas for the equilibrium measures yields $f (x, \eta_g)$ as:

$$
f(x, \eta_g) = \left( \frac{x}{x_g} \right)^{\frac{1}{\theta + 1}} h \left[ 1 - (1 - \alpha) T_g - \alpha \eta_g \right] - \alpha \eta_g E_{gl}
$$

$$
(1 - \alpha) \left\{ (1 - T_{gh}) E_{gl} + (1 - T_{gl}) E_{gh} \right\} \eta_g - \left( \frac{x}{x_g} \right)^{\frac{1}{\theta + 1}} h \left[ (1 - T_{gh}) (1 - \eta_g) - (1 - T_{gl}) \eta_g \right]
$$

It can be seen that $f$ is decreasing in $\eta_g$ and increasing in $x$. The next step is to use $f (x, \eta_g)$ in one of the absolute labor market conditions above to derive $x \left( w_a, \eta_g \right)$. We choose the $H^d_a = H^n_a$ condition and obtain

$$
\frac{\partial \Gamma_g}{\partial w_a} \left( (1 - T_{gh}) (1 - \alpha h) + h (1 - (1 - \alpha) T_{gl}) \right) + \left( \frac{\partial x}{\partial w_a} \right) \left( 1 - \alpha \right) h [(1 - T_{gh}) - T_{gl}] = \left[ ((1 - T_{gh}) (1 - \alpha h) + h (1 - (1 - \alpha) T_{gl}) - \alpha \eta_g (h - 1) [(1 - T_{gh}) - T_{gl}] \right].
$$

For a given $\eta_g$, the right hand side is constant. Under assumption 1, one can show that the terms on the left hand side are positive. Therefore, an increase in $x$ implies an increase in $w_a$. That is, $\frac{\partial \psi}{\partial \psi} > 0$. Further simplification yields

$$
x = \left\{ \frac{\alpha}{1 - \alpha} (h - 1) \eta_g \left( \frac{w_a}{\theta} \right)^{\frac{1}{\theta + 1}} - \left[ h \left( \frac{w_a}{\theta} \right)^{\frac{1}{\theta + 1}} - (\Gamma_g)^{\frac{1}{\theta + 1}} \right] \frac{\alpha}{1 - \alpha} + \frac{1 - T_{gh}}{1 - T_{gh} (1 - T_{gh})} \right\}^{1 - \theta}
$$

It can be seen that $\frac{\partial \psi}{\partial \psi} > 0$. Therefore, we obtain:

$$
x = \kappa_2(w_a, \eta_g),
$$

38
a function strictly increasing in \( w_a \) and \( \eta_g \). After some algebra, we can verify that when \( f = 1 \), 
\[ \kappa_2(w_{a1}, \eta_g) = w_{a1}, \kappa_1(\eta_g). \]
That is, the \( x^*(w_a) \)-schedule is continuous at \( w_a = w_{a1} \).

For case 2 to be relevant, we need \( x_2 > 1 \) and \( x_1 < \lambda \); otherwise the \( x^*(w_a) \)-schedule is described by one of the regions that follow.

**Case 3:** Equilibrium relative wages satisfy \( \frac{w_a}{w_n} \in (\Gamma_g, \Gamma_v) \). As in case 1 above, supply determines relative quantities at \( \frac{H_a}{H_n} = \Xi^2 \). In order for relative wages to fall into the region \((\Gamma_g, \Gamma_v)\), the growth rate of the economy has to satisfy \( x \in (x_2, x_3) \), where \( x_3 \) is defined as \( x_3 = [\Xi^2]^{1-\theta} \Gamma_v \). As in case 1, we use this equilibrium wage premium, the labor supplied to the adopting sector from (23), and demand from (12), and derive the \( x^* \)-schedule as:

\[ x^* = w_a \kappa_3(\eta_g), \]

where,

\[ \kappa_3(\eta_g) = \frac{1}{\theta} \left[ \eta_g \left( \alpha T_{gl} + h(1 - T_{gl}) + (1 - \alpha)(1 - T_{gl} - T_{gh}) \right) \right]^{1-\theta}. \]

Hence for \( x \in (x_2, x_3) \), the \( x^*(w_a) \)-schedule is linearly increasing in \( w_a \), and strictly increasing in \( \eta_g \).

For case 3 to be relevant, we need \( x_3 > 1 \) and \( x_2 < \lambda \). Again, one can show that the \( x^*(w_a) \)-schedule is continuous at \( w_a = w_{a2} \). We denote the endpoints for this case as \( w_{a2}, w_{a3} \).

**Case 4:** Finally, suppose equilibrium relative wages satisfy \( \frac{w_a}{w_n} = \Gamma_v \). The equilibrium relative labor allocation, from the labor supply equation, satisfies \( \frac{H_a}{H_n} \left( \frac{w_a}{w_n} \right) \geq \Xi^2 \). From labor demand, in order to rationalize equilibrium wages \( \frac{w_a}{w_n} = \Gamma_v \) we need the growth rate to satisfy \( x \geq x_3 \).

At these relative wages all agents except possibly some of the high-skilled agents with vocational education work in the adopting sector. These agents are indifferent between both sectors. In order to obtain absolute labor inputs and hence absolute wages in both sectors we have to determine the fraction \( f = I_{vh} \) of high-skilled vocationally educated agents working in the adopting sector. Equilibrium in both labor markets require

\[ \left( \frac{\theta}{w_n} \right)^{\frac{1}{\theta - 1}} = H_n^d = H_n^* = \bar{\mu}_{vh}(1 - f)h \]

\[ \left( \frac{\theta x}{w_a} \right)^{\frac{1}{\theta - 1}} = H_a^d = H_a^* = \bar{\mu}_{gl}E_{gl} + \bar{\mu}_{vp}E_{vp} + \bar{\mu}_{gh}E_{gh} + f \bar{\mu}_{vh}E_{vh} \]

From the labor market clearing condition in the nonadopting sector and the equilibrium stationary measures, one obtains, after several algebraic steps:

\[ f(w_a, \eta_g) = \frac{(1 - (1 - \alpha)T_{vl}) \left[ h(1 - \eta_g) - \left( \frac{\theta \Gamma_v}{w_n} \right)^{\frac{1}{\theta - 1}} \right]}{(1 - \alpha)(1 - T_{vh}) \left( \frac{\theta \Gamma_v}{w_n} \right)^{\frac{1}{\theta - 1}} + (1 - (1 - \alpha)T_{vl})h(1 - \eta_g)}. \]

Next we use the equilibrium condition from the labor market \( \frac{H_a}{H_n} = \left( \frac{w_a}{w_n} \right)^{\frac{1}{\theta - 1}} = \left( \frac{x}{\Gamma_v} \right)^{\frac{1}{\theta - 1}} \), and the
expression for \( f \) to arrive at:

\[
x^* = \left[ \left( \frac{w_a}{\theta} \right) \eta_g (\Lambda_g - \Lambda_v) + \Lambda_v \left( \left( \frac{w_a}{\theta} \right) \frac{1}{h} - \frac{(\Gamma_v)^{\frac{1}{T_v}}}{h} \right) \right]^{1-\theta}
\]

where:

\[
\Lambda_g = \frac{\alpha E_{gl} + (1 - \alpha) [(1 - T_{gh}) E_{gl} + (1 - T_{gh}) E_{gh}]}{1 + (1 - \alpha) [(1 - T_{gh}) - T_{gl}]}\]

is the effective amount of labor contributed by each general education worker to \( H_a \) and:

\[
\Lambda_v = \frac{\alpha E_{vl} + (1 - \alpha) [(1 - T_{ch}) E_{vl} + (1 - T_{ch}) E_{ch}]}{1 + (1 - \alpha) [(1 - T_{ch}) - T_{vl}]}\]

is the maximum possible effective amount of labor contributed by each general education worker to \( H_a \) (which will occur when \( f = 1 \)). Note that \( \Lambda_g \) and \( \Lambda_v \) only depend on the underlying parameters of the model. Also, under assumption 1 one can show that \( \Lambda_g > \Lambda_v \). Therefore \( \kappa_4(w_a, \eta_g) \) is strictly increasing in both \( w_a \) and \( \eta_g \). Furthermore it can be verified when \( f = 0 \), that \( \kappa_4(w_{a3}, \eta_g) = w_{a3} \kappa_3(\eta_g) \) so that the \( x^*(w_a) \)-schedule is continuous at \( w_a = w_{a3} \). For case 4 to be relevant, we need \( x_3 \leq \lambda \).

### A.6 Proof of Lemma 5:

**Proof.** Fix \( \eta_g \). First we define the wage rate \( w_a \in (0, \infty) \) such that \( x^*(w_a) = 1 \). If \( x_1 > 1 \) then obviously \( w_a = w_{a1} \). If \( x_1 \leq 1 \) and \( x_2 > 1 \), let \( w_a = \kappa_2^{-1}(x = 1, \eta_g) \) where \( \kappa_2^{-1} \) is the inverse of \( \kappa_2 \) with respect to its first argument (which exists since \( \kappa_2 \) is strictly increasing in \( w_a \)). If \( x_2 \leq 1 \) and \( x_3 > 1 \), let \( w_a = \frac{1}{\kappa_3(\eta_g)} \). Finally, if \( x_3 \leq 1 \), let \( w_a = \kappa_2^{-1}(x = 1, \eta_g) \).

From the previous section we know that \( x^*: [w_a, \infty) \rightarrow [1, \infty) \) is a continuous, strictly increasing function that by construction of \( w_a \) satisfies \( x^*(w_a) = 1 \) and that tends to \( \infty \) as \( w_a \) tends to \( \infty \). The function \( x: [w_a, \infty) \rightarrow (1, \lambda) \) is continuous and strictly decreasing, with \( x(w_a) > 1 \) and \( \lim_{w_a \rightarrow \infty} x(w_a) = 1 \). Thus, for a given fraction \( \eta_g \in (0, 1) \) of agents obtaining general education there exists a unique BGP growth rate \( x^*(\eta_g) \in (1, \lambda) \).

### A.7 Proof of Lemma 6:

**Proof.** Given the four distinct segments of \( w_a(x) \) and the two segments of \( w_a^A(x) \) there are eight possible combinations for the intersection (depending on parameter values). However, no matter

\[\text{For } w_a > w_{a3} \text{ we have equilibrium wages } \frac{w_a}{w_{a3}} = \Gamma_v, \text{ and thus } w_{a3} \text{ tends to } \infty \text{ with } w_a. \text{ Thus implies that } H_n \rightarrow 0 \text{ and thus } H_a \text{ tends to } H_a = \sum_{i=g,v} \sum_{H=h} \bar{\mu}_{iH} E_i H \]

From labor demand, this requires that \( x \) tends to \( \infty \).
where the intersection is, when \( \eta_g \) increases, there is no change in \( x(w_a) \); for \( x^*(w_a) \), the cutoff points \( x_1,x_2 \) and \( x_3 \) move up and the slopes of the linear segments increase. Therefore the whole schedule shifts up. Therefore, the equilibrium \( x \) either increases (if the intersection was in the downward sloping portion of the \( x(w_a) \) curve or remains at \( \lambda \) (if the intersection was in the horizontal portion of the \( x(w_a) \) curve). Thus the equilibrium growth rate is (weakly) increasing in \( \eta_g \). This is subject to the proviso that we remain in the same labor market case with the new \( \eta_g \). The shift obtains even when the cutoffs change and the labor market case changes, but we omit the proof here for brevity; it is available from the authors upon request.

### A.8 Proof of Lemma 7:

**Proof.** We derive explicit formulas for the thresholds \( \eta_{gi} \) that separate the various cases described above. Let’s start with \( \eta_{g3} \), i.e. with the threshold \( \eta_g \) above which Case 1 applies.

**Case 1:** From the labor market we have 

\[
\left( \frac{H_i}{H_n} \right)(\eta_g)^{-1-\theta} = \frac{1}{\Xi^1(\eta_g)}.
\]

There are two possibilities for the equilibrium growth rate for case 1.

Case 1a: \( x(\eta_g) = \lambda \), i.e. the intersection of the \( x(w_a) \) and \( x^*(w_a) \) curve occur in the flat portion of the \( x(w_a) \)-schedule. Then 

\[
\left( \frac{w_a}{w_n} \right)(\eta_g) = \frac{\lambda}{\Xi^1(\eta_g)}
\]

and since \( \Xi^1 \) is strictly increasing in \( \eta_g \), \( \frac{w_a}{w_n} \) is strictly decreasing in \( \eta_g \) and strictly increasing in \( \lambda \).

Case 1b: \( x(\eta_g) < \lambda \), i.e. the intersection occurs in the downward sloping part of the \( x(w_a) \) schedule. Equilibrium wages and growth rates \( w_a \) and \( x \) satisfy:

\[
x = w_a \kappa_1(\eta_g)
\]

\[
\left( \frac{x}{w_a} \right)^{\frac{\theta}{1-\theta}} = x - 1.
\]

Solving these equations yields:

\[
w_a(\eta_g) = \theta \kappa_1(\eta_g)^{-\frac{2\theta}{1-\theta}} + \kappa_1(\eta_g)^{-1}
\]

\[
x(\eta_g) = \theta \kappa_1(\eta_g)^{\frac{\theta}{1-\theta}} + 1,
\]

where, since \( \kappa_1 \) is strictly increasing in \( \eta_g \) and from the assumption \( \theta < \frac{1}{2} \), one obtains that \( w_a(\eta_g) \) is strictly decreasing and \( x(\eta_g) \) is strictly increasing. Thus:

\[
\left( \frac{w_a}{w_n} \right)(\eta_g) = \frac{\theta \kappa_1(\eta_g)^{\frac{\theta}{1-\theta}} + 1}{\Xi^1(\eta_g)}
\]

\[
= \theta^{\frac{1+2\theta}{1-\theta}} \left[ \frac{1 - (1 - \alpha) T_{gl} - \alpha \eta_g}{\Xi^1(\eta_g)} \right]^{1-\theta}.
\]

\[
\left( \frac{1}{\alpha \left( 1 - T_{gl} (1 - \frac{1}{h}) \right) \eta_g} \right)^{1-\theta} + \left( \frac{1}{\left( 1 - (1 - \alpha) T_{gl} \right)} \right)^{\frac{\theta}{1-\theta}} \cdot \left( \frac{1}{\alpha \left( 1 - T_{gl} (1 - \frac{1}{h}) \right) \eta_g} \right)^{\theta(1-\theta)}
\]

\[
= B_1 \left( B_2 (\eta_g)^{\theta-1} + B_3 (\eta_g)^{2\theta-1} \right) = B(\eta_g)
\]

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where:

\[ B_1 = \theta \left( 1 - (1 - \alpha)T_{gl} - \alpha \eta_g \right)^{1-\theta} > 0 \]

\[ B_2 = \left( \frac{1}{\alpha (1 - T_{gl}(1 - \frac{1}{h}))} \right)^{1-\theta} > 0 \]

\[ B_3 = \left( \frac{h}{1 - (1 - \alpha) T_{gl}} \right)^{\theta} \cdot \left( \alpha \left( 1 - T_{gl}(1 - \frac{1}{h}) \right) \right)^{2\theta - 1} > 0. \]

The first term of the product is positive and strictly decreasing in \( \eta_g \) and the second term is positive and strictly decreasing in \( \eta_g \) because of the assumption that \( \theta < \frac{1}{2} \). Thus \( \left( \frac{w_a}{w_n} \right)(\eta_g) \) is strictly decreasing in \( \eta_g \) (and evidently constant in \( \lambda \)). Conditional on being in case 1 (i.e. \( \eta_g > \eta_{g3} \)) we can find conditions on \( \eta_g \) such that case 1.a or case 1.b. prevails. For case 1.a. we need

\[ w_a = \frac{\lambda}{\kappa_1(\eta_g)} \leq \bar{w}_a, \]

which, after some algebra and using the expression for \( \kappa_1(\eta_g) \) and \( \bar{w}_a \) translates into

\[ \eta_g \geq \frac{1 - (1 - \alpha)T_{gl}}{\alpha h(1 - T_{hl}(1 - \frac{1}{h}))}(\lambda - 1) = \bar{\eta}_g, \]

whereas case 1.b. requires the same inequality reversed. Note that, given the restriction for being in case 1, subcases 1.a or 1.b or both may occur as we increase \( \eta_g \).

In summary, we have:

\[ \left( \frac{w_a}{w_n} \right)(\eta_g) = \begin{cases} \left( \frac{\lambda}{\kappa_1(\eta_g)} \right)^{\theta} & \text{if } \eta_g > \bar{\eta}_g^1 \\ \left( \frac{\lambda}{\kappa_1(\eta_g)} \right)^{\theta} + 1 & \text{if } \eta_g \leq \bar{\eta}_g^1 \end{cases} \]

Turning to the determination of the thresholds, since \( \kappa_1(\eta_g) \) is strictly increasing and \( \bar{\eta}_g^1 \) solves exactly the equation \( \lambda = \theta \kappa_1(\eta_g)^{\frac{\theta}{\theta-\alpha-1}} + 1 \), we can rewrite this as

\[ \left( \frac{w_a}{w_n} \right)(\eta_g) = \frac{\min \left\{ \lambda, \theta \kappa_1(\eta_g)^{\frac{\theta}{\theta-\alpha-1}} + 1 \right\}}{\left( \Xi(\eta_g) \right)^{1-\theta}}. \]

Note that \( \left( \frac{w_a}{w_n} \right)(\eta_g) \) is strictly decreasing and continuous in \( \eta_g \). In order to be in case 1 we require

\[ \left( \frac{w_a}{w_n} \right)(\eta_g) < \Gamma_g. \]

Thus the threshold \( \eta_{g3} \) solves:

\[ \left( \frac{w_a}{w_n} \right)(\eta_{g3}) = \Gamma_g. \]

Since \( \left( \frac{w_a}{w_n} \right)(\eta_g) \) is strictly decreasing in \([0, 1] \), this equation has at most one solution in the unit interval. Note that \( \left( \frac{w_a}{w_n} \right)(\eta_g = 0) = \infty \) and

\[ \left( \frac{w_a}{w_n} \right)(\eta_g = 1) = \left( \frac{(1 - \alpha)(1 - T_{gl})}{\alpha(h - T_{gl}(h - 1))} \right) \min \left\{ \lambda, \theta \frac{\lambda^{\theta-1}}{\alpha h(1 - T_{gl}(1 - \frac{1}{h}))} \right\}, \]

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So either \( \left( \frac{w_a}{w_n} \right)^1 (\eta_g = 1) \leq 1 \), in which case there is a unique solution for \( \eta_{g3} \in [0, 1] \) and case 1 does occur for all \( \eta_g \in [\eta_{g3}, 1] \) or \( \left( \frac{w_a}{w_n} \right)^1 (\eta_g = 1) > 1 \) in which case case 1 is irrelevant. Also note that this condition is purely on the fundamentals of the economy. Furthermore, as a function of \( \lambda \), \( \eta_{g3} \) is increasing in \( \lambda \) (also note that \( \tilde{\eta}_g^3 \) is strictly increasing in \( \lambda \)).

**Case 3:** The analysis is similar to case 1. From the labor market we have \( \left( \frac{w_a}{w_n} \right) (\eta_g) = \eta_g \). Again two possibilities emerge.

Case 3a: \( x(\eta_g) = \lambda \). Then \( \left( \frac{w_a}{w_n} \right) (\eta_g) = \lambda \) and since \( \Xi^2 \) is strictly increasing in \( \eta_g \), \( \frac{w_a}{w_n} \) is strictly decreasing in \( \eta_g \) and strictly increasing in \( \lambda \).

Case 3b: \( x(\eta_g) < \lambda \). Equilibrium wages and growth rates \( w_a \) and \( x \) satisfy:

\[
\begin{align*}
x &= w_a \kappa_3(\eta_g) \\
\left( \frac{x}{w_a} \right)^{\frac{\theta}{\theta + 1}} &= x - 1.
\end{align*}
\]

Solving these equations yields:

\[
\begin{align*}
w_a(\eta_g) &= \theta \kappa_3(\eta_g)^{-\frac{1 - 2\theta}{\theta}} + \kappa_3(\eta_g)^{-1} \\
x(\eta_g) &= \theta \kappa_3(\eta_g)^{\frac{\theta}{\theta + 1}} + 1.
\end{align*}
\]

Again \( w_a(\eta_g) \) is strictly decreasing and \( x(\eta_g) \) is strictly increasing. Thus

\[
\left( \frac{w_a}{w_n} \right) (\eta_g) = \frac{\theta \kappa_3(\eta_g)^{\frac{\theta}{\theta + 1}} + 1}{\Xi^2(\eta_g)^{1 - \theta}} = \frac{(\eta_g)^{\theta - \frac{1 - 2\theta}{\theta}} \left[ (\alpha T_{gh} + h(1 - T_{gh}) + (1 - \alpha)(1 - T_{gh})) \right]^\theta + 1}{\Xi^2(\eta_g)^{1 - \theta}} = \frac{C_1 \left( C_2 (\eta_g)^{2\theta - 1} (1 - \eta_g)^{-\theta} + \left( \frac{1}{\eta_g} - 1 \right)^{1 - \theta} \right)}{\Xi^2(\eta_g)^{1 - \theta}} = C(\eta_g),
\]

where:

\[
\begin{align*}
C_1 &= \frac{\left( (1 - (1 - \alpha)T_{gh}) (1 - T_{gl}(1 - \frac{\bar{h}}{\bar{h}})) + (1 - \alpha)(1 - T_{gl}) (1 - (1 - T_{gh})(1 - \frac{1}{\bar{h}})) \right)}{\left( (1 - (1 - \alpha)T_{gh}) + (1 - \alpha)(1 - T_{gh}) \right)^{\theta - 1}} > 0, \\
C_2 &= \frac{\theta - \frac{1 - 2\theta}{\theta} \left[ \frac{\alpha T_{gl} + h(1 - T_{gl}) + (1 - \alpha)(1 - T_{gl}(1 - \frac{1}{\bar{h}}))}{1 + (1 - \alpha)(1 - T_{gl} - T_{gh})} \right]^\theta}{\Xi^2(\eta_g)^{1 - \theta}} > 0.
\end{align*}
\]

Hence \( \left( \frac{w_a}{w_n} \right) (\eta_g) \) is strictly decreasing in \( \eta_g \) and evidently constant in \( \lambda \). Conditional on being in case 3 (i.e. \( \eta_g \in [\eta_{g1}, \eta_{g2}] \)) we can find conditions on \( \eta_g \) such that case 3.a or case 3.b. prevails. For case 3.a. we need:

\[
w_a = \frac{\lambda}{\kappa_3(\eta_g)} \leq \tilde{w}_a,
\]

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which, after some algebra and using the expression for $\kappa_3(\eta_g)$ translates into

$$
\eta_g \geq \frac{1 + (1 - \alpha) (1 - T_{gl} - T_{gh})}{(1 - (1 - \alpha) T_{gh}) E_{gl} + (1 - \alpha) (1 - T_{gl}) E_{gh}} (\lambda - 1)^{\frac{3}{2}} = \overline{\eta}_g^3,
$$

whereas case 3.b. requires that the same inequality is reversed. Note that given the restriction for being in case 3, subcases 3.a or 3.b or both may occur as we increase $\eta_g$.

The same logic as in case 1 applies to find the cutoffs. By the same argument as in case 1 we can write

$$
\left( \frac{w_a}{w_n} \right)^3 (\eta_g) = \min \left\{ \lambda, \theta \kappa_3(\eta_g) \frac{w}{w_n} + 1 \right\},
$$

which is strictly decreasing and continuous in $\eta_g$. To be in case 3 requires

$$
\left( \frac{w_a}{w_n} \right)^3 (\eta_g) \in (\Gamma_g, \Gamma_v).
$$

Define the cutoffs $\eta_{g1}, \eta_{g2}$ as solutions to

$$
\left( \frac{w_a}{w_n} \right)^3 (\eta_{g1}) = \Gamma_v
$$

$$
\left( \frac{w_a}{w_n} \right)^3 (\eta_{g2}) = \Gamma_g.
$$

Note that $\left( \frac{w_a}{w_n} \right)^3 (\eta_g = 0) = \infty$ and $\left( \frac{w_a}{w_n} \right)^3 (\eta_g = 1) = 0$, so there exist unique $\eta_{g1}, \eta_{g2} \in (0, 1)$. This assures that case 3 (and case 2 and 4) do occur for some $\eta_g \in [0, 1]$. Since $\Gamma_v > \Gamma_g$ and $\left( \frac{w_a}{w_n} \right)^3 (\eta_g)$ is strictly decreasing we have that $\eta_{g1} < \eta_{g2}$. Since $\Xi^2(\eta_g) > \Xi^1(\eta_g)$ and $\left( \frac{\kappa_3(\eta_g)}{\Xi^2(\eta_g)} \right) \frac{w}{w_n} < \left( \frac{\kappa_3(\eta_g)}{\Xi^1(\eta_g)} \right) \frac{w}{w_n}$ for all $\eta_g \in (0, 1)$ (after some tedious algebra) for all $\eta_g$ we have that $\eta_{g2} < \eta_{g3}$. Also note that $\eta_{g1}, \eta_{g2}$ are increasing as functions of $\lambda$ and that $\overline{\eta}_g^3$ is strictly increasing in $\lambda$. 

![Diagram](image_url)
The dependence of case 3 on \( \lambda \) is illustrated in the above figure. The one for case 1 is similar. The initial thresholds are \( \eta_{31}, \eta_{32}^3 \), and \( \eta_{32} \). An increase in \( \lambda \), leaves case 3b unaltered, while shifting the relative wage for case 3a, and shifts \( \eta_{32}^3 \), and \( \eta_{32}^2 \) to \( \eta_{32}^3 \), and \( \eta_{32}^2 \). The dependence of the relative wage schedule and thus the utility differential is relevant for the proofs that follow. ■

A.9 Derivation of utility differential \((v_{gl} - v_{ch})(\eta_g)\):

Recall that the utility differential depends on \( \eta_g \) only via its dependence on \( \frac{w}{w_n} \). Given the expressions for \( \frac{w}{w_n} \) derived in the above proof, the expressions for the utility differential given in the text follow. The expressions for the constants \( B_1, B_2, B_3, C_1, \) and \( C_2 \) are as given in the above proof.

Note in particular, for case 1a, \( \frac{w}{w_n} \) and hence the utility differential is strictly decreasing in \( \eta_g \) and strictly increasing in \( \lambda \). For case 1b, \( \frac{w}{w_n} \) and thus the utility differential is strictly decreasing in \( \eta_g \) and evidently constant in \( \lambda \). Identical observations follow for cases 3a, and 3b respectively. We establish that an increase in \( \lambda \) shifts the utility-differential curve, \((v_{gl} - v_{ch})(\eta_g)\), (weakly) upward: it leaves utility differentials constant for all cases but cases 1a. and 3a. for which utility differentials strictly increase. Furthermore it makes lower cases (i.e. higher utility differentials) more prevalent by (weakly) increasing the cutoff levels \( \{\eta_{g1}, \eta_{g2}, \eta_{g3}\} \) and \( \{\tilde{\eta}_{g1}, \tilde{\eta}_{g2}\} \).

A.10 Proof of Lemma 8:

**Proof.** Given the functional form assumed for \( e \), the equilibrium condition (24) can be restated as \( e(1-\eta_g) = (v_{gl} - v_{ch})(\eta_g) \). From (27) we can see that the segments corresponding to cases 2 and 4 are constant with respect to \( \eta_g \). The segments corresponding to cases 1 and 3 are strictly decreasing in \( \eta_g \). Furthermore the differentials are continuous in \( \eta_g \), so that the function \((v_{gl} - v_{ch})(\eta_g)\) is a continuous decreasing function, with \((v_{gl} - v_{ch})(0) > 0 \) and \((v_{gl} - v_{ch})(1) < \infty \). Given the properties of \( e(1-\eta_g) \) it therefore follows that there exists a unique \( \eta_g^* \in (0,1) \) and associated ability level \( a^* \) such that the agent with ability \( a^* \) is indifferent between opting for general or vocational education. ■

A.11 Proof of Lemma 9:

**Proof.** The first part follows from the properties of \( C^{US}(\eta_g), C^{EUR}(\eta_g) \) (or \( \tilde{C}^{US}(\eta_g), \tilde{C}^{EUR}(\eta_g) \) correspondingly) and the properties of \((v_{gl} - v_{ch})(\eta_g)\). Since the \( x^*(w_a)\)-schedule is independent of \( \eta_g \) and the \( x(w_a)\)-schedule shifts up with \( \eta_g \) (i.e. for all \( w_a \) we have \( x(w_a, \eta_g) > x(w_a, \tilde{\eta}_g) \) if \( \eta_g > \tilde{\eta}_g \)), the second result follows: either the intersection takes place in the flat region of the \( x(w_a)\)-schedule for both the US and EUR (in which case both regions have the same growth rate \( x = \lambda \)) or the intersection for Europe takes place in the decreasing part of the \( x(w_a)\)-schedule, with \( x^{EUR} < \lambda \). From the first result it then follows that \( x^{US} > x^{EUR} \). The third part follows from the second part and the definition of the growth gap. ■
A.12 Proof of Proposition 1:

Proof. By construction the cost functions $e(1 - \eta_g)$ or $C(\eta_g)$ or $\tilde{C}(\eta_g)$ do not change with $\lambda$. The $(v_{gl} - v_{eh})(\eta_g)$-function shifts (weakly) up. This is due to two factors. First, it is strictly increasing for cases 1.a. and 3.a. Second, the thresholds shift to the right with an increase in $\lambda$. Thus equilibrium $\eta_g$ increases or remains the same (if the original intersections occur in cases 1.b, 2, 2.b or 4 and remain there).

A.13 Proof of Proposition 2:

Proof. It is sufficient to show that $d\Delta(\lambda) = 0$ under the assumption that we don’t switch cases.

If we are in cases 1.b,2,2.b or 4 the equilibrium $\eta_g$ does not change, so the $x^*(w_a)$-schedule remains the same. The horizontal portion of the $x(w_a)$-schedule moves up and $\tilde{w}_a$ becomes smaller.

Two potential situations can happen:

1. The original growth rate satisfies $x(\lambda) < \lambda$, in which case $dx(\lambda)/d\lambda = 0$ and $d\Delta(\lambda)/d\lambda = 1$

2. The original intersection between $x(w_a)$ and $x^*(w_a)$ is exactly at the kink. Then $x(\lambda) = \lambda$, but again $dx(\lambda)/d\lambda = 0$ and $d\Delta(\lambda)/d\lambda = 1$

3. The original growth rate satisfies $x(\lambda) = \lambda$, but the intersection is not at the kink. Then $x(\lambda) = \lambda$, and $dx(\lambda)/d\lambda = 1$ and $d\Delta(\lambda)/d\lambda = 0$.

Note that the proposition looks at a marginal change in $\lambda$, inframarginal changes will in general satisfy $d\Delta(\lambda)/d\lambda \in [0,1]$

If we are in cases 1.a.or 3.a then we know that $x(\lambda) = \lambda$ (that is how these cases distinguish themselves from cases 1.b and 3.b, respectively). Since an increase in $\lambda$ increases $\eta_g$ and thus shifts the $x^*(w_a)$-schedule to the left, this pushes the intersection between the two curves only deeper into the region for which $x(\lambda) = \lambda$. Therefore $dx(\lambda)/d\lambda = 1$ and $d\Delta(\lambda)/d\lambda = 0$.

A.14 Proof of Proposition 3:

Proof. The first part we prove by contradiction. Suppose $d\Delta^{EUR}(\lambda)/d\lambda < d\Delta^{US}(\lambda)/d\lambda$. From the previous result this requires $d\Delta^{EUR}(\lambda)/d\lambda = 0 < 1 = d\Delta^{US}(\lambda)/d\lambda$. But this requires $x^{EUR} = \lambda$ and $x^{US} < \lambda$, a contradiction to lemma 9.

The second part follows from the arguments in the proof of the previous proposition.
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