SIEPR Discussion Paper No. 06-20

Optimal Tax Treatment of Families with Children

By
Kevin J. Mumford
Stanford University
February 2007

Stanford Institute for Economic Policy Research
Stanford University
Stanford, CA 94305
(650) 725-1874

The Stanford Institute for Economic Policy Research at Stanford University supports research bearing on economic and public policy issues. The SIEPR Discussion Paper Series reports on research and policy analysis conducted by researchers affiliated with the Institute. Working papers in this series reflect the views of the authors and not necessarily those of the Stanford Institute for Economic Policy Research or Stanford University.
The Optimal Tax Treatment of Families with Children

Kevin J. Mumford†

January 2007

Abstract

In the United States, the value of child tax benefits in the federal income tax have increased dramatically since 1992 and now exceed $140 billion annually. This paper examines the efficiency implications of child tax benefits. Using a representative agent framework, it lays out conditions under which a child subsidy is part of an optimal tax policy. The key finding is that child tax benefits are not part of an optimal tax policy if children and leisure (time not spent doing market work) are complements or weak substitutes. The cross-price substitution effect for children and leisure is estimated using data on female labor supply and birth histories in the National Longitudinal Survey of Youth. The results imply that children and leisure are complements and thus child subsidies are not optimal. The sign of the optimal tax result remains unchanged when the model is extended to allow for time costs associated with raising children, but the optimal child tax is likely lower. Explicitly including quality-producing expenditure on children as a fourth good in the model leads to the result that child subsidies likely reduce the average quality of children. The model is also extended to allow for externalities associated with children in calculating the optimal tax treatment. Distributional considerations may play an important role in providing a justification for child subsidies, although this paper suggests that this is only true at the lower range of the income distribution. This is shown formally in a two-agent model with ability differences.

JEL Codes: H21, J13

†Email: mumford@stanford.edu. Address: Stanford University, Department of Economics, 579 Serra Mall, Stanford, CA 94305-6072. Web: www.stanford.edu/people/mumford

Acknowledgments: I thank Kenneth J. Arrow, Michael J. Boskin, John B. Shoven, and members of the public finance and labor economics groups at Stanford, particularly Colleen Flaherty, Gopi Shah Goda, and Anita Alves Pena. I am grateful to the John M. Olin Program for Law and Economics at Stanford University for financial support and to the Kapnick Dissertation Fellowship provided through the Stanford Institute for Economic Policy Research.
1 Introduction

Families with children receive preferential treatment in the U.S. federal income tax. These child tax benefits are quite large, generally in the $1,500 to $3,000 range per child. Between 1993 and 2004, the real value of child tax benefits approximately doubled due to the expansion of existing tax provisions and the creation of new provisions. The expansion of child tax benefits over the last decade has had strong support from both political parties. The real value of child tax benefits could continue to increase as tax provisions designed to aid working families with children are expanded.¹

This paper examines the efficiency implications of child tax benefits. A simple representative agent model with endogenous children is considered and data from the National Longitudinal Survey of Youth (NLSY) is used to estimate key parameters of the model. The optimal tax results from the representative agent model are used with the estimated parameters to show that the optimal tax treatment of children would be a tax on children rather than a child subsidy. However, distributional considerations may justify tax benefits to families with children if the benefits exceed the efficiency costs. In fact, the results presented in this paper indicate that child tax benefits may be optimal in the lower range of the income distribution.

The optimal tax results are also derived for three different extensions to the basic model. The first extension considers the time costs associated with raising children. The second considers the role of child quality. The third allows for externalities associated with children (positive or negative). It is the claim of positive externalities that has generally driven child subsidy programs outside of the United States.

The United States has never had an explicit child subsidy program; child benefits are hidden in the complexity of the tax code. Most other developed countries have explicit child subsidy programs and in recent years, many of these have been expanded in an effort to increase fertility. Among other demographic concerns, many Western European governments are increasingly worried that low birth rates will further reduce the ratio of workers to retired persons which will make it more difficult to finance promised retirement benefits, implying a mix of higher taxes and lower benefits. In addition, since most developed countries get between 60 to 80 percent of government revenue from income and payroll taxes, a lower worker to total population ratio would likely make it more difficult to finance other government functions as well.

¹A September 10, 2006 article in the New York Times by David Brooks described a growing push by some conservative, pro-family groups to increase the child tax credit to $5,000.
Fertility rates in many countries have declined to levels below the replacement rate. This is especially apparent in Western Europe where, given age-specific fertility rates, an average of only 1.5 children are born per woman (see Figure 1).

Figure 1: Western Europe Total Fertility Rate: 1960-2004

An obvious, but increasingly unpopular, way to increase the ratio of workers to retired persons is immigration. Through immigration, a country can increase its population of workers at what may be a much lower cost than that of raising and educating children. However, as birth rates have fallen, rather than increase immigration, Western European countries have set limits. After the 2005 riots in France, the government announced that immigration officials would begin strict enforcement of requirements for immigrants seeking 10-year resident permits or French citizenship. The requirements include mastery of the French language and a high level of education or job-skills. The effect has been a dramatic decline in the number of immigrants. The Netherlands recently passed a law requiring immigrants pass a Dutch

---

2The total fertility rate measures the average number of children that would be born to a woman over her lifetime if she were to experience the current age-specific fertility rates through her lifetime. It is computed by summing the age-specific birthrates (live births per 1,000 women in each specific age group). The replacement rate is usually defined as the fertility rate at which the average woman has two children that survive to age 15. In developed countries the replacement rate is estimated to be slightly less than 2.1. The degree to which the replacement rate exceeds 2 is heavily influenced by child mortality rates.
Western Europe is increasingly turning to child subsidies in an attempt to increase the birth rate. Child subsidy programs have been around for decades, but in the last few years they have expanded dramatically, often with the explicitly announced goal of increasing birth rates. For example, France targeted a recent change in its child subsidy programs at increasing the number of families that have 3 children. The change entitles the mother of a third child to a $750 monthly subsidy for one year and then a $230 monthly subsidy until the child is grown. This is in addition to several child related income tax deductions and other government benefits for families with children.

Child subsidies are certainly not unique to Western Europe. South Korea has experienced a recent and very dramatic decline in its total fertility rate (see Figure 2). In response, the government announced an official goal of increasing fertility and then expanded its child subsidy program by $7 billion annually, approximately 0.9% of GDP. Russian fertility has also declined significantly over the past 20 years. In May 2006 Russia announced a large child subsidy program that is scheduled to begin making payments in 2007. The program offers a lump sum subsidy of about $9,000 for the birth of a second child and about $1,500 each year after. The annual cost of this program is estimated at 1% of GDP (equivalent to about $120 billion in the United States).

In the United States, the generous child benefits in the federal income tax do not have a stated goal of increasing fertility. Rather, they are seen as a way to decrease family income inequality or to compensate families for their child related expenditure. Several federal income tax provisions depend on the number of dependent children, primarily: the child tax credit, the dependent exemption, the earned income tax credit, the child and dependent care credit, and the head of household filing status. These and other tax provisions combine to give large benefits to families with children. Figure 3 shows the level of child tax benefit by number of children and income for a married couple in 2004. These benefits are not trivial; Figure 3 shows that a married couple with two children and a family income of $20,000 would receive $5,000 in child tax benefits. Child tax benefits remain high and do not decline significantly until nearly $150,000 of adjusted gross income.

Child tax benefits in the United States have grown substantially over the past decade, as shown

---

3There is evidence that immigrants to Western Europe, the majority of which are from North Africa and Turkey, have had trouble integrating (Banton 1999). Lack of integration by immigrants has increasingly been blamed for all sorts of troubles from urban blight to crime and terrorism.

4Russia has both a low fertility rate and a high mortality rate (male life expectancy is 58 years). Russia’s population is currently declining at a rate of 700,000 per year. The World Bank estimates that Russia’s population will fall from its present level of about 140 million to under 100 million by 2050. In contrast to the Western European concern of an aging population with few workers, the average Russian male does not live long enough to retire. Given that the decline in the worker to retired person ratio due to lower birthrates will be more moderate in Russia, it appears that Russia’s concerns...
Figure 2: Total Fertility Rate for Selected Countries: 1960-2004

![Graph showing total fertility rate for selected countries from 1960 to 2004.]


Figure 3: 2004 U.S. Child Tax Benefits (married couple)

![Graph showing U.S. child tax benefits for different numbers of children in 2004.]

The growth of these child tax benefits is very expensive. Table 1 gives an estimate of the budgetary cost of child tax benefits for selected years.\(^6\) It is clear from the table that the increase in government

\(^6\)The budgetary cost of child tax benefits is the government expenditure on refundable child tax benefits combined with the tax expenditure of child tax benefits. The tax expenditure for a tax policy is a measure of the loss of government revenue due to the policy. The term “tax expenditure” is attributed to Harvard Law professor, Stanley S. Surrey who first measured federal tax expenditures as an Assistant Secretary of the U.S. Treasury in 1967. See Surrey and McDaniel (1985) for a comprehensive treatment of the tax expenditure concept. See Burman (2003) for discussion of recent measurement issues.
spending on child tax benefits is large: a $95 billion increase over just fourteen years. In real terms, child tax benefits approximately doubled from 1992 to 2004.\footnote{Adjusting for inflation, the 1992 total is $57.3 billion in 2004 dollars.}

Much of the political dialogue around the creation and expansion of the child tax credit has been an assertion that American families (or children) are struggling and need the government’s help. By early 1995, the White House and leaders in Congress had publicly committed to providing a child tax credit. President Clinton proposed a $500 credit for children under age 13 in his 1995 State of the Union address (January 24, 1995). This amount became a focal point; most other proposals adopted a $500 credit and varied only on the child age limit and on phase-in and phase-out points for the credit. Indeed, during the 1996 presidential campaign, the only difference between the child tax credits proposed by the two parties was the child age limit (Democrats proposed 13, Republicans proposed 17).

However, while the White House and Congress had publicly committed to providing a child tax credit in 1997, “privately many of the members of both branches express misgivings that a child credit is not the best use of federal dollars” (Steuerle, 1997). The political discussion on child tax benefits has primarily focused on how much the government can afford to help families with children. Often, an increase in child subsidies is assumed to be desirable and there seems to be no public discussion of whether or not child subsidies are a good use of public funds. Kamin and Greenstein (2004) argue that politicians should explicitly consider the trade-off between higher child benefits now and the ability to provide child benefits in the future. While it is true that policy makers should consider intertemporal trade-offs, one might first ask if the income tax should include any child subsidies at all.

\begin{table}[h]
\centering
\caption{Estimated Budgetary Cost of Child Tax Benefits ($ billions)}
\begin{tabular}{lccccc}
\hline
\hline
Dependent Exemption & 24.1 & 30.7 & 35.8 & 36.4 & 35.9 \\
Earned Income Credit & 13.0 & 28.2 & 31.3 & 38.0 & 40.2 \\
Child Tax Credit & – & – & 19.9 & 31.2* & 56.2 \\
Child Care Expenses & 2.5 & 2.6 & 3.0 & 3.6 & 2.7 \\
Head of Household Status & 3.0 & 3.5 & 3.7 & 3.9 & 4.1 \\
\hline
TOTAL & 42.6 & 65.0 & 93.7 & 113.1 & 139.1 \\
\hline
Number of Children (millions) & 64.25 & 68.09 & 70.36 & 73.73 & 74 \\
Expenditure per Child & $663 & $955 & $1,332 & $1,540 & $1,880 \\
\hline
\end{tabular}
\footnote{* does not include the early child tax credit payments made in 2003}
\end{table}

Sources: OMB analytical perspectives tables 5-1 and 19-1 various years, IRS statistics of income publication 1304, US Census Bureau population estimates, and author’s calculations
It is clear that current child tax benefits have an important distributional impact; they represent a large income transfer from taxpayers without children to those with children.\(^8\) That parents, to a large extent, determine the number of children in the family is rarely included in this discussion. Government subsidization of children at this magnitude may have an effect on fertility choices. Child subsidies may distort fertility behavior and thus have an effect on efficiency. This paper’s primary goal is to describe the conditions under which a child subsidy is optimal. To do this we examine a series of models, each with a specific focus, and derive the optimal tax treatment of families with children.

Section 2 lays out conditions for optimal child subsidization in a representative agent model where parents decide how much time to spend working and how many children to have. The key parameter in determining the optimal tax treatment of children is the cross-price substitution effect for leisure and children. This substitution effect is estimated in Section 3 using data from the NLSY. Section 4 calculates the optimal tax treatment of children implied by the estimates. Some distributional implications are also considered. Section 5 extends the representative agent model from Section 2 to include time costs of children, child quality, and externalities associated with children. Section 6 briefly considers a two-agent model to show that distributional considerations can justify child tax benefits. Section 7 concludes.

2 Optimal Tax Treatment of Children

The optimal tax treatment of families with children is examined in a representative agent model. Social welfare in this model is represented by the following utility function

\[ U(C, L, N) \]  

where \( L \) is leisure (time endowment less market work time), \( N \) is the number of children, and \( C \) is the consumption of other goods. By assumption, each argument of the utility function is a good, meaning that children are a net source of enjoyment to their parents. The government has the ability to impose a linear income tax and can either subsidize or tax children, but must raise revenue \( R \). There are no lump-sum taxes or subsidies and consumption is untaxed.\(^9\)

The goal of this section is to derive conditions under which it is optimal to subsidize, rather than tax,

\(^8\) The details of these transfers are documented by Ellwood and Liebman (2001) and Bell and Steuerle (2006).
\(^9\) The price of consumption is the numeraire. This model with a linear income tax is equivalent to a model with a consumption tax and no income tax. One can either think of an income tax that decreases the price of leisure relative to the price of consumption, or a consumption tax that increases the price of consumption relative to the price of leisure. The tax or subsidy of children is simply disproportionate taxation of children relative to consumption goods.
the presence of children in a family while ignoring externalities (considered in Section 5) and distributional considerations (considered in Section 6). Because there is a single representative agent and the government must raise revenue $R$, the optimal tax policy is simply the policy that is most efficient at raising the required revenue. As will be shown, the optimal tax treatment of children in this simple model primarily depends on the cross-price substitution effect between leisure and children.

The efficiency cost of a tax policy is measured by its excess burden, that is, the loss of utility greater than would have occurred had the tax revenue been collected as a lump sum (Rosen, 1978). In other words, the excess burden of a tax policy is the loss in social welfare due to the distortion in relative prices only and not that which is due to the tax-induced loss of income. Exact measures of excess burden are defined by Diamond and McFadden (1974) and Auerbach and Rosen (1980). However, in general, it is necessary to have an explicitly specified utility function in order to calculate the exact excess burden. Rather than assume a particular utility function, we will use the well-known approximation developed by Hotelling (1938), Hicks (1939), and Harberger (1964):

$$EB = -rac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} t_i t_j S_{ij}$$

where $t_i$ is the tax rate on good $i$ and $S_{ij}$ is the substitution effect for good $i$ given an increase in the price of good $j$.

The pioneering analysis of the excess burden of a tax policy in a representative agent model with three goods is Corlett and Hague (1954). The analysis in the three-good case is extended by Boskin and Sheshinski (1983). The particular application was the optimal tax treatment of the labor supply for married couples where the three goods were consumption, the leisure of the husband, and the leisure of the wife. Here we follow the same Boskin-Sheshinski framework in that the optimal tax policy is the one that minimizes the Hotelling-Hicks-Harberger approximation of excess burden while still raising revenue $R$.

Assuming that the prices of all goods are constant in the relevant range and that there are no distortions in the economy (other than those caused by taxation), the excess burden of a tax policy is approximated by the following expression:

$$EB = -\frac{1}{2} (\Delta P_C \Delta C^e + \Delta P_L \Delta L^e + \Delta P_N \Delta N^e) .$$

\footnote{Auerbach (1985) provides a comprehensive mathematical and graphical descriptions of the excess burden and its relation to optimal tax theory.}
The excess burden is simply the sum of the three compensated deadweight loss triangles for consumption, leisure, and children. By assumption the only price distortions in the economy are those caused by the tax policy.\textsuperscript{11} Consumption is untaxed (see footnote 9), thus the change in the price of consumption is always zero. This enables us to drop the first of the three compensated deadweight loss triangles in the expression. The remaining compensated demands for leisure and children are both potentially affected by changes in either price:

\[
EB = -\frac{1}{2} \left[ \Delta P_L \left( \frac{\partial L^c}{\partial P_L} \Delta P_L + \frac{\partial L^c}{\partial P_N} \Delta P_N \right) + \Delta P_N \left( \frac{\partial N^c}{\partial P_N} \Delta P_N + \frac{\partial N^c}{\partial P_L} \Delta P_L \right) \right].
\]

(4)

If the compensated demand curves are highly linear, equation (4) will be a close approximation of the true excess burden of the tax policy.\textsuperscript{12} Once again using the assumption that the only distortions in the economy are those caused by the tax policy, the price change for leisure is given by \(\Delta P_L = (1-\tau)P_L - P_L = -\tau P_L\), where \(\tau\) is the income tax rate. Similarly for children, \(\Delta P_N = (1 + \theta)P_N - P_N = \theta P_N\), where \(\theta\) defines the tax treatment of children. A positive value of \(\theta\) is a tax on children whereas a negative value of \(\theta\) is a child subsidy.

While it is natural to think of the price of leisure as the after-tax wage, it is less clear what is meant by the “price of children”. We will take \(P_N\) to represent the level of expenditure necessary to raise a child. By assumption, the agent cannot chose to spend less than \(P_N\) for each child and any child-related expenditure above the necessary level will be considered consumption. This is equivalent to assuming that the agent has no ability to increase the quality of children by choosing to increase child-related expenditure. Expanding the model to allow for investment in child quality is considered in Section 5.

The agent has an endowment of time, \(T\), that can be taken as leisure \(L\), or market work, \(H\). The assumption that \(T - H = L\) is less restrictive than it first appears because \(T\) can be defined to exclude time required for household and personal maintenance including sleep. However, we are assuming that \(T\) is given exogenously and does not depend on the number of children. This must be taken to mean that the agent considers time spent raising children as leisure because it is unlikely that the amount of time spent raising children does not increase with the number of children. The implications of relaxing this assumption by allowing an explicit time cost of raising children are also explored in Section 5.

Returning to the approximation of excess burden, the symmetry of the Slutsky matrix allows (4) to

\textsuperscript{11} All goods are produced under constant returns by competitive firms employing labor as the only input, so there are no profits.

\textsuperscript{12} Green and Sheshinski (1979) show when this method provides an accurate approximation (even for large tax changes) and derive an approximation built from a higher order Taylor series expansion.
be rewritten as:

\[ EB = -\frac{1}{2} \left( (-\tau P_L)^2 S_{LL} + (\theta P_N)^2 S_{NN} + 2 S_{LN} (-\tau P_L) (\theta P_N) \right). \]  

(5)

For analytical convenience, we will scale the units of each good so that prices are unity. This means that consumption, leisure, and children are all expressed in dollar terms. For example, two children implies a value for \( N \) of \( 2(1 + \theta)P_N \). This normalization allows us to express the agent’s “full income” budget constraint as:

\[ T(1 - \tau) = C + L(1 - \tau) + N(1 + \theta). \]  

(6)

The optimal tax policy is the one that minimizes the excess burden of the tax policy subject to raising government revenue \( R \):

\[ \min_{\tau, \theta} \left\{ -\frac{1}{2} \left[ \tau^2 S_{LL} + \theta^2 S_{NN} - 2 \tau \theta S_{LN} \right] - \lambda \left[ \tau H + \theta N - R \right] \right\}. \]  

(7)

The first order conditions with respect to \( \tau \) and \( \theta \) can be solved to yield:

\[ \tau = -\frac{\lambda (S_{NN} H + S_{LN} N)}{S_{NN} S_{LL} - (S_{LN})^2} \]  

(8)

\[ \theta = -\frac{\lambda (S_{LL} N + S_{LN} H)}{S_{NN} S_{LL} - (S_{LN})^2} \]  

(9)

where \( \lambda \) is the multiplier on the government budget constraint. It is clear that the denominator for either expression is non-negative because it is the determinant of a second order principal minor of the Slutsky matrix which is negative semidefinite. It is also clear that the multiplier \( \lambda \) is positive because the excess burden of a tax policy increases in the revenue requirement, \( R \).\(^{13}\) Determining whether it is optimal to subsidize or tax the presence of children in a family is then reduced to signing the following expression:

\[ S_{LL} N + S_{LN} H \]  

(10)

\(^{13}\)The optimal tax policy as a function of the required government revenue, \( R \), is given by the following:

\[ \tau = \frac{R (S_{LN} N + S_{NN} H)}{S_{NN} H^2 + 2 S_{LN} H N + S_{LL} N^2} \]  

and

\[ \theta = \frac{R (S_{LL} N + S_{LN} H)}{S_{NN} H^2 + 2 S_{LN} H N + S_{LL} N^2} \]  

These expressions are used later in the paper as part of a back of the envelope calculation of the optimal tax treatment of children.
A child subsidy is optimal if and only if $S_{LL}N + S_{LN}H > 0$. Both $N$ and $H$ are constrained to be non-negative and $S_{LL}$ is non-positive by definition, so it is the value of $S_{LN}$ that is key in determining if a child subsidy is optimal. Thus, a necessary condition for the optimal tax policy to include child subsidies is that children and leisure be substitutes. This is Result 2.1.

**Result 2.1.** If leisure and children are complements ($S_{LN} < 0$) then it is not optimal to subsidize children.

An intuitive explanation for this result comes from considering how the compensated demand for each good is affected by the tax policy. By totally differentiating the first order conditions from the optimal tax problem, we can derive how the agent’s demand for leisure and children are affected by $\tau$ and $\theta$:

$$\frac{\partial L}{\partial \tau} = -S_{LL} - Hi_L$$
$$\frac{\partial L}{\partial \theta} = S_{LN} - Ni_L$$
$$\frac{\partial N}{\partial \tau} = -S_{LN} - Hi_N$$
$$\frac{\partial N}{\partial \theta} = S_{NN} - Ni_N$$

The income effects are not relevant in the excess burden measure because the same level of tax revenue, $R$, is raised under any policy considered. The sign of $S_{LN}$ is crucial in signing the effect of the tax policy. If leisure and children are complements ($S_{LN} < 0$) then an increase in $\tau$ increases the compensated demand for both leisure and children whereas an increase in $\theta$ decreases the compensated demand for both leisure and children. The income tax distortions are reduced by imposing a tax on children ($\theta > 0$) because it pushes the compensated demands back in the opposite direction. A tax on children also raises revenue, enabling the government to raise $R$ with a lower income tax rate.

If leisure and children are substitutes ($S_{LN} > 0$), an increase in $\tau$ increases the compensated demand for leisure but decreases the compensated demand for children. The income tax distortions are reduced by giving a child subsidy ($\theta < 0$). Providing child tax benefits is costly in that $\tau$ must be increased in order to finance the benefits, so only when leisure and children are strong substitutes (as defined below) is it optimal to provide child tax benefits.

Result 2.1 implies that $S_{LN} > 0$ is a necessary condition for the optimal tax policy to include a child subsidy. However, leisure and children as substitutes is not a sufficient condition for the optimality of a child subsidy unless the demand for children is zero under a lump sum tax of size $R$. Result 2.2 shows how strongly substitutable leisure and children must be for a child subsidy to be optimal.
**Result 2.2.** If \( S_{LN} > -S_{LL} \left( \frac{N}{H} \right) \) then it is optimal to subsidize children.

From Result 2.2 we see that subsidizing children is more likely to be optimal if there is a low demand for children and a high supply of labor. In fact, if work effort, \( H \), is zero then regardless of the value of \( S_{LN} \) it is never optimal to subsidize children.\(^{14}\) The intuition for Result 2.2 is that the income tax distortion can be somewhat offset by a child tax subsidy if leisure and children are substitutes. Figure 5 shows this graphically by depicting how the optimal child tax treatment varies with the cross-price substitution effect for leisure and children.

Figure 5: Optimal Child Tax Treatment.

\[
\theta = \frac{RNS_{LL}}{N^2S_{LL} + H^2S_{NN}} \\
S_{LN} = -S_{LL} \frac{N}{H}
\]

It appears from Result 2.2 that if labor is very inelastically supplied (\( S_{LL} \) close to zero), this would make the optimal tax policy more likely to include a child subsidy instead of a tax. Note however that the properties of the Slutsky matrix put bounds on the relationship between \( S_{LN} \) and \( S_{LL} \). We know that \( S_{NN}S_{LL} > (S_{LN})^2 \) which implies that inelastic labor supply is associated with a smaller absolute value of \( S_{LN} \). A similar reasoning about the demand for children is expressed as Result 2.3.

\(^{14}\)This does not imply that a particular tax unit with no earned income should receive no child subsidy. This result only applies in the representative agent model and so implies that if no one in the economy is working, it would not be optimal to subsidize children. Of course, with no earned income, the income tax would produce no revenue and children would have to be taxed in order to meet the revenue requirement.
Result 2.3. If \(|S_{NN}| < \left|S_{LL}\left(\frac{N}{H}\right)^2\right|\) then it is not optimal to subsidize children.

It is generally accepted that the demand for children is not very price sensitive. Result 2.3 points out that if this is correct, the theoretically implied bound on \(S_{LN}\) may make it impossible for leisure and children to be strong substitutes (as defined in Result 2.2). In fact, if the compensated demand for children is completely inelastic then the optimal policy is to tax children quite heavily. In this case, a child tax would have the ability to raise a large amount of revenue while causing no distortions.

While this simple model of labor supply and fertility choice abstracts from various characteristics of children, externalities associated with children, and differences across families, Results 2.1 - 2.3 are useful as a starting point to examine child tax policy. In this model, the case for subsidizing children primarily depends on the cross-price substitution effect for leisure and children, a quantity that has received little attention in the empirical literature. The following section attempts to estimate this cross-price substitution effect. The resulting estimate is then used in a numerical calculation of the optimal child tax treatment.

3 Data and Estimation

The data for this exercise is a sample of women from the National Longitudinal Survey of Youth 1979 (NLSY). The NLSY contains detailed labor supply and fertility information for each respondent from 1979 to 2004. The sample is restricted to women who were 16 to 20 when first interviewed in 1979. This restriction enables labor supply and earnings histories to be constructed from age 19 until age 43. The number of children born to each woman by age 43 is also obtained. While it would be preferable to extend the age range, data availability limits this. Some women in the sample may have children after age 43 and we would ideally have this in the data as well. However, there will likely be only a very small number of births after this age. The U.S. National Center for Health Statistics reports that less than 1 percent of women have a child after age 40 and that only 0.05 percent of women have a child after age 45 (Martin et al., 2005).

The women in the sample were personally interviewed annually from 1979 to 1994 and then biennially from 1996 to 2004. Missed interviews do not necessarily prevent the construction of a complete labor supply history because interviewers attempt to ask questions from the previous interviews if missed. However, multiple missed interviews, especially if consecutive, do prevent the creation of the labor supply

\footnote{For the youngest cohort, birth at age 42 is assumed if pregnancy is reported in 2004.}
history. Women for which it is not possible to construct a complete labor supply or fertility history are dropped from the sample.

The decision to use a sample of women rather than a sample of married couples is motivated by the fact that approximately one-third of all births in the United States are to unmarried women. This is not simply due to teenage mothers. While about 90 percent of teenage mothers are unmarried at the time they give birth, teenage mothers make up less than a quarter of the total number of births to unmarried women each year. Births to unmarried women of age 20 or more accounted for 26 percent of total births in 2003 (Martin et al., 2005).

Three relationship categories are defined: married, partnership, single. Nearly all of the women in the sample are single at age 19 and about 88 percent are married for some period of time between age 19 and 43. Only 8 percent never report being married or in a partnership. Some women move between relationship categories several times. A more complete model would allow the relationship status to be influenced by the woman’s choices, however, in this exercise, the relationship history is taken as exogenous. Like the relationship status, the labor supply and earnings of husbands and partners is also taken as exogenous. This requires the addition of nonwage income to the model of Section 2. The agent’s budget constraint becomes:

\[
T(1 - \tau) + M(1 - \tau) = C + L(1 - \tau) + N(1 + \theta)
\]  

where \(M\) is male earnings and other nonwage income (including transfer payments). The government budget constraint is also adjusted to reflect this addition:

\[
R = \tau H + \tau M + \theta N
\]

In the model, male and other income is subject to the linear income tax, but does not respond to changes in tax treatment. This common assumption is often justified by the finding that the labor supply elasticity for men is very low (some claim approximately zero) and thus while changes in tax treatment have an influence on female labor supply, men are less responsive. For example, MaCurdy, Green, and Paarsch (1990) estimate that both the substitution and income effects for male labor supply are close to zero. However, there is considerable evidence that investment and other non-wage income is quite responsive to the tax treatment, thus a worthwhile extension of the model would be to explicitly model
investment behavior. The model presented in Section 2 does not allow for saving, thus investment income is simply taken as exogenous for the estimation.

Because the representative agent model of Section 2 is a static model of labor supply and fertility, the choice variables are interpreted as the average labor supply of the woman (age 19 to 43) and the total number of children that she has. The average hours of market work over this period is not the answer to a particular survey question asking how many hours per week she works when employed. Rather, it is created from a series of more than a thousand questions asking how many hours she worked week by week over the previous period. This measure of the average hours of work cannot distinguish between a part-time worker who is employed continuously from age 19 to 43 and a full-time worker who is employed for only half that time period.

After removing observations that do not have complete birth, work, and earnings histories, the sample used in this analysis consists of 4,169 of the 6,283 women in the NLSY. All dollar amounts are inflation adjusted to year 2000 dollars using the CPI-U before averaging. An implied wage is calculated as the real average annual earnings divided by the average annual hours (see Figure 6). The summary statistics for this sample are given in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Sd.Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly Hours</td>
<td>4169</td>
<td>26.43</td>
<td>11.71</td>
<td>0</td>
<td>65.67</td>
</tr>
<tr>
<td>Annual Earnings</td>
<td>4169</td>
<td>18084.51</td>
<td>13435.37</td>
<td>0</td>
<td>104252.40</td>
</tr>
<tr>
<td>Nonwage Income</td>
<td>4169</td>
<td>27867.33</td>
<td>19347.70</td>
<td>131.47</td>
<td>123649.40</td>
</tr>
<tr>
<td>Hourly Wage</td>
<td>4059</td>
<td>12.45</td>
<td>6.48</td>
<td>4.81</td>
<td>65.14</td>
</tr>
<tr>
<td>Children</td>
<td>4169</td>
<td>1.958</td>
<td>1.340</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Married</td>
<td>4169</td>
<td>0.566</td>
<td>0.327</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Partner</td>
<td>4169</td>
<td>0.071</td>
<td>0.137</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Single</td>
<td>4169</td>
<td>0.363</td>
<td>0.306</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>White</td>
<td>4169</td>
<td>0.776</td>
<td>0.417</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Hispanic</td>
<td>4169</td>
<td>0.062</td>
<td>0.242</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Black</td>
<td>4169</td>
<td>0.144</td>
<td>0.351</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Other Race</td>
<td>4169</td>
<td>0.017</td>
<td>0.131</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Rural</td>
<td>4169</td>
<td>0.219</td>
<td>0.414</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Moved</td>
<td>4169</td>
<td>0.570</td>
<td>0.495</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Mother Education</td>
<td>4169</td>
<td>11.54</td>
<td>2.734</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Both Parents (14)</td>
<td>4169</td>
<td>0.618</td>
<td>0.486</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Immigrant Parents</td>
<td>4169</td>
<td>0.086</td>
<td>0.281</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The reported summary statistics are computed using sample weights.

The married, partner, and single variables in Table 2 are the fraction of time age 19 until 43 that
the individual is either married, living with a partner, or single. The income of the husband or partner as well as any nonwage income, including welfare benefits, are combined into a single nonwage income variable. The moved variable indicates whether the individual’s family moved to a different town while she was growing up. The summary statistics for variables indicating whether the individual lived with both biological parents until age 14 and whether either parent is an immigrant are also listed.

Figure 6 plots the real average annual earnings against the average hours of work expressed in hours per week. As indicated in the summary statistics, the average hourly wage is $12.45, represented as a line extending from the origin. The other two lines represent the 95th and the 5th percentile wages. For those women with no reported hours of market work over the full time period, no wage calculation can be made. This is unfortunate because an observed wage rate is essential in estimating the cross-price substitution effect for leisure and children. Therefore, those with no hours of market work are dropped from the sample. The characteristics of the full sample, those with positive hours of work, and those with no market work are shown in Table 3.

It is apparent from Table 3 that the women for whom it is not possible to calculate an average wage
Table 3: Sample Average by Hours of Work

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Hours &gt; 0</th>
<th>Hours = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>4,169</td>
<td>4,059</td>
<td>110</td>
</tr>
<tr>
<td>Sample Weight</td>
<td>1</td>
<td>0.985</td>
<td>0.015</td>
</tr>
<tr>
<td>Weekly Hours</td>
<td>26.43</td>
<td>26.83</td>
<td>0</td>
</tr>
<tr>
<td>Annual Earnings</td>
<td>18,084.51</td>
<td>18,362.11</td>
<td>0</td>
</tr>
<tr>
<td>Nonwage Income</td>
<td>27,867.33</td>
<td>28,001.11</td>
<td>19,152.40</td>
</tr>
<tr>
<td>Hourly Wage</td>
<td>12.45</td>
<td>12.45</td>
<td>-</td>
</tr>
<tr>
<td>Children</td>
<td>1.958</td>
<td>1.944</td>
<td>2.856</td>
</tr>
<tr>
<td>Married</td>
<td>0.566</td>
<td>0.569</td>
<td>0.366</td>
</tr>
<tr>
<td>Partner</td>
<td>0.071</td>
<td>0.070</td>
<td>0.123</td>
</tr>
<tr>
<td>Single</td>
<td>0.363</td>
<td>0.361</td>
<td>0.511</td>
</tr>
<tr>
<td>White</td>
<td>0.776</td>
<td>0.782</td>
<td>0.394</td>
</tr>
<tr>
<td>Hispanic</td>
<td>0.062</td>
<td>0.060</td>
<td>0.191</td>
</tr>
<tr>
<td>Black</td>
<td>0.144</td>
<td>0.140</td>
<td>0.382</td>
</tr>
<tr>
<td>Other Race</td>
<td>0.017</td>
<td>0.017</td>
<td>0.033</td>
</tr>
<tr>
<td>Rural</td>
<td>0.219</td>
<td>0.220</td>
<td>0.132</td>
</tr>
<tr>
<td>Moved</td>
<td>0.570</td>
<td>0.571</td>
<td>0.517</td>
</tr>
<tr>
<td>Mother Education</td>
<td>11.54</td>
<td>11.57</td>
<td>9.20</td>
</tr>
<tr>
<td>Both Parents (14)</td>
<td>0.616</td>
<td>0.618</td>
<td>0.439</td>
</tr>
<tr>
<td>Immigrant Parents</td>
<td>0.086</td>
<td>0.086</td>
<td>0.111</td>
</tr>
</tbody>
</table>

The reported summary statistics are computed using sample weights.

rate are quite different from the remaining sample. This is particularly true with respect to their family
background and fertility choices. No effort is made to correct for the selected nature of the remaining
sample. Therefore, estimates are interpreted as only applying to the 99 percent of women in the full
sample with an observable wage.\(^{16}\)

A common approach to estimating a substitution effect in a static model is to estimate a linear demand
equation that includes a wage variable and a nonwage income variable. This is particularly common in
the labor supply literature where an econometrician estimates a labor supply function of this form:

\[
\text{Hours}_i = \alpha_0 + \alpha_1 \text{Wage}_i + \alpha_2 \text{Nonwage Income}_i + \alpha_3 X_i + \epsilon_i. \tag{13}
\]

The vector \(X_i\) represents the predetermined characteristics of the individual that are observed by the
econometrician and \(\epsilon_i\) represents those unobserved characteristics that affect labor supply. A true labor
supply function would depend not only on the wage, but also on the prices of all other goods. In practice,

\(^{16}\)Sample selection correction using a Heckit method would require the specification of a participation equation, a wage
equation, and an exclusion restriction. Because nearly 99 percent of the sample has an observable wage, the sample selection
correction would not likely yield much additional insight, but would certainly add to the complexity.
other prices are not frequently included in the regression equation, particularly when using cross-sectional data. The usual assumption used to justify this is that the prices for all other goods are not individual specific. Among other things, this assumption implies that there are no geographical differences in other prices. If this assumption holds, there is no omitted variable bias in the estimates of $\alpha_1$ and $\alpha_2$.

In the labor supply equation, $\alpha_1$ is the change in labor supply due to a one dollar increase in the hourly wage. This total effect is comprised of an income effect and a substitution effect as given by the Slutsky equation:

$$\frac{\partial \text{Hours}}{\partial \text{Wage}} = \left( \frac{\partial \text{Hours}}{\partial \text{Wage}} \right)^c + \text{Hours} \frac{\partial \text{Hours}}{\partial \text{Income}}.$$  \hspace{1cm} (14)

This same approach to estimating an own-price substitution effect for labor supply (or leisure demand) can be used to estimate the cross-price substitution effect for leisure and children. We specify a linear child demand function that depends on the wage, nonwage income, a vector of predetermined and observed characteristics, and an error term that represents those unobserved characteristics that affect child demand:

$$\text{Children}_i = \beta_0 + \beta_1 \text{Wage}_i + \beta_2 \text{Nonwage Income}_i + \beta_3 X_i + \eta_i.$$ \hspace{1cm} (15)

Ideally, the price of children should also be included in this specification. The absence of this variable is cause for some concern because the demand of a good clearly depends on its own price. It is the lack of data on the cost of children that prevents it from being included in the regression equation. The high degree of uncertainty about the level of expenditure required to raise a child—and how this level changes with family size, family income, and other factors—severely complicates determining an individual specific cost of raising a child. Similar to the argument for the exclusion of other prices in a labor supply equation, one could argue that there is little individual specific variation in the direct cost of raising a child. However, differences in child tax benefits by income level, family economies of scale, and geographical differences in the cost of food, housing, and health care suggest that this may not be the case.

For this exercise, we make the assumption that there are no individual differences in the monetary cost of children. This does not rule out differences in the opportunity cost of raising children (considered in Section 5). Rather, the assumption is that each woman faces the same out-of-pocket expenditure necessary to raise a child. If it were possible to determine an individual specific $P_{Ni}$, this variable could be used in an alternative method for identifying the cross-price substitution effect for leisure and children: a regression of labor supply on $P_{Ni}$, the wage rate, and nonwage income.
For the identification of $\beta_1$, there is a serious concern that the decision to have a child has a direct influence on the wage. This is similar to concerns in the labor supply literature that the choice of hours of work directly affects the wage. For the child demand equation, a valid instrument is a predetermined characteristic that affects the individual’s wage but not the demand for children (or be correlated with other factors that affect the demand for children). Several observed characteristics like the month and year of birth and measures of the reading habits of the individual’s parents satisfy the definition in this sample. However, instrumental variables estimation gives very similar estimates of $\beta_1$ as OLS and using a Hausman test of endogeneity, we fail to reject that the wage is exogenous. This is not conclusive evidence given the suspect nature of the instruments; however, the assumption that the wage is exogenous is maintained in the discussion that follows.

Using the estimates for $\beta_1$ and $\beta_2$, the cross-price substitution effect for leisure and children are given by $(\beta_1 - \text{Hours} \beta_2)$ as indicated by the Slutsky decomposition:

$$\frac{\partial \text{Children}}{\partial \text{Wage}} = \left( \frac{\partial \text{Children}}{\partial \text{Wage}} \right)^c + \text{Hours} \frac{\partial \text{Children}}{\partial \text{Income}}. \quad (16)$$

In the representative agent model of Section 2, we normalize the units of all goods so that prices are unity. This same normalization is easily made to the total, income, and substitution effects by multiplying through by the appropriate prices:

$$S_{LN} = P_N \left( P_L \hat{\beta_1} - H \hat{\beta_2} \right). \quad (17)$$

Given this normalization, $S_{LN}$ is the change in the compensated demand for children for a doubling of the wage. At the average wage in the sample, a 100 percent wage increase places it above the 95th percentile (see Figure 6).

The estimation of equation (15) is given in Table 4. The region controls include indicators for the region of the country the individual was raised in, either the Northeast, South, Central, or West. The family controls provide information about the family during the woman’s growing-up years, whether it was a rural or urban area, if the family had access to a local library, and if the family ever moved. Also included is the number of siblings and indicators for oldest or youngest child, immigrant parents, and

\[17\] The two-stage least squares results are available from the author by request. In each regression, the estimate of $\beta_1$ was larger in magnitude (more negative, although not statistically different) using instrumental variables estimation than the corresponding OLS regression. If having an additional child (not the timing of the birth) had a direct effect on the wage, one would expect the instrumental variables estimation to produce estimates of $\beta_1$ that were smaller in magnitude.
Table 4: Linear Child Demand Estimation

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>-0.0308</td>
<td>-0.0232</td>
<td>-0.0160</td>
<td>-0.0157</td>
<td>-0.0229</td>
</tr>
<tr>
<td></td>
<td>(0.0031)**</td>
<td>(0.0032)**</td>
<td>(0.0031)**</td>
<td>(0.0031)**</td>
<td>(0.0032)**</td>
</tr>
<tr>
<td>Nonwage Income (thousands)</td>
<td>0.0191</td>
<td>0.0206</td>
<td>0.0080</td>
<td>0.0078</td>
<td>0.0203</td>
</tr>
<tr>
<td></td>
<td>(0.0011)**</td>
<td>(0.0011)**</td>
<td>(0.0013)**</td>
<td>(0.0013)**</td>
<td>(0.0011)**</td>
</tr>
<tr>
<td>Hispanic</td>
<td>0.603</td>
<td>0.316</td>
<td>0.383</td>
<td>0.374</td>
<td>0.309</td>
</tr>
<tr>
<td></td>
<td>(0.086)**</td>
<td>(0.092)**</td>
<td>(0.089)**</td>
<td>(0.093)**</td>
<td>(0.096)**</td>
</tr>
<tr>
<td>Black</td>
<td>0.499</td>
<td>0.331</td>
<td>0.615</td>
<td>0.575</td>
<td>0.281</td>
</tr>
<tr>
<td></td>
<td>(0.061)**</td>
<td>(0.062)**</td>
<td>(0.062)**</td>
<td>(0.065)**</td>
<td>(0.065)**</td>
</tr>
<tr>
<td>Other race</td>
<td>-0.029</td>
<td>-0.229</td>
<td>-0.059</td>
<td>-0.020</td>
<td>-0.182</td>
</tr>
<tr>
<td></td>
<td>(0.154)</td>
<td>(0.152)</td>
<td>(0.147)</td>
<td>(0.156)</td>
<td>(0.161)</td>
</tr>
<tr>
<td>Married</td>
<td>1.372</td>
<td>1.372</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.080)**</td>
<td>(0.080)**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partner</td>
<td>0.571</td>
<td>0.593</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.153)**</td>
<td>(0.153)**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.701</td>
<td>1.996</td>
<td>1.156</td>
<td>1.208</td>
<td>1.918</td>
</tr>
<tr>
<td></td>
<td>(0.072)**</td>
<td>(0.133)**</td>
<td>(0.084)**</td>
<td>(0.165)**</td>
<td>(0.162)**</td>
</tr>
</tbody>
</table>

Region controls: yes yes yes yes yes
Family controls: no yes yes yes yes
Religion controls: no no no yes yes

Observations | R-squared |
--- | --- |
4059 | 0.1005 |
4059 | 0.1422 |
4059 | 0.2020 |
4059 | 0.2063 |
4059 | 0.1473 |

Total Effect -0.383 -0.289 -0.199 -0.195 -0.285
Income Effect 0.351 0.378 0.147 0.143 0.373
Cross-Price Substitution Effect 
-0.734 -0.667 -0.346 -0.339 -0.658

* significant at 5% ** significant at 1%. The reported values are computed using sample weights. Standard errors in parentheses.

Region controls include: northeast, central, and south. Family controls include: number of siblings, youngest child indicator, oldest child indicator, biological parents (14), immigrant parents, mother’s education level, rural, moved, and a library indicator. Religion controls include: Catholic, Baptist, Methodist, Lutheran, Presbyterian, Pentecostal, Episcopalian, Jewish, Other Christian Religion, Non-Christian Religion, and a measure of frequency of attendance.

if the woman grew up in a home with both biological parents.\textsuperscript{18} Religion controls are for the religion that the individual was raised in and also a measure of how often the family went to religious services.\textsuperscript{19}

The religion categories (in order of size in the sample) are: Catholic, Baptist, Other Christian Religion, Methodist, Lutheran, Presbyterian, None, Pentecostal, Episcopalian, Jewish, and Non-Christian Religion.

There are several issues to consider in this type of estimation. Concern is often expressed about the measurement of nonwage income. The NLSY income variables are top-coded which puts a downward bias...

\textsuperscript{18}Youngest child is an indicator that the individual has at least one sibling and was the youngest child in her family; oldest child indicates that the individual has at least one sibling and was the oldest child in her family.

\textsuperscript{19}Included in these variables are indicator for being raised in a family that went to religious services twice a month or more and an indicator of having converted to a religion other than the religion raised in.
on both the average wage and the average nonwage income for an individual. The cutoff at which top-coding occurs and the procedure have changed over the 25 years of the survey. However, the number of individuals affected by top-coding is small. Another second concern is that the procedure for calculating nonwage income is to subtract female earnings from total family income. This means that transfer payments that may not be independent of female earnings are included in the measure of nonwage income instead of altering the wage. A third issue is the problem of nonresponse to income questions. Nearly all respondents in the NLSY report their own income and their spouse’s income, however, approximately 30 percent of respondents living with a partner in a given year do not report their partner’s income. Even if the individual keeps her finances completely separate from her partner’s, living together suggests that they probably share some common expenses like rent or house payments. Quite often, a woman who refuses to answer the partner’s income question will answer the question in the next year and since the average woman in the sample spends only about 7 percent of her time in a partnership, the nonresponse bias is likely small. A regression of the percentage of the time living with a partner on other factors including wage, race, region, and religion reveals very little correlation between time living with a partner and any other characteristics.

The assumption of a linear child demand function implies a specific form of the utility function. An alternate functional form assumption for the direct utility could yield different estimates for the substitution effects. Using OLS to estimate the child demand function may also be inappropriate because of the nature of the dependent variable. It is not possible for a woman to have a negative number of children. As Figure 7 shows, there is more bunching at zero than would be expected if children were distributed normally. While this suggests the possibility of censoring at zero (because individuals cannot have a negative number of children), the more serious issue is that the dependent variable that is not even approximately continuous and thus the normally distributed error term is probably not reasonable. The dependent variable in this exercise takes on only 12 different values, the natural numbers from 0 to 11. A discrete distribution, such as the Poisson is commonly used for this type of count data.

The Poisson distribution is determined by a single parameter, $\mu$ that indicates the intensity of the

---

20 In the NLSY, partner income is not included in the constructed total family income variable. For this study, partner income was added to the constructed family income variable and then the respondent’s earnings were subtracted to give nonwage income. Note that from 1979-1994 the partner income variables are separate from spouse income variables, but after 1994, these variables are combined.

21 This regression shows that women who live in the west and those who have lower wage rates spend more time living with a partner. None of the religion variables were significantly different than zero. The R-squared from the regression was 0.021, implying that the variables used in this analysis are not strongly correlated with time spent living with a partner.

22 See Pencavel (1986) for a discussion of the direct and indirect utility functions implied by linear demand equations.
Figure 7: Number of Children Histogram

Density computed using NLSY sample weights.

Poisson process. The value of \( \mu \) is both the mean and the variance of the distribution. Therefore, we only need to specify \( \mu = E(\text{children}_i|\text{wage}_i, \text{nonwage income}_i, X_i) \), which we assume is given by the exponential function:

\[
E(\text{children}_i|\text{wage}_i, \text{nonwage income}_i, X_i) = e^{(\beta_0 + \beta_1 \text{wage}_i + \beta_2 \text{nonwage income}_i + \beta_3 X_i)}.
\] (18)

This specification assures that \( \mu \) will be positive for all values of wage, nonwage income, and \( X \). In this model, the probability that the number of children equals the value \( j \), conditional on the independent variables, is

\[
\Pr(\text{children} = j|X) = \frac{e^{-e^{(X\beta)}}(e^{(X\beta)})^j}{j!} \quad \text{for } j = 0, 1, 2, \ldots
\] (19)

where \( X \) represents the full set of explanatory variables, including wage and nonwage income. Maximum likelihood estimation is used to obtain the parameter estimates. The estimated marginal effects evaluated at the mean are reported as the first two columns of Table 5.

The Poisson regression model is often criticized because it requires that the conditional mean and variance be equal. The unconditional mean and variance for children are actually quite close in value. The importance of this restriction is often evaluated by also fitting a negative binomial regression model because it allows for overdispersion. However, because the sample variance for children is slightly less than the sample mean, the data will be even more underdispersed once regressors are included. Using the negative binomial regression model would be inappropriate because it can only accommodate overdispersion, not underdispersion (Cameron, 1998).

As an alternative, an ordered probit model is also considered. The model is built around the concept
Table 5: Poisson and Ordered Probit Child Demand Estimation

<table>
<thead>
<tr>
<th></th>
<th>Poisson</th>
<th>Poisson</th>
<th>Ordered Probit</th>
<th>Ordered Probit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>-0.0170</td>
<td>-0.0252</td>
<td>-0.0148</td>
<td>-0.0208</td>
</tr>
<tr>
<td></td>
<td>(0.0051)**</td>
<td>(0.0056)**</td>
<td>(0.0040)**</td>
<td>(0.0042)**</td>
</tr>
<tr>
<td>Nonwage Income (thousands)</td>
<td>0.0074</td>
<td>0.0185</td>
<td>0.0074</td>
<td>0.0183</td>
</tr>
<tr>
<td></td>
<td>(0.0013)**</td>
<td>(0.0011)**</td>
<td>(0.0013)**</td>
<td>(0.0012)**</td>
</tr>
<tr>
<td>Hispanic</td>
<td>0.347</td>
<td>0.281</td>
<td>0.319</td>
<td>0.244</td>
</tr>
<tr>
<td></td>
<td>(0.081)**</td>
<td>(0.082)**</td>
<td>(0.068)**</td>
<td>(0.066)**</td>
</tr>
<tr>
<td>Black</td>
<td>0.646</td>
<td>0.282</td>
<td>0.545</td>
<td>0.235</td>
</tr>
<tr>
<td></td>
<td>(0.075)**</td>
<td>(0.066)**</td>
<td>(0.059)**</td>
<td>(0.054)**</td>
</tr>
<tr>
<td>Other race</td>
<td>0.006</td>
<td>-0.168</td>
<td>-0.005</td>
<td>-0.162</td>
</tr>
<tr>
<td></td>
<td>(0.216)</td>
<td>(0.194)</td>
<td>(0.197)</td>
<td>(0.183)</td>
</tr>
<tr>
<td>Married</td>
<td>1.450</td>
<td>1.356</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.093)**</td>
<td>(0.086)**</td>
<td>(0.183)</td>
<td></td>
</tr>
<tr>
<td>Partner</td>
<td>0.792</td>
<td>0.635</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.204)**</td>
<td>(0.171)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region controls</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Family controls</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Religion controls</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Observations</td>
<td>4059</td>
<td>4059</td>
<td>4059</td>
<td>4059</td>
</tr>
</tbody>
</table>

* significant at 5% ** significant at 1%. The reported values are computed using sample weights. Standard errors in parentheses.

Region controls include: northeast, central, and south. Family controls include: number of siblings, youngest child indicator, oldest child indicator, biological parents (14), immigrant parents, mother’s education level, rural, moved, and a library indicator. Religion controls include: Catholic, Baptist, Methodist, Lutheran, Presbyterian, Pentecostal, Episcopalian, Jewish, Other Christian Religion, Non-Christian Religion, and a measure of frequency of attendance.

of a latent demand for children that, conditioning on \( X \), is normally distributed. This latent demand for children is continuously distributed with nothing preventing it from taking on negative values. However, this latent demand is not observed. What we do observe is the actual number of children \((0, 1, 2, \ldots)\) selected by each individual. A set of cutoff values, for example \({0.5, 1.5, 2.5, 3.5, \ldots}\), for the latent variable determine the observed number of children. If the latent variable were to have a value that falls between 0.5 and 1.5 (given the cutoff values in the example above) then the individual would choose to have one child. The cutoff values and parameter estimates are obtained by maximum likelihood estimation. The estimated marginal effects evaluated at the mean are reported as the third and fourth columns of Table 5.

The results reported in Table 5 are consistent with those of Table 4. A one dollar increase in the
life-time average hourly wage (evaluated at the mean wage) is associated with a decrease in the number of children between 0.014 and a 0.031. The estimated income effect is positive, meaning that children are a normal good. Thus, the negative total effect implies that the substitution effect is larger in magnitude than the income effect.

For some, a positive value for the estimated income effect is perhaps surprising. It is sometimes claimed that high-income countries have lower fertility rates due to a negative income effect. These results are contrary to that claim. In fact, it is apparent from an examination of the unconditional average number of children by income level that the number of children increases with income (see Figure 8). Instead, these results support the claim that higher female wages are an important factor in explaining fertility decline.\footnote{Butz and Ward (1979), Schultz (1985), and Heckman and Walker (1990) all find evidence that fertility is decreasing in female wage rates. While it seems clear that rising female wages in the last half of the 20th Century is an important explanation for the decline in fertility rates, it should be noted that fertility rates in the United States began to decline in the late 19th Century, long before any sizable increase in female wages.}

Figure 8: Average Number of Children by Life-Time Income

This estimation exercise suggests that leisure and children are complements. As you recall from Section 2, the optimal tax treatment of children if leisure and children are complements is a tax on children rather than a child subsidy. Some may consider this to be a troubling result, arguing that it is anti-family or regressive in nature. In contrast, it is simply an indication of the efficiency cost of providing child tax benefits. Not only does provision of these benefits require higher income tax rates than would be necessary in order to raise an equal level of government revenue, but child tax benefits also augment the distortion of family behavior. However, simply showing that child subsides have an efficiency cost does not
imply that they should be eliminated. Externalities, redistributive goals, ability-to-pay considerations, and other factors may provide a rational for child tax benefits. The result from the representative agent model that children should be taxed and not subsidized indicate that careful consideration should be given to whether the benefits from providing child tax benefits outweigh the efficiency costs.

4 Numerical Example

The sign of the estimated cross-price substitution effect for leisure and children implies that they are complements. This, in turn, implies that an optimal tax policy would involve a child tax rather than a subsidy. This section attempts to determine the magnitude of the optimal child tax given the estimated substitution effect for leisure and children.24

The addition of nonwage income to the model implies only minor changes to the optimal child tax treatment equation derived in Section 2:

\[
\theta = \frac{R (S_{LL}N + S_{LN}(H + M))}{S_{NN}(H + M)^2 + 2NS_{LN}(H + M) + S_{LL}N^2}.
\] (20)

Evaluating this equation requires estimates of each of the parameters. The sample means for annual earnings, nonwage income, and children as reported in Table 3 provide values for some of the parameters. However, we still lack estimates for \(S_{LL}, S_{NN}, P_N, \) and \(R.\)

An estimate for the own-price substitution effect for leisure can be obtained by following the method used to obtain the estimate of the cross-price substitution effect for leisure and children. We assume that labor supply is given by a linear function as in equation (13). The estimation results are reported in Table 6.

The results indicate that a dollar increase in a woman’s average hourly wage leads to an increase of approximately 0.48 hours per week of market work. As shown in Table 6, this total effect is decomposed into a negative income effect and a positive substitution effect. The substitution effect reported in the table is normalized so that the interpretation of the effect applies for a doubling of the hourly wage. Since the time endowment is simply the sum of hours of work and leisure time, the own-price substitution effect for leisure demand is the negative of the own-price substitution effect for labor supply.

The female labor supply elasticities implied by these results seem reasonable. Using the column (4)
Table 6: Linear Labor Supply Estimation

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>0.508</td>
<td>0.473</td>
<td>0.482</td>
<td>0.478</td>
<td>0.470</td>
</tr>
<tr>
<td></td>
<td>(0.026)**</td>
<td>(0.027)**</td>
<td>(0.027)**</td>
<td>(0.027)**</td>
<td>(0.027)**</td>
</tr>
<tr>
<td>Nonwage Income (thousands)</td>
<td>-0.149</td>
<td>-0.157</td>
<td>-0.177</td>
<td>-0.178</td>
<td>-0.160</td>
</tr>
<tr>
<td></td>
<td>(0.009)**</td>
<td>(0.009)**</td>
<td>(0.011)**</td>
<td>(0.011)**</td>
<td>(0.009)**</td>
</tr>
<tr>
<td>Hispanic</td>
<td>-1.836</td>
<td>-0.457</td>
<td>-0.367</td>
<td>-0.552</td>
<td>-0.634</td>
</tr>
<tr>
<td></td>
<td>(0.716)*</td>
<td>(0.783)</td>
<td>(0.783)</td>
<td>(0.814)</td>
<td>(0.814)</td>
</tr>
<tr>
<td>Black</td>
<td>-3.037</td>
<td>-2.344</td>
<td>-1.950</td>
<td>-2.198</td>
<td>-2.587</td>
</tr>
<tr>
<td></td>
<td>(0.508)**</td>
<td>(0.527)**</td>
<td>(0.548)**</td>
<td>(0.572)**</td>
<td>(0.551)**</td>
</tr>
<tr>
<td>Other race</td>
<td>-0.692</td>
<td>0.189</td>
<td>0.469</td>
<td>0.427</td>
<td>0.164</td>
</tr>
<tr>
<td></td>
<td>(1.286)</td>
<td>(1.291)</td>
<td>(1.292)</td>
<td>(1.366)</td>
<td>(1.364)</td>
</tr>
<tr>
<td>Married</td>
<td>2.002</td>
<td>1.879</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.702)**</td>
<td>(0.703)**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partner</td>
<td>-0.738</td>
<td>-0.695</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.341)</td>
<td>(1.343)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.602)**</td>
<td>(1.127)**</td>
<td>(1.225)**</td>
<td>(1.443)**</td>
<td>(1.372)**</td>
</tr>
<tr>
<td>Region controls</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Family controls</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Religion controls</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Observations</td>
<td>4059</td>
<td>4059</td>
<td>4059</td>
<td>4059</td>
<td>4059</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.1328</td>
<td>0.1528</td>
<td>0.1551</td>
<td>0.1627</td>
<td>0.1606</td>
</tr>
<tr>
<td>Total Effect</td>
<td>6.325</td>
<td>5.889</td>
<td>6.001</td>
<td>5.951</td>
<td>5.852</td>
</tr>
<tr>
<td>Own-Price Substitution Effect</td>
<td><strong>9.061</strong></td>
<td><strong>8.772</strong></td>
<td><strong>9.251</strong></td>
<td><strong>9.220</strong></td>
<td><strong>8.789</strong></td>
</tr>
</tbody>
</table>

* significant at 5% ** significant at 1%. The reported values are computed using sample weights. Standard errors in parentheses.

Region controls include: northeast, central, and south. Family controls include: number of siblings, youngest child indicator, oldest child indicator, biological parents (14), immigrant parents, mother’s education level, rural, moved, and a library indicator. Religion controls include: Catholic, Baptist, Methodist, Lutheran, Presbyterian, Pentecostal, Episcopalian, Jewish, Other Christian Religion, Non-Christian Religion, and a measure of frequency of attendance.

Coefficient estimates, the compensated wage elasticity is 0.344 and the uncompensated wage elasticity is 0.224. While they are well within the range of estimated elasticities in the female labor supply literature, they lie in the lower segment of the range (Killingsworth and Heckman, 1986).

Difficulty measuring the labor supply and wage rate in this type of analysis is common. Fortunately, the NLSY has very good labor supply data. Rather than the average hours of work per year, the individuals are asked to record the number of hours they work each week. The wage rate is calculated by dividing the real average annual earnings by the average annual hours. This can lead to the well-known division bias if the hours of work are measured with error. Errors in measuring the true hours of work.
will influence the calculated wage rate which causes a spurious negative correlation between wages and hours. However, because of the precision of the NLSY hours of work data, division bias is not likely to be a significant problem.

Estimation of the own-price substitution effect for children, $S_{NN}$, is not possible with this data because there is no information about the cost of children. Instead, we will turn to a recent estimate in the literature. Laroque and Salanié (2005) use a structural model of labor supply and fertility to explain the fertility response of families in France. They estimate the uncompensated cost elasticity of the demand for children to be about 0.2. If we accept this value and apply it to the NLSY sample used in this analysis, it implies that doubling the cost of children would reduce the demand by about 0.392 children. This uncompensated effect is the sum of the substitution effect, $S_{NN}$, and the income effect for children. In the estimation of the demand for children we obtained an estimate of the income effect for children. Using the Laroque and Salanié (2005) uncompensated effect and the income effect estimated from the NLSY data we can calculate the implied own-price substitution effect.

However, completing this exercise requires an estimate of the cost of children, $P_N$. There are various estimates of $P_N$ in the literature that cover a wide range. A convenient starting point is the U.S. federal poverty thresholds. The poverty threshold increases with the size of the family; therefore it gives an implicit valuation of the cost of an additional child. The 2004 poverty threshold for married couples placed the implied annual cost of a second child at $3,952 and of a third child at $3,386. The child cost derived from the poverty thresholds is near the bottom of the range of estimates of the annual cost of children. The USDA Expenditures on Children (2005) gives a much larger estimate of the cost of a child, placing the value at about $10,000 for a middle income family (Lino, 2006). Housing and transportation costs account for much of the difference between these two estimates. Rather than select an average value, we will perform the calculation twice, once using a value of $3,700 for $P_N$ and then using a value of $10,000 for $P_N$.\(^{25}\)

Figure 9 depicts the optimal child tax implied by these parameter values. The two lines extending from the origin show how the optimal child tax level depends on the required level of revenue, $R$, for the two assumed values of $P_N$. A higher cost of children implies a larger optimal child tax. The Congressional Budget Office reports that taxpayers in 2003 paid an average of $6,100 in federal income tax.\(^{26}\) Taking

\(^{25}\)For $P_N = 3,700$, $S_{LN} = -1.253$ and $S_{NN} = -1.234$. For $P_N = 10,000$, $S_{LN} = -3.387$ and $S_{NN} = -2.404$. The parameter values imply $S_{LL} = -5.969$ regardless of the assumed value of $P_N$.

\(^{26}\)See the CBO document “Historical Effective Federal Tax Rates: 1979 to 2003” located on the web at: http://www.cbo.gov/ftpdocs/70xx/doc7000/12-29-FedTaxRates.pdf. The CBO reports that taxpayers in 2003 paid an average of $14,200 in federal taxes, 43 percent of which was due to the individual income tax. This publication also shows
this value for \( R \), the implied optimal tax level per child is between $600 and $1,230.

In this numerical example, the optimal income tax rate is approximately 10 percent. As an illustration, in order to provide child tax benefits of $1,500 per child and still raise tax revenue of $6,100, the income tax rate would need to be set at 20 percent. The United States would be unlikely to ever adopt a system of explicit child taxes. A child tax would likely be seen as regressive and fundamentally unfair. Public response would perhaps be similar to the 1988 head tax in the United Kingdom. Of course, there is no need for a child tax to be regressive; it could be designed to increase with taxable income. A flat tax per child (a child head tax) would be regressive because while it is true that higher income families tend to have more children, they do not have proportionally more children.

Table 7 presents the correlation between the number of children under age 18 in families and the before-tax total family income for various groups in a cross section of the population. The sample is 47,675 families in the 2004 Survey of Income and Program Participation (SIPP). Family income is the sum of all income from whatever source to members of the family. The family income distribution is quite skewed with a median of $37,092 and a mean of $62,262 using the sample weights. The results show that in the full sample of all families, the correlation between income and children is positive and tax liability by income quintiles. the average federal tax liability in the middle quintile is $7,100, 20 percent of which was due to the individual income tax.
Table 7: Correlation between Family Income and Children

<table>
<thead>
<tr>
<th></th>
<th>(a) All</th>
<th>(b) 25-55</th>
<th>(c) Married</th>
<th>(d) Single</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child Income</td>
<td>35.1408</td>
<td>36.1274</td>
<td>44.1374</td>
<td>13.3625</td>
</tr>
<tr>
<td>Correlation</td>
<td>(0.357)**</td>
<td>(0.420)**</td>
<td>(0.575)**</td>
<td>(0.308)**</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.1709</td>
<td>0.2045</td>
<td>0.2165</td>
<td>0.0719</td>
</tr>
</tbody>
</table>

Family income measured in thousands, results computed using SIPP sample weights.

One might suspect that the correlation between children and income found for the whole population is driven by low-income retired persons or young singles that the data categorize as one-person families. To demonstrate that this is not driving the results, the correlation is also computed for the subsets of families where the adult answering the survey questions is between the ages of 25 and 55, married, and single. In each case there is a positive and significantly different than zero correlation between children and family income. Figure 10 shows the average number of children in family income bands for each of the groups considered in Table 7. These histograms show that families with more income tend to have more children living at home.27 The increased variability for singles at the upper end of the income distribution is due to the low number of observations.28

Therefore, the results from the representative agent model, the estimation exercise, and the numerical example should not be interpreted to mean that the income tax should include no child tax benefits. In the representative agent model, the child tax result hinges on the assumption that the government needs to raise a positive level of revenue. If instead, the government was deciding how best to transfer money to the agent, the optimal policy would include a child subsidy. Recall from Figure 9 that a negative value of $R$ implies a child subsidy rather than a child tax. This suggests that any individual who faces negative average and marginal income tax rates should be given a child subsidy for purely economic efficiency reasons. This is important because families at the lower end of the income distribution in the United States generally have a negative federal income tax liability. Figure 11 shows that a large and increasing fraction of taxpayers receive a net subsidy from the federal income tax.

---

27The general perception that low income families have more children than high income families may be based on life-cycle observations rather than simple cross sections. Figure 8 shows that women with low life-time family income tend to have more children than those with median levels of life-time income. In a cross section, low income couples are likely younger and thus probably have fewer children than older higher income couples. However, in a given year, high income families have more children and thus child subsidies provide more benefits to higher income families.

28Less than 5 percent of the one-person families in the sample have annual incomes above $100,000.
In the current federal income tax, families at the low end of the income distribution face negative marginal and effective tax rates. Figure 12 shows how the marginal and effective tax rates are affected by the number of children in the family. The figure indicates that a married couple with two children receives a net subsidy if family income is less than $48,000. If maximizing social welfare involves making some level of transfer payment to families making less than this amount, the most efficient way to transfer the income, if lump-sum transfers are not feasible, is through the combination of a negative tax rate and a child subsidy. However, high income families that face positive effective tax rates could be made better off by eliminating child tax benefits and reducing the marginal tax rate in such a way as to keep the effective tax rate constant.

A brief formal treatment of distributional considerations is given in Section 6. The desire for some level of redistribution may provide justification for child tax benefits. This somewhat blunts the child tax result from the representative agent model. A strong objection to the model itself is that it assumed
away many of the important characteristics of children. The next section extends the representative agent model to incorporate some of these characteristics and discusses the implications with respect to the optimal child tax treatment.

5 Children in an Economic Model

While the results so far indicate that child tax benefits are not optimal, the model used to derive these results was very restrictive in that it does not treat children any differently than other goods. Reinterpreting $N$ as some other good, cars for example, would have no effect on the optimal tax results of Section 2. If cars and leisure are complements, then a positive vehicle registration fee (or car tax) is optimal. Only if individuals worked a great deal, spent very little on cars, and thought of leisure and vehicles as substitutes would a car subsidy be optimal. But, children are fundamentally different than cars and other goods.

This section alters the basic representative agent model of Section 2 to include time costs of children, child quality, and externalities associated with children. Each of these features is considered individually and the implications for the optimal tax treatment of children are discussed.
5.1 Time Cost of Raising Children

Children cannot be purchased like other goods. A ban against buying or selling children (or any humans) is now fairly universal. Instead, children must be produced (raised) by the household using time and other goods as inputs.\textsuperscript{29} Housing, food, clothing, and medical care constitute the primary other goods used as inputs in raising children. Failure to provide for a child’s basic needs is defined as child neglect, a crime.\textsuperscript{30} We will assume that child production requires a certain level of time and a certain level of money. Additional time and money spent on children is treated as additional leisure or consumption.

\textsuperscript{29}Adoption is simply an alternative production technology that requires a different mix of inputs.
\textsuperscript{30}In the U.S., states are responsible for providing their own definitions of child neglect. It is frequently defined in terms of a failure to provide for the child’s basic needs, such as adequate food, clothing, shelter, supervision, and medical care. In most states child neglect is a felony if the child is younger than a specified age and a misdemeanor if the child is older.
Adding a time cost of raising children to the model presented in Section 2 is a simple modification. The model presented in Section 2 is altered so that the agent now faces a time constraint that depends explicitly on children:

$$TE = H + L + g(N).$$  \hspace{1cm} (21)

The function \(g(N)\) represents the time cost of raising children. The time cost likely increases in the number of children, \(g'(N) > 0\), but due to economies of scale in families, \(g''(N) < 0\). For convenience, we scale the units of each good so that the wage and the monetary costs of children are unity. The agent maximizes equation (1), subject to the following full income budget constraint:

$$TE(1 - \tau) = L(1 - \tau) + g(N)(1 - \tau) + C + N(1 + \theta)$$  \hspace{1cm} (22)

where \(M\) is non-labor income.

By totally differentiating the first order conditions from the optimal tax problem (see the appendix), we derive how the agent’s leisure and child choices are affected by \(\tau\) and \(\theta\).

$$\frac{\partial L}{\partial \tau} = -S_{LL} - g'(N)S_{LN} - Hi_L$$  \hspace{1cm} (23)

$$\frac{\partial N}{\partial \tau} = -S_{LN} - g'(N)S_{NN} - Hi_N$$  \hspace{1cm} (24)

$$\frac{\partial L}{\partial \theta} = S_{LN} - Ni_L$$  \hspace{1cm} (25)

$$\frac{\partial N}{\partial \theta} = S_{NN} - Ni_N$$  \hspace{1cm} (26)

If children and leisure are complements \((S_{LN} < 0)\), then from equations (23) and (24) it is clear that increasing the income tax increases the compensated demand for both leisure and children. Equations (25) and (26) again show that a child tax would decrease the compensated demand for both leisure and children.

The estimation of the cross-price substitution effect for leisure and children in Section 3 and for the own-price substitution effect for leisure in Section 4 were not estimates of \(S_{LN}\) and \(S_{LL}\) in this time-cost model. Instead, the estimate of what was called the cross-price substitution effect for leisure and children is, in fact, an estimate of the combined term \(S_{LN} + g'(N)S_{NN}\). The estimate of the own-price substitution effect for leisure is instead an estimate of the combined term \(S_{LL} + g'(N)S_{LN}\). 33
The true value of $S_{LN}$ is likely much smaller in magnitude than what we estimated in Section 3. Section 4 presented evidence that the magnitude of the combined term $S_{LN} + g'(N)S_{NN}$ ranged from about 40 percent larger than $S_{NN}$ to only 1 percent larger than $S_{NN}$ (see footnote 25). This implies that leisure and children may be substitutes if $g'(N) > 1$. If children and leisure are substitutes then while it is still true that the income tax distorts behavior toward more leisure and more children, a child subsidy would augment only the child distortion and would actually reduce the leisure distortion. The intuition is that the effect on the compensated demand for children due to a change in the price of leisure is no longer the same as the effect on the compensated demand for leisure due to a change in the price of children. By requiring time in child production, any change in the value of time directly changes the price of children.

Since we have estimates of $S_{LL} + g'(N)S_{LN}$, $S_{LN} + g'(N)S_{NN}$, and $S_{NN}$, solving for $S_{LL}$ and $S_{LN}$ would be possible if we knew the value of $g'(N)$. Recall that the term $g'(N)$ measures the increase in time (measured in dollars) needed to raise an additional child. If an additional child costs more in time than in money then $g'(N) > 1$, implying that $S_{LN}$ is possibly greater than zero. If, on the other hand, necessary child-related expenditure for an additional child is larger than the opportunity cost of the time required to raise an additional child, then $g'(N) < 0$ and $S_{LN}$ is likely less than zero.

To formally derive the optimal tax policy, we again use the excess burden approximation given in equation (3). As before, the change in the price of children depends on the child tax treatment, but it now also depends on the income tax. Specifically, $\Delta P_N = (1 + \theta) + g'(N)(1 - \tau) - 1 - g'(N) = \theta - \tau g'(N)$.

We then arrive at the following expression for the excess burden:

$$EB = -\frac{1}{2} \left[ \tau^2 \left( S_{LL} + 2g'(N)S_{LN} + g'(N)^2S_{NN} \right) - 2\theta \tau \left( S_{LN} + g'(N)S_{NN} \right) + \theta^2S_{NN} \right].$$

(27)

The derivation of equation (27) and the method for solving the optimal tax problem are given in the appendix. The optimal tax policy, given below, is the one that minimizes equation (27) subject to raising the required government revenue $R$.

$$\tau = \frac{-\lambda(g'(N)S_{NN} + S_{NN}H + S_{LN}N)}{S_{NN}S_{LL} - (S_{LN})^2}$$

(28)

$$\theta = \frac{-\lambda \left( 2g'(N)S_{LN}N + S_{LN}H + g'(N)^2S_{NN}N + g'(N)S_{NN}H + NS_{LL} \right)}{S_{NN}S_{LL} - (S_{LN})^2}$$

(29)
The expressions for the optimal tax policy given in equations (28) and (29) above are similar to those from Section 2. In fact, for \( g'(N) = 0 \), equations (28) and (29) reduce to equations (8) and (9). Both \( \tau \) and \( \theta \) are unambiguously positive for \( R > 0 \) and \( S_{LN} < 0 \). This leads to the following result, analogous to Result 2.1:

**Result 5.1.** In a model with time costs of children, if leisure and children are complements \( (S_{LN} < 0) \) then it is not optimal to subsidize children.

The introduction of time costs to the model does not alter the central result that child subsidies are not optimal if leisure and children are complements, but it does imply that the size of the optimal child tax is less than what we computed in Section 4. The precise cutoff value is given as Result 5.2.

**Result 5.2.** With time costs of children, if \( S_{LN} > \frac{-\left(g'(N)^2S_{NN}N + g'(N)S_{NN}H + NS_{LL}\right)}{2g'(N)N + H} \) then it is optimal to subsidize children.

With time costs of children incorporated into the model, the case for taxing children is somewhat reduced. The central optimal tax result is relatively unchanged, but reinterpreting the empirical evidence using this model leads us to believe that leisure and children are not nearly as strong of complements as implied by Section 3. The model suggests that unless the opportunity cost of raising an additional child is much greater on average than the monetary cost, children and leisure are complements. Even if children and leisure are substitutes, they would need to be strong substitutes in order for the optimal tax policy to include a child subsidy.

### 5.2 Child Quality

There is a large sociology literature on the trade-off between the number of children in a family and the quality of those children. The negative relationship between academic success (educational attainment, grades, and test scores) and the number of siblings was recognized more than a century ago. A recent review of the sociology literature, Steelman et al. (2002), reports that this negative relationship is consistently found in studies and is quite strong, comparable in importance to gender, race, or family income. The common explanation for this empirical regularity is that the amount of family resources that can be allocated to any given child depends not only on family income, wealth, and parental time allocation, but also on the number of children. Commonly called the resource dilution model, the hypothesis claims that increasing the number of children in a family requires fixed resources to be divided over more individuals.
and hence average child quality is reduced, even after taking into account the economies of scale (Blake 1981).31

Steelman et al. (2002) reports that multiple empirical studies confirm that resources are diluted as the number of children increases. These studies find that time with a child, educational materials per child, educational expenditure per child, and child participation in sports and cultural activities all decline in family size. However, the survey notes that a link between diluted resources and decreased child quality has yet to be established.32 Some sociologists now interpret the inability to link expenditure on children and measures of child quality as evidence that parents have little ability to increase the quality of their children through expenditure. They claim that parental resources are less important in determining child quality than other factors unrelated to parental expenditure on children (i.e. genetic endowment).

The empirical economics literature on the subject seems to hold a similar position. Angrist, Lavy, and Schlosser (2005) find, using an IV strategy, that the link between family size and child quality, as measured by education, is not causal. They suggest that the correlation between family size and measures of child quality are due to factors that affect both fertility and the home environment, such as parental education. The decision to have an additional child does not seem to directly decrease the average quality of children in the family. Their explanation for the absence of a causal link between the quantity and the quality of children is that marginal expenditure on children is primarily consumption rather than investment. That is, additional expenditure on children contributes little to human capital development even though it is valued by the parents, the children, or both. For example, Caceres (2004) finds that children in smaller families are more likely to have their own room, but that this has no effect on measures of child quality.

However, there is other evidence in the economics literature that indicates that some types of additional expenditure can increase child quality. For example, Conley (2001) shows that financial support for a child’s college education—a resource that is diluted by additional children—increases the likelihood of college graduation. This suggests that parents can increase child quality through certain expenditures, but at the margin allocate most expenditure on children to consumption goods.33

31The Becker-Lewis (1973) model of child quantity and quality is more general in that it allows family resources to change with family size. The resource dilution model could be considered a special case of the Becker-Lewis (1973) model in which labor supply is fixed.
32While the assumed link between diluted resources and decreased intellectual performance is not well established, there appears to be a causal link between parental financial assistance provided for college and college graduation (Conley 2001).
33Parents can have a very strong negative effect on their children by not providing the basic necessities for development (neglect) or by abuse. Duncan et al. (1998) report that children raised in very low-income families are disadvantaged in terms of health, cognitive development, school achievement, and emotional wellbeing.
If we think of child quality as measured along a single dimension (some composite of various measures
of child quality), the evidence discussed above suggests that parents do have some ability to produce
child quality, primarily through financing college education. We extend the basic model of Section 2 to
include a fourth good, average child quality. The utility function is given by

\[ U(C, L, N, Q) \]  

where \( Q \) represents the average quality of children. Child expenditure is thus divided into three parts,
necessary child expenditure, \( N \), expenditure that is categorized as investment because it increases child
quality, \( Q \), and remaining child expenditure that is considered consumption, included in \( C \). It is often
argued that the value that society places on child quality is larger than the private valuation of parents.
We will examine the quality of children from the parental point of view only, and deal with externalities
associated with children in Section 5.3. As in the basic model of Section 2, social welfare is given by the
representative agent’s utility function.

The model presented here treats child quality in a similar manner to the models of Cigno (1986),
(estate tax, lump-sum subsidy, income tax, and child subsidy) and models parents as receiving utility
from both current expenditure on children and from future bequests to their children. It also allows child
quality to depend both on the purchase of goods and on time spent with children. However, Cigno (1986)
focuses only on the trade-off between consumption and children and ignores the labor supply decision
which limits its analysis of the income tax and complicates a direct comparison with the results presented
here.

While Becker (1991) and Becker and Tomes (1976) do not address taxation, we will follow their
treatment of quality as a function of those types of child expenditure that can be characterized as
investment. Additional children increase the price of quality, \( P_Q \), because total quality expenditure
would need to increase in order to maintain a constant level of quality. Therefore, \( P_Q'(N) \) is greater
than zero. However, because there are likely to be economies of scale in some expenditure (like housing),
\( P_Q''(N) \) is assumed to be less than zero.

This treatment of child quality allows the budget constraint to be expressed as follows:

\[ H w(1 - \tau) = C + (1 + \theta)P_N N + P_Q(N) Q. \]  

37
After normalizing prices, the excess burden of the tax policy is given by the following expression:

\[
EB = -\frac{1}{2} \left[ \tau^2 \left( S_{LL} + 2S_{LQ}S_{LN}P_Q'(N) + (S_{LN})^2S_{QQ}P_Q'(N)^2 \right) \\
- 2\tau\theta \left( S_{LN} + (S_{NQ}S_{LN} + S_{LQ}S_{NN})P_Q'(N) + S_{LN}S_{QQ}S_{NN}P_Q'(N)^2 \right) \\
+ \theta^2 \left( S_{NN} + 2S_{NQ}S_{NN}P_Q'(N) + (S_{NN})^2S_{QQ}P_Q'(N)^2 \right) \right].
\]

(32)

The derivation is contained in the appendix.

This excess burden approximation contains three terms that require discussion: \(S_{QQ}\), \(S_{NQ}\), and \(S_{LQ}\).

Own price substitution effects are always negative, so \(S_{QQ} < 0\). Becker (1991) argues that children and quality are likely substitutes because of the direct effect of children on the price of quality. If the cost of raising children increases, the demand for children falls and this directly decreases the cost of quality. I am not aware of any empirical evidence that addresses the substitutability of non-market time and expenditure on child quality. It seems reasonable to assume that there is some trade off between spending money on children and spending time with children, suggesting that \(S_{LQ} > 0\).

The complete solution to the optimal tax problem of minimizing equation (32) subject to the government budget constraint, \(H\tau + N\theta = R\), is found in the appendix. Here, we focus on determining if the addition of quality to the model changes the qualitative results from Section 2. Assuming that \(S_{LN} = 0\), we can show that the optimal tax policy involves a child tax if \(S_{NQ} > 0\) and \(S_{LQ} > 0\) . The ratio of the optimal \(\theta\) and \(\tau\) assuming \(S_{LN} = 0\) is given by the following:

\[
\frac{\theta}{\tau} = \frac{S_{LQ}P_Q'(N)S_{NN}H + NS_{LL}}{S_{NN}P_Q'(N)\left(2S_{NQ}H + S_{NN}S_{QQ}P_Q'(N)H + S_{LQ}N\right) + HS_{NN}}.
\]

(33)

From equation (33) it is clear that \(\theta\) is positive if \(S_{LQ} > 0\) and \(S_{NQ} > 0\). In fact, this is true for any \(S_{LN} \leq 0\). This leads to a result similar to Result 2.1.

**Result 5.3.** With child quality, if \(S_{LN} \leq 0\), \(S_{NQ} > 0\), and \(S_{QL} > 0\) then it is not optimal to subsidize children.

The violation of any of the conditions in Result 5.3 does not mean that it is optimal to subsidize children. For example, the precise value of \(S_{LQ}\) at which the optimal value of \(\theta\) is zero for \(S_{LN} = 0\) is significantly less than zero. Assuming \(S_{LN} = 0\), it is optimal to subsidize children if \(S_{LQ} > -\frac{NS_{LL}}{P_Q'(N)S_{NN}H}\).

---

34Becker (1991) argues that while children and quality must be substitutes they cannot be close substitutes.
However, that the addition of child quality to the model does not significantly alter the optimal tax result is not the most interesting application of the model. We can use this model to ask how a tax policy would affect the average quality of children. This can be derived by totally differentiating the first order conditions of the optimal tax problem. Given below is the change in the compensated demand for child quality due to changes in tax policy:

\[
dQ^c = (S_{NQ} + S_{QQ}P'_Q(N)S_{NN}) d\theta - (S_{LQ} + S_{QQ}P'_Q(N)S_{LN}) d\tau.
\] (34)

Under the assumption that \( S_{LQ} > 0 \) and \( S_{LN} < 0 \), an increase in the income tax, \( \tau \), leads to a decrease in the compensated demand for child quality. Under the assumption that \( S_{NQ} > 0 \), an increase in child tax benefits leads to a decrease in the compensated demand for child quality. The intuition is that an increase in the income tax decreases the price of leisure so that if leisure and child quality are substitutes, the compensated demand for child quality decreases. The same is true for child tax benefits; increasing child tax benefits decreases the cost of raising children so that if children and child quality are substitutes, the compensated demand for child quality decreases. However, there is an additional effect from the relationship between the number of children and the price of child quality. As the number of children increases, the price of average child quality increases. Thus, lowering the price of leisure increases the demand for children if \( S_{LN} < 0 \) and this increases the price of quality. Similarly, decreasing the cost of children increase the demand for children and this increases the price of quality. The combination of these two effects is stated as Result 5.4

**Result 5.4.** Child quality is decreasing in \( \theta \) if \( S_{NQ} < -S_{QQ}S_{NN}P'_Q(N) \) and \( S_{LQ} < -S_{QQ}S_{LN}P'_Q(N) \).

Result 5.4 states that child quality would decrease as a result of a reduction in child tax benefits only if children and child quality are strong complements and leisure and child quality are strong complements. As argued above, it seems reasonable to assume that in both cases they are substitutes, not complements. This suggests that a policy of reducing child tax subsidies and income tax rates together (leaving government tax revenue unchanged) would increase the average level of investment expenditure on children.

This result is similar to the result from a model of intergovernmental grants with a matching grant. Child tax benefits increase the cost of child quality relative to other goods and thus lower the average level of child investment expenditure. However, it has been shown that the design of the child subsidy program can affect this. Lundberg, Pollak, and Wales (1997) show that when the child tax allowance in
the UK was changed so that the child benefit was payable directly to the mother, instead of lumping it together in a tax refund, child expenditures increased. This is similar to the “flypaper effect” often found in empirical studies of public expenditure. A child subsidy program could perhaps be designed so that the subsidy would increase compensated investment expenditure on children. However, the current U.S. system of child tax benefits is unlikely to make child subsidies “stick” to child expenditures in this way.

5.3 Externalities Associated with Children

The third extension of the basic optimal tax model is to incorporate externalities associated with children and show how the optimal tax results are affected. The claim of positive externalities associated with children is the primary justification for child subsidy programs in many countries. By deriving the optimal child tax treatment implied for any level of externalities associated with children, we will be able to analyze the level of positive externalities required to justify child subsidies. Whether externalities associated with children are positive, negative, or even whether any important externalities exist at all has been a long-running debate in the economics literature. This section does not contribute any new evidence to the externality debate. Rather, it briefly presents some of the arguments concerning externalities and then derives the optimal tax treatment of children in the representative agent model showing what tax treatment is implied for any level of externalities.

The debate began with Thomas Malthus’s *An Essay on the Principle of Population* (1798) that claimed a negative externality associated with population growth. He argued that population growth causes living standards to decline because the additional people would consume more resources over their life than they would produce. His model of a geometric population growth rate and an arithmetic food production growth rate predicted famines and other “checks” on the population level. Without “moral restraint”, the growing population of workers would drive down wages below the subsistence level (a pecuniary externality). Malthus was strongly opposed to the poor laws in England and the new system of child subsidies introduced at the time. He argued that the poor laws in England only made the poor more numerous, not better off.

Many environmentalists claim negative externalities associated with population growth because the consumption of environmental goods have a marginal social cost that exceeds the marginal private cost. Because additional people are able to consume these goods without having to produce something of equal social value in exchange, population growth makes society worse off. The consumption of polluting
goods and non-renewable resources for which the social cost exceeds the private cost are indications of market failure for which the direct solution is a corrective tax or a permit scheme. However, belief that government involvement is and will be incomplete in this area indicates a negative externality associated with population growth. Other claims of a social cost from additional children cite crowding or the burden on society of additional poor living on public assistance or incarcerated as the mechanism.

Those who believe that the externalities associated with children are positive claim that the marginal individual creates more resources than she consumes. For example, we generally assume that workers are only paid their marginal product, not their total product; this leaves a large surplus. The argument follows that while additional workers would decrease the marginal product of labor for a fixed level of capital, the increased return on capital would drive up investment and thus increase the marginal product of labor. If labor is a scarce resource, then its value is certainly reflected in its price. It is then telling that the data indicate wages have increased at a higher rate than the prices of nearly all other resources.

As discussed in the introduction, many developed countries have introduced large child subsidy programs in an effort to increase the worker to retired person ratio. Many governments have argued that pro-natalist policies are essential to the long-run sustainability of their social programs. For example, the internal rate of return for a social security system is increasing in the rate of population growth. Folbre (1994) argues that all citizens enjoy significant claims on the future earnings of children, even individuals who devote relatively little time or energy to child-rearing. Thus, because individuals can free-ride on the child-rearing efforts of others she claims that fertility is likely much lower than optimal.

There is also the argument that innovation incentives increase with population size because while there is a fixed development cost, the returns are proportional to the size of the market. There is a literature that argues that an increase in population density causes increases in social capital, social organization, and networks. The argument follows that each of these has a high social value and leads to faster technological innovation.35

Today, most economists would probably agree that the issue of whether the externality associated with children is positive or negative can be described as “controversial” rather than “settled” on one side or the other. There are clearly arguments with merit on each side, however it seems that the importance of the various arguments would depend on the particular economic conditions, immigration, and current fertility rate for a country. The question of whether the marginal individual contributes more to society

35The idea that faster population growth leads to an increased rate of technological innovation simply because more people produce more good ideas is credited to Julian Simon who has a very good review of the history of economic thought on the topic of the consequences of population growth (Simon, 1993).
that she consumes is a difficult one with a long history of attempts to provide an answer.

Consider the standard representation of externalities associated with children, either positive or negative (see Figure 13). We can make a strong argument that at the lower extreme \((N = 0)\), the externality associated with a child would be positive. To claim otherwise is to argue that social welfare would be higher if there were no people. At the other extreme, it is not difficult to imagine a densely-populated country with such a high population growth rate that additional children would clearly be associated with large external costs.

**Figure 13: Standard Graphical Representation of an Externality**

To model this, consider a marginal social benefit curve that starts above the marginal private benefit curve and then declines at a greater rate than the marginal private benefit. The function \(V(N)\) represents the total external cost or benefit associated with children. As depicted in Figure 14 there is a single maximum that occurs at \(N^*\). To be clear, \(N^*\) is not the socially optimal number of children, rather it is the point at which there is neither a positive or a negative externality. As discussed above, the function \(V(N)\), and thus the value of \(N^*\), likely depends on the characteristics of the country, its size, current level of population, ease of immigrant integration, level of technology, etc.

As in the model in Section 2, the representative agent seeks to maximize

\[ U(C, L, N) \]  

(35)
Figure 14: Externalities Associated with Children
while social welfare is given by
\[ U(C, L, N) + V(N). \] (36)

The excess burden of a tax policy in this setting is similar to equation (5) of Section 2. The only addition is an extra term, \( V(N^*) - V(N + \Delta N) \), to account for the difference between the total externality at the actual level of \( N \) and \( N^* \):
\[ EB = -\frac{1}{2} \left( \tau^2 S_{LL} - 2\tau \theta S_{LN} + \theta^2 S_{NN} \right) + \left( V(N^*) - V(N + \theta S_{NN} - \tau S_{LN}) \right) \] (37)

Note that for \( N \neq N^* \) equation (37) would not be minimized by a lump-sum tax policy with \( \tau = \theta = 0 \). A revenue-neutral tax policy change that increases the distortion of the representative agent’s behavior may actually improve social welfare if it moves \( N \) closer to \( N^* \).

The optimal tax problem is:
\[
\min_{\tau, \theta} \left\{ -\frac{1}{2} \left[ \tau^2 S_{LL} + \theta^2 S_{NN} - 2\tau \theta S_{LN} \right] + V(N^*) - V(N + \theta S_{NN} - \tau S_{LN}) - \lambda \left[ \tau H + \theta N - R \right] \right\}. \] (38)

If \( N < N^* \) then \( V'(N) > 0 \) and if \( N > N^* \) then \( V'(N) < 0 \). For convenience, we will assume that \( V'(N) \) is constant over the relevant range. In practice, we will need to check that the optimal tax policy does not reverse the sign of \( V'(N) \), because given the linear assumption, nothing in the first order conditions would prevent utility gains from overshooting \( N^* \). The first order conditions for the optimal tax problem are given by:
\[
\begin{align*}
\frac{\partial L}{\partial \tau} &= -\tau S_{LL} + \theta S_{LN} + V'(N)S_{LN} - \lambda H = 0 \quad (39) \\
\frac{\partial L}{\partial \theta} &= \tau S_{LN} - \theta S_{LN} - V'(N)S_{NN} - \lambda N = 0 \quad (40) \\
\frac{\partial L}{\partial \lambda} &= -\tau H - \theta N + R = 0 \quad (41)
\end{align*}
\]

which can then be solved for the optimal child tax policy:
\[ \theta = \frac{-\lambda \left( S_{LL}N + S_{LN}H \right)}{S_{LL}S_{NN} - S_{LN}^2} - V'(N). \] (42)

This expression is similar to equation (9) with the \( V'(N) \) simply shifting the curve up or down. For \( V'(N) < 0 \), the optimal child tax would be higher than before. For \( V'(N) > 0 \), the optimal child tax
would be lower, possibly even negative if the positive externalities are strong enough.\textsuperscript{36} The case of a positive externality is represented graphically by Figure 15. If leisure and children are complements, the positive externality represented in the figure is not large enough to justify child subsidies although it does lower the optimal child tax level.

In the case of negative externalities, we can show that it is not optimal to subsidize children if leisure and children are complements. This is given as Result 5.5.

Result 5.5. If $N \geq N^*$ and if leisure and children are complements ($S_{LN} < 0$) then it is not optimal to subsidize children.

This result is quite obvious. With no externalities associated with children, Result 2.1 stated that

\begin{equation}
\theta = \frac{(V'(N) + R)S_{LN}H - H^2V'(N)S_{NN} + RN_{SLL}}{S_{LL}N^2 + 2HS_{LN}N + H^2S_{NN}}
\end{equation}
it is not optimal to subsidize children if leisure and children are complements. Now adding negative externalities associated with children can only increase the optimal child tax. The more interesting case is if we assume that there are positive externalities associated with children. From Figure 15 it is clear that the presence of positive externalities associated with children does not alone justify child subsidies. How large these positive externalities must be to justify child subsidies is given as Result 5.6.

Result 5.6. If \( V'(N) > \frac{R(S_{LL}N+S_{LN}H)}{H(S_{LN}N+S_{NN}H)} \) then it is optimal to subsidize children.

The expression \( R(S_{LL}N+S_{LN}H)/H(S_{LN}N+S_{NN}H) \) is clearly greater than zero for \( S_{LN} \leq 0 \). The condition indicates that for higher values of \( R \), the level of \( V'(N) \) necessary to justify a child subsidy is larger. Doing calculations similar to those in Section 4 reveals that the value of \( V'(N) \) necessary to justify a child subsidy is highly dependent on \( P_N \), the assumed cost of children. For \( P_N = $3,700 \), the annual externality from an additional child would only need to be over $1,000 to justify a child subsidy. For \( P_N = $10,000 \), the annual externality from an addition child would need to be over $7,900. Some may argue that the external benefit from the marginal individual is much larger than these annual values. This is essentially the argument being made by Western European governments. This analysis shows that if the claim of large positive externalities is true, then it is a strong justification for child subsidies.

We should consider one final point in this section. One of the most important characteristics of children is that most of us care about the current and future wellbeing of all children, even those to which we are in no way related. As long as no one else is affected, if I decide to damage or wreck my car, the rest of society probably would not care. But the same is not true for children. Laws that attempt to prevent parents from “wrecking” their own children or punishing those who do, are clear evidence of this. This type of externality is somewhat different than those considered in this section because the externality has to do with the wellbeing of the child instead of simply the number of children. A representative agent model is not well-equipped to handle this issue because all children are treated identically by the representative agent.

5.4 Other Child Characteristics

The three characteristics considered above—time costs of raising children, child quality, and externalities associated with children—are only a small subset of the differences between children and the types of goods normally considered in economic models. These particular characteristics were chosen for analysis because of their importance, but also because they could be analyzed using a static representative agent.
model. While not given formal treatment, three other categories of child characteristics that have received significant attention in the economics literature are briefly discussed.

**Children as an Investment**

Children can contribute to family income by performing household chores, working on the family farm, or by supplying child labor. That children have earning potential significantly lowers their net cost to parents. This is probably a very important characteristic of children in many developing economies, but is arguably much less important in the U.S. and other developed countries today. Becker (1991) points out that the contribution of children to family income has dramatically declined as agriculture has become more mechanized. This combined with child labor laws and mandated schooling has greatly reduced the earning potential of children.

Even though children bring in little to no family income while young, they are future workers. Even in developed countries where the payoff is decades away, parents may see children as a type of investment. A risk-adverse agent that faces uncertain longevity and has no access to annuity insurance would be willing to give a large fraction of wealth to have access to a fair annuity market (Kotlikoff and Spivak, 1981). Children can act as a type of annuity insurance through an agreement with their parents. In exchange for providing financial support in old age, parents leave their remaining wealth when they die to their children. The deal can be crafted so that the expected utility of both the parents and the children is improved, even when both are risk-adverse.

Kotlikoff, Shoven, and Spivak (1987) compare economies with perfect annuity insurance to economies with no insurance. In economies with no insurance, parents are able to pool longevity risk with their children to their mutual advantage. In an economy with annuity insurance (or a Social Security system) parents do not need children to provide them with annuity insurance and thus don’t have an explicit need for bequests as payment for this insurance. While the welfare gains to the current generation from moving to a system with annuity insurance can be very large, the welfare of future generations can be much lower because of the decrease in savings due to the lack of bequests. In this model, the introduction of a social security system or the creation of life annuities removes the investment characteristic from children.
Biological Constraints

Some car collectors have hundreds of vehicles. A standard economic model of the demand for automobiles would require the satisfaction of a budget constraint, but would place no other limit on the number of cars that an individual can purchase. A model of children should explicitly recognize that there is a biological constraint on the number of children a woman may have in her lifetime. The risk of infertility is another form of biological constraint that could affect the analysis of the tax treatment of children.

It is important to point out that what is commonly called the demand for children usually means the number of children desired subject to budget and time constraints only, and not subject to biological constraints. While a maximum biological limit to the total number of children that the average woman could carry certainly exists, it is almost never binding and thus is usually ignored in economic models. However, the evidence on infertility is mixed.

Heckman and Walker (1990) use longitudinal data from Sweden in an analysis of fertility and finds that fecundity has little influence on fertility. They assert that “behavior swamps biology” in developed economies. Becker (1960) claimed that “sterility” (infertility) played a major role in explaining the declining birth rates in most developed countries. However, Becker (1991) reverses this position and allows infertility only a minor role in explaining national birth rates.

Studies on the incidence of infertility claim that a large fraction of couples have reduced ability to have children. The most recent data from the U.S. National Center for Health Statistics estimates that 2.1 million couples in the U.S. are infertile (Abma et al. 1997). If infertility is exogenous, then child tax policy would have no efficiency implications with respect to infertility. If however, infertility risk increases with age or with certain behavior, tax policies may have efficiency implications. This type of analysis would require a dynamic model that allows individuals to age and plan the timing of their children. As the age at first birth continues to increase, we may find that biological constraints are becoming more, not less, important.

Timing of Births

In the static framework, the agent’s only choice with respect to children is how many to have and how much to spend on them. In a dynamic setting, the timing of births becomes a choice variable as well. Many of the characteristics of children that we would like to capture in a model have to do with the timing of births. For example, the cost of raising children may change both with child age and parent
Younger children are generally thought to require more time, while older children generally require greater expenditure. Miller (2005) shows that delaying the birth of a child for one year is associated with a 10% increase of career earnings for the mother, implying that the timing of births can have a large impact on finances for a family.

All durable goods age, but unlike other goods, children are irreversible—they can not be resold or returned. Because children are non-transferable, they cannot be redistributed by the government nor can they be used as collateral to secure a loan. The second implication means that it is generally not possible to borrow to finance having a child. This is irrelevant in a static model because the agent faces a single life-time budget constraint. But in a dynamic setting, agents may be credit-constrained when young which could affect the timing of births.

A change in the timing of births can have a significant effect on population growth. To demonstrate this, consider a simple overlapping generations model with a stable population. Each agent has exactly one child at age 25 and dies at age 80. We begin at a steady state with an equal number of agents of each age. Consider the effect of a small change in the age at which the agents have children, say from age 25 to age 24. To make the change smooth, we assume that the birth timing change takes place gradually; each year an additional $\frac{1}{50}$th of the agents at age 24 decide to have their only child at age 24 instead of age 25. After 50 years, the gradual transition from age 25 to age 24 is complete.

Even though no agent in the model has more that one child, the transition to earlier births increases the population. The solid line in Figure 16 shows the population over time due to this birth timing change. The population increases gradually and stabilizes at a level slightly more than 4 percent above the baseline population. The population growth from the birth timing change is, in the short-term, comparable to the population growth that would result if 2 percent of the agents had two children instead of one. This
is shown as the dashed line in Figure 16. An increase from one to two children for 2 percent of the population corresponds to an increase in the total fertility rate from 2.00 (the replacement rate in this model) to 2.04.

While the timing of births has an effect on population growth, it is not nearly as important as changes to average number of children per woman. This is particularly true in the long-run. Tax policy may have a much larger influence on the timing of births than it does on the average number of births per woman, but this difference would need to be quite large for the first effect to be more important than the second. This, coupled with the additional difficulties associated with a dynamic model, require that the optimal tax treatment of children in a dynamic model be left for future work.

6 Some Distributional Considerations

With the exception of the discussion in Section 4, this paper has focused on economic efficiency. However, most people are probably more concerned with issues of fairness in the tax treatment of families with children. The efficiency results are important in that they show that the benefits from a “fairer” treatment of families with children must be balanced with the distortionary costs. One of the key points of Section 4 is that for negative values of $R$, the efficiency argument strongly supports child subsidies.

One of the dominant principles of tax equity in modern tax policy analysis is the ability-to-pay principle. A common argument for child subsidies is that they are a useful method for redistributing to those with less ability to pay. The claim is that when controlling for income, families with more children have less “effective income” than families with fewer children. It is argued that necessary expenditure on children provides no direct utility and should be deducted from the income measure in the form of an exemption. This makes the tax liability proportional to the family’s true “effective income”, a measure of their ability to pay.

Closely related is the argument that ability-to-pay begins only after an allowance has been made for personal expenses deemed sufficient to maintain the family according to some minimum standard. Because the level of expenditure needed to maintain the minimum standard rises with the number of children, relative ability-to-pay should depend on the number of dependents. Citing this reasoning, Pigou (1928) argues that personal exemption allowances are pertinent at all levels of income and should moreover increase with income (p. 101-103). But, while it is true that children are costly, should the personal decision to have a child allow a family to pay less taxes? At low income levels, part of the answer
is that without a tax break, the after-tax income may not be sufficient to provide for the minimum needs of the family. However, as income rises, being able to meet this minimum standard is no longer a concern.

Clearly, raising a child is costly. However, whether or not the cost of raising a child should be deducted from taxable income is not immediately obvious. To illustrate this, consider the expenses associated with the use and maintenance of a car. Should these expenses be deducted from taxable income? One can make the same argument that there is no direct utility gain from the expenditure associated with car maintenance, and so some deduction should be given in the income tax. Of course, the income tax allows for no such deductions.

The tax code is quite explicit in describing what deductions are allowed. The default is that all income “from whatever source derived” is taxable unless there is a specific statute that exempts it from taxation. Before the adoption of the provision allowing a credit for certain child care expenses of a working parent, the U.S. Tax Court ruled that child expenses were not deductible as business expenses (Smith v. Commissioner 113 F.2d 114). The petitioner was a married couple; both husband and wife worked out of the home and they had one child. They argued that the court should apply the “but for” test and that the proximate cause of the child care expenses was that both parents worked. The petitioner’s argument was that but for the child care expenses, at least one parent would need to stay at home to care for the child and would earn no income. The court argued that but for the child, the couple would have no need of child care and ruled that child care expenses were personal and not deductible. The petitioner wanted to be compared to another couple with a child but with one stay-at-home parent. The court compared the petitioner to another couple with both spouses employed but with no children. The court’s position was that having children is a choice, not an exogenously imposed condition, and as such should not be given any special tax treatment.

From a utilitarian perspective, a justification for child subsidies is that they may improve the efficiency of redistributive transfers. Consider a model with two types of agents, a low ability type and a high ability type. These agents have the same utility function and $w_h > w_l$. We first restrict the government so that it only has the ability to impose a linear income tax and either a child subsidy or tax. This set-up is equivalent to a flat consumption tax with special child tax treatment or a linear income tax with a dependent exemption (or negative exemption). These restrictions on the government’s ability to have anything other than a flat tax are more applicable in a developing country that does not have the structure in place to impose a progressive income tax.
Suppose the agents’ utility is given by:

\[ U(C, L, N) \]  

(43)

and the budget constraint be given by:

\[ T E w_i (1 - \tau) = L_i w_i (1 - \tau) + (1 + \theta) N_i + C_i. \]  

(44)

Social welfare is given by the weighted sum of the agents’ utilities:

\[ W = \gamma_l U_l + \gamma_h U_h \]  

(45)

and the government budget constraint is:

\[ R = \tau \sum w_i H_i + \theta \sum N_i. \]  

(46)

Using this quite limited set of tax instruments, the case for providing a child subsidy is quite strong. This is true even though the high ability agent will likely have more children than the low ability agent. The intuition is that while the high income family may have more children than the low income family, the high income family does not have proportionally more children. This means that increasing the flat tax rate and increasing child subsidies redistributes from the high type to the low type.

To show this, consider a tax policy where \( \theta = 0 \) and \( \tau = R / (w_l H_l + w_h H_h) \). By assumption, the marginal utility for the low type is larger than the marginal utility for the high type: \( U_l (C_l, L_l, N_l) > U_h (C_h, L_h, N_h) \). Thus if

\[ \gamma_l \left( \frac{\partial U_l}{\partial C_l} \frac{\partial C_l}{\partial \theta} + \frac{\partial U_l}{\partial L_l} \frac{\partial L_l}{\partial \theta} + \frac{\partial U_l}{\partial N_l} \frac{\partial N_l}{\partial \theta} \right) < \gamma_h \left( \frac{\partial U_h}{\partial C_h} \frac{\partial C_h}{\partial \theta} + \frac{\partial U_h}{\partial L_h} \frac{\partial L_h}{\partial \theta} + \frac{\partial U_h}{\partial N_h} \frac{\partial N_h}{\partial \theta} \right) \]  

(47)

then some level of redistribution from the high type to the low type is desirable. Because the government is limited in its choice of tax instruments, the only method for redistribution is to increase the flat tax rate and increase the provision of child subsidies. This result is reversed only if the low ability type has even fewer than \( N_h (w_l H_l) / (w_h H_h) \) children.

In developed countries, the government has the ability to allow the tax treatment of children to depend
on income. Here we will model this by allowing the government to set a different child tax treatment for the two agents. In the case where the government is still restricted to a flat tax, it will choose to set \( \theta_l < \theta_h \) so that the low type receives a child subsidy, or at least pays a smaller child tax per child.

If the government has the ability to set individual specific income and child tax rates, then we can reformulate the problem as one of simply selecting a value \( R_i \) for each of the two types to maximize social welfare where \( R_l + R_h = R \). From Section 4 it is clear that the optimal tax treatment will include a child subsidy only if \( R_i < 0 \). However, a model where the government can specify income and child tax treatment by type is not reasonable because the government cannot observe ability, only income and the number of children.

Instead, we will reformulate the problem as one of selecting the optimal two-bracket income tax schedule:

\[
\max_{\tau_l, \tau_h, \theta_l, \theta_h} \left\{ \gamma_l U_l + \gamma_h U_h \right\} \quad \text{s.t.} \quad R = \tau_l w_l H_l + \tau_l B + \tau_h (w_h H_h - B) + \theta_l N_l + \theta_h N_h.
\]

We do not allow the government to choose the income level at which the brackets meet given by \( B \).

The first order condition with respect to \( \theta_l \) is given by:

\[
\gamma_l \left( \frac{\partial U_l}{\partial C_l} \frac{\partial C_l}{\partial \theta_l} + \frac{\partial U_l}{\partial L_l} \frac{\partial L_l}{\partial \theta_l} + \frac{\partial U_l}{\partial N_l} \frac{\partial N_l}{\partial \theta_l} \right) = \mu \left( \tau_l w_l \frac{\partial H_l}{\partial \theta_l} + N_l + \theta_l \frac{\partial N_l}{\partial \theta_l} \right).
\]

This first order condition does not directly depend on any of the decisions or tax parameters for the high ability type. However, the optimal choice of \( \tau_l \) does have an important role in that the high ability type has a total income tax liability of \( \tau_l B + \tau_h (w_h H_h - B) + \theta_h N_h \). The optimal \( \tau_l \) is higher than it would be if the government were able to directly observe the ability of the agents. However, if the optimal tax policy is such that \( \tau_l < 0 \) then the optimal tax policy includes \( \theta_l < 0 \) as well.

The first order condition with respect to \( \theta_h \) is given by:

\[
\gamma_h \left( \frac{\partial U_h}{\partial C_h} \frac{\partial C_h}{\partial \theta_h} + \frac{\partial U_h}{\partial L_h} \frac{\partial L_h}{\partial \theta_h} + \frac{\partial U_h}{\partial N_h} \frac{\partial N_h}{\partial \theta_h} \right) = \mu \left( \tau_h w_h \frac{\partial H_h}{\partial \theta_h} + N_h + \theta_h \frac{\partial N_h}{\partial \theta_h} \right).
\]

Similar reasoning applies for the optimal child tax treatment of the high type. If \( \tau_l > 0 \) and \( \tau_h > 0 \) then the optimal child tax treatment for the high type is a child tax. This does not rule out the possibility of optimal child tax subsidies for the low type, but the conditions under which \( \theta_l < 0 \) when \( \tau_l > 0 \) are not immediately obvious.
7 Conclusion

This paper developed a representative agent model that was used to show that if children and leisure (non-market) time are complements that child subsidies are not part of an optimal child tax policy. Estimation of a child demand function using NLSY data indicated that children and leisure are complements. A back of the envelope calculation of the optimal child tax implied by the estimates was in the $600 to $1,200 per child range for the representative agent.

When considering an expanded model that allows for time costs of raising children, the argument for child taxation is somewhat weakened, although not reversed. When considering child quality, the argument for child taxation is unchanged, but the results indicate that the provision of child subsidies likely decrease child expenditure categorized as quality-producing investment. The effect on the optimal tax results from the presence of externalities associated with children is also considered. As expected, positive externalities associated with children decrease the optimal child tax, while negative externalities associated with children increase the optimal child tax. If positive externalities associated with children are large enough (over $8,000 per child per year), a strong case can be made for subsidizing children.

This paper shows that the efficiency cost of child tax benefits is large, and suggests that the nearly universal provision of child benefits in the current federal income tax code may be suboptimal. However, negative average tax rates for a large fraction of taxpayers is evidence that child tax benefits should be present at the lower range of the income distribution. Analysis of a two-agent model suggest that child subsidies should be provided to low income families, but not high income families.

There is a strong need for additional research on the optimal design of child subsidies. The child tax benefits in the United States are large and expensive and have the potential to grow even larger over the next few years. This is a research topic with too little formal analysis on which to base policy recommendations. Careful thinking about the efficiency costs associated with child subsidies is only the first step. Further analysis of the consequences of child subsidies, especially for different ranges in the income distribution, is needed.

References


BECKER, G. S. (1960): “An Economic Analysis of Fertility,” in *Demographic and Economic Change in Developed Countries*. NBER.


Appendix

Derivation for Section 5.1: Time Costs of Raising Children

After scaling \(N\) and \(L\), the agent's Lagrangean is:

\[
\mathcal{L} = U(C, L, N) + \lambda \left[ TE (1 - \tau) + M - L (1 - \tau) - g(N)(1 - \tau) - C - N (1 + \theta) \right]
\]  

(A-1)
The first order conditions are:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial \mathcal{C}} &= U_{\mathcal{C}} - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial \mathcal{L}} &= U_{\mathcal{L}} - \lambda (1 - \tau) = 0 \\
\frac{\partial \mathcal{L}}{\partial \mathcal{N}} &= U_{\mathcal{N}} - \lambda (1 + g'(N)(1 - \tau) + \theta) = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= TE (1 - \tau) + M - L (1 - \tau) - g(N)(1 - \tau) - C - N (1 + \theta) = 0
\end{align*}
\]  

(A-2)  

(A-3)  

(A-4)  

(A-5)

Totally differentiating (A-2) - (A-5) and placing in matrix form, the system of differential equations is:

\[
\begin{bmatrix}
U_{\mathcal{C}\mathcal{C}} & U_{\mathcal{C}\mathcal{L}} & U_{\mathcal{C}\mathcal{N}} & -1 & \vdots \\
U_{\mathcal{L}\mathcal{C}} & U_{\mathcal{L}\mathcal{L}} & U_{\mathcal{L}\mathcal{N}} & (\tau - 1) & \vdots \\
U_{\mathcal{N}\mathcal{C}} & U_{\mathcal{N}\mathcal{L}} & U_{\mathcal{N}\mathcal{N}} & (\tau - 1) & \vdots \\
-1 & (\tau - 1) & g'(N)(\tau - 1) - (1 + \theta) & 0 & \vdots \\
\end{bmatrix}
\begin{bmatrix}
d\mathcal{C} \\
d\mathcal{L} \\
d\mathcal{N} \\
d\lambda \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
-\lambda d\tau \\
-\lambda g'(N)d\tau + \lambda d\theta \\
(TE - L - g(N))d\tau + Nd\theta + (1 - \tau)dTE + dM
\end{bmatrix}
\]  

(A-6)

The matrix above is called the bordered Hessian and is symmetric by Young’s Theorem. The solution to the system is found by taking the inverse of the bordered Hessian matrix

\[
\left(\text{Bordered Hessian}\right)^{-1} = \frac{1}{D} \begin{bmatrix}
D_{11} & D_{12} & D_{13} & D_{14} \\
D_{12} & D_{22} & D_{23} & D_{24} \\
D_{13} & D_{23} & D_{33} & D_{34} \\
D_{14} & D_{24} & D_{34} & D_{44}
\end{bmatrix}
\]  

(A-7)

where \(D\) is the determinant of the bordered Hessian and \(D_{ij}\) is the cofactor found by taking the determinant of the bordered Hessian after deleting row \(i\) and column \(j\). Note that the inverse of the bordered Hessian is also symmetric. The solution to the system of equations is given by the following expressions:

\[
\begin{align*}
d\mathcal{C} &= -\left[\lambda \frac{\partial D_{ij}}{\partial \mathcal{C}} + \lambda g'(N) \frac{\partial D_{ij}}{\partial \mathcal{N}} - H \frac{\partial D_{ij}}{\partial \lambda} \right] d\tau + \left[\lambda \frac{\partial D_{ij}}{\partial \mathcal{L}} + N \frac{\partial D_{ij}}{\partial \mathcal{N}} \right] d\theta + \left[(1 - \tau) \frac{\partial D_{ij}}{\partial \mathcal{L}} \right] dTE + \left[\frac{\partial D_{ij}}{\partial \mathcal{N}} \right] dM \\
d\mathcal{L} &= -\left[\lambda \frac{\partial D_{ij}}{\partial \mathcal{C}} + \lambda g'(N) \frac{\partial D_{ij}}{\partial \mathcal{N}} - H \frac{\partial D_{ij}}{\partial \lambda} \right] d\tau + \left[\lambda \frac{\partial D_{ij}}{\partial \mathcal{L}} + N \frac{\partial D_{ij}}{\partial \mathcal{N}} \right] d\theta + \left[(1 - \tau) \frac{\partial D_{ij}}{\partial \mathcal{L}} \right] dTE + \left[\frac{\partial D_{ij}}{\partial \mathcal{N}} \right] dM \\
d\mathcal{N} &= -\left[\lambda \frac{\partial D_{ij}}{\partial \mathcal{C}} + \lambda g'(N) \frac{\partial D_{ij}}{\partial \mathcal{N}} - H \frac{\partial D_{ij}}{\partial \lambda} \right] d\tau + \left[\lambda \frac{\partial D_{ij}}{\partial \mathcal{L}} + N \frac{\partial D_{ij}}{\partial \mathcal{N}} \right] d\theta + \left[(1 - \tau) \frac{\partial D_{ij}}{\partial \mathcal{L}} \right] dTE + \left[\frac{\partial D_{ij}}{\partial \mathcal{N}} \right] dM
\end{align*}
\]  

(A-8)  

(A-9)  

(A-10)
The first order conditions are:

\[ S_{LC} = \lambda \frac{\partial P_i}{\partial P_j}, \quad S_{NC} = \lambda \frac{\partial P_i}{\partial P_j}, \quad i_C = -\frac{\partial P_i}{\partial P_j}, \]
\[ S_{LL} = \lambda \frac{\partial P_i}{\partial P_j}, \quad S_{NL} = \lambda \frac{\partial P_i}{\partial P_j}, \quad i_L = -\frac{\partial P_i}{\partial P_j}, \]
\[ S_{LN} = \lambda \frac{\partial P_i}{\partial P_j}, \quad S_{NN} = \lambda \frac{\partial P_i}{\partial P_j}, \quad \text{and} \quad i_N = -\frac{\partial P_i}{\partial P_j}. \]

Note that the symmetry of the inverse of the bordered Hessian matrix means that the Slutsky matrix is also symmetric. This implies that the effect of a small increase in the price of good \( i \) on the compensated demand of good \( j \) is identical to the effect of an equivalent increase in the price of good \( j \) on the compensated demand of good \( i \) \((S_{ij} = S_{ji})\).

The excess burden approximation is given by:

\[ EB = -\frac{1}{2} \left[ \Delta P_i \left( \frac{\partial L^c}{\partial \tau} \tau + \frac{\partial L^c}{\partial \theta} \theta \right) + \Delta P_N \left( \frac{\partial N^c}{\partial \theta} \theta + \frac{\partial N^c}{\partial \tau} \tau \right) \right] \quad (A-11) \]

Substituting in the compensated demand effects from equations (23) - (26) we obtain:

\[ EB = -\frac{1}{2} \left[ -\tau \left( (-S_{LL} - g'(N)S_{LN}) \tau + S_{LN} \theta \right) + \left( \theta - \tau g'(N) \right) \left( S_{NN} \theta + (-S_{LN} - g'(N)S_{NN}) \tau \right) \right]. \quad (A-12) \]

By simply rearranging terms, we arrive at equation (27) in the text.

The Lagrangean for the optimal tax problem is:

\[ \mathcal{L} = EB - \lambda (\tau H + \theta N - R) \quad (A-13) \]

The first order conditions are:

\[
\frac{\partial \mathcal{L}}{\partial \tau} = -\lambda \left( -S_{LN} + \tau g(N) \left( -S_{LN} - g'(N)S_{NN} \right) + g'(N)S_{NN} \theta \right)
- \frac{1}{2} \left( \theta - \tau g'(N) \right) \left( -S_{LN} - g'(N)S_{NN} \right) - \tau \left( S_{LL} + g'(N)S_{LN} \right)
= -\frac{1}{2} \left( S_{NN} \theta - \tau S_{LN} + \tau \left( -S_{LN} - g'(N)S_{NN} \right) - S_{NN} \left( \theta - \tau g'(N) \right) \right) - \lambda N
\]
\[
\frac{\partial \mathcal{L}}{\partial \theta} = \tau H + \theta N - R
\]

Solving these first order conditions for \( \tau \) and \( \theta \) give the optimal tax policy expressions in the text.

\[
\tau = \frac{R \left( g'(N)S_{NN}N + S_{NN}H + S_{LN}N \right)}{N^2S_{LL} + 2N^2g'(N)S_{LN} + N^2g'(N)^2S_{NN} + 2NS_{LN}H + 2Ng'(N)S_{NN}H + S_{NN}H^2}
\quad (A-14)
\]
\[
\theta = \frac{R \left( 2g'(N)S_{LN}N + S_{LN}H + g'(N)^2S_{NN}N + g'(N)S_{NN}H + NS_{LL} \right)}{N^2S_{LL} + 2N^2g'(N)S_{LN} + N^2g'(N)^2S_{NN} + 2NS_{LN}H + 2Ng'(N)S_{NN}H + S_{NN}H^2}
\quad (A-15)
\]
Derivation for Section 5.2: Child Quality

After scaling $N$ and $L$ so that prices are unity, the agent’s Lagrangean is:

$$\mathcal{L} = U(C, L, N, Q) + \lambda \left( TE (1 - \tau) + M - L (1 - \tau) - C - N (1 + \theta) - P_Q(N) Q \right)$$  \hspace{1cm} (A-16)

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C} = U_C - \lambda = 0 \hspace{1cm} (A-17)$$
$$\frac{\partial \mathcal{L}}{\partial L} = U_L - \lambda (1 - \tau) = 0 \hspace{1cm} (A-18)$$
$$\frac{\partial \mathcal{L}}{\partial N} = U_N - \lambda (1 + \theta) - \lambda P_Q' (N) Q = 0 \hspace{1cm} (A-19)$$
$$\frac{\partial \mathcal{L}}{\partial Q} = U_Q - \lambda P_Q (N) = 0 \hspace{1cm} (A-20)$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = TE (1 - \tau) + M - L (1 - \tau) - C - N (1 + \theta) - P_Q(N) Q = 0 \hspace{1cm} (A-21)$$

Totally differentiating (A-17) - (A-21) and placing in matrix form, the system of differential equations is:

$$\begin{bmatrix}
U_{CC} & U_{CL} & U_{CN} & U_{CQ} & -1 \\
U_{LC} & U_{LL} & U_{LN} & U_{LQ} \\
U_{NC} & U_{NL} & U_{NN} - \lambda P_Q'(N) Q & U_{NQ} - \lambda P_Q'(N) Q - (1 + \theta) \\
U_{QC} & U_{QL} & U_{QN} - \lambda P_Q(N) & U_{QQ} - P_Q(N) \\
-1 & (\tau - 1) & g'(N)(\tau - 1) - (1 + \theta) & -P_Q(N) & 0
\end{bmatrix}
\begin{bmatrix}
dC \\
dL \\
dN \\
dQ \\
d\lambda
\end{bmatrix}
= \begin{bmatrix}
0 \\
-\lambda d\tau \\
\lambda d\theta \\
0 \\
(TE - L) d\tau + Nd\theta + (1 - \tau)dTE + dM
\end{bmatrix}$$  \hspace{1cm} (A-22)

The solution to the system of equations is given by the following expressions:

$$dC = [-S_{LC} - H i_C]d\tau + [S_{NC} - N i_C]d\theta + [S_{CQ} - Q i_C]dP_Q(N) + ... \hspace{1cm} (A-23)$$
$$dL = [-S_{LL} - H i_L]d\tau + [S_{NL} - N i_L]d\theta + [S_{LQ} - Q i_L]dP_Q(N) + ... \hspace{1cm} (A-24)$$
$$dN = [-S_{LN} - H i_N]d\tau + [S_{NN} - N i_N]d\theta + [S_{NQ} - Q i_N]dP_Q(N) + ... \hspace{1cm} (A-25)$$
$$dQ = [-S_{LQ} - H i_Q]d\tau + [S_{NQ} - N i_Q]d\theta + [S_{QQ} - Q i_Q]dP_Q(N) + ... \hspace{1cm} (A-26)$$
The excess burden approximation is given by:

\[
EB = -\frac{1}{2} \left( \Delta L \Delta P_L + (\Delta N \Delta P_N) + (\Delta Q \Delta P_Q) \right) \quad (A-27)
\]

Under the assumption that all prices are constant, the only prices changes are those due to the tax policy: \(\Delta P_L = -\tau\) and \(\Delta P_N = \theta\). However, the price of child quality depends explicitly on the demand for children: \(\Delta P_Q = P_Q(N_1) - P_Q(N_0) \approx P_Q'(N) \Delta N\). Therefore,

\[
EB = -\frac{1}{2} \left[ -\tau \left( \frac{\partial L^c}{\partial \tau} \tau + \frac{\partial L^c}{\partial \theta} \theta \right) + \theta \left( \frac{\partial N^c}{\partial \theta} \theta + \frac{\partial N^c}{\partial \tau} \tau \right) + P_Q'(N) \Delta N \left( \frac{\partial Q^c}{\partial \tau} \tau + \frac{\partial Q^c}{\partial \theta} \theta \right) \right] \quad (A-28)
\]

Substituting in the compensated demand effects from equations \((A-23) - (A-26)\) we obtain

\[
EB = -\frac{1}{2} \left[ \tau^2 \left( S_{LL} + 2S_{LQ}S_{LN}P_Q'(N) + (S_{LN})^2 S_{QQ}P_Q'(N)^2 \right) \right.
\]

\[
- 2 \tau \theta \left( S_{LN} + (S_{NQ}S_{LN} + S_{LN}S_{NN})P_Q'(N) + S_{LN}S_{QQ}S_{NN}P_Q'(N)^2 \right)
\]

\[
+ \theta^2 \left( S_{NN} + 2S_{NN}S_{NN}P_Q'(N) + (S_{NN})^2 S_{QQ}P_Q'(N)^2 \right) \right]
\]

The Lagrange for the optimal tax problem is:

\[
\mathcal{L} = EB - \lambda (\tau H + \theta N - R) \quad (A-30)
\]

The first order conditions are:

\[
\frac{\partial \mathcal{L}}{\partial \tau} = 0 = S_{LN}S_{QQ} (S_{NN}H + S_{LN}N) P_Q'(N)^2 + P_Q'(N) (S_{LQ}S_{NN}H + S_{LN} (2S_{LN}N + S_{NQ}H)) + S_{LN}H + S_{LL}N \]

\[
\frac{\partial \mathcal{L}}{\partial \theta} = \tau S_{NN}P_Q'(N)S_{LN} - \theta P_Q'(N)^2 S_{NN}S_{QQ} + S_{NN} \left( \tau S_{LN}P_Q'(N)^2 S_{QQ} + P_Q'(N) (\tau S_{LQ} + 2\theta S_{NQ}) - \theta \right) + \tau S_{LN} - \lambda N \]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = \tau H + \theta N - R \]

Solving for \(\tau\) and \(\theta\), the ratio is give by:

\[
\frac{\theta}{\tau} = \frac{S_{NN}S_{QQ} (S_{NN}H + S_{LN}N) P_Q'(N)^2 + P_Q'(N) (S_{LQ}S_{NN}H + S_{LN} (2S_{LN}N + S_{NQ}H)) + S_{LN}H + S_{LL}N}{S_{NN}S_{QQ} (S_{NN}H + S_{LN}N) P_Q'(N)^2 + P_Q'(N) (S_{NQ}S_{LN}S_{NN} + S_{NN} (S_{LQ}N + 2S_{NQ}H)) + S_{NN}H + S_{LN}N} \quad (A-31)
\]

The optimal tax treatment of children is given by:

\[
\theta = R \frac{P_Q'(N)^2 S_{LN}S_{QQ} (S_{NN}H + S_{LN}N) + P_Q'(N) (H (S_{NQ}S_{LN} + S_{LN}S_{NN}) + 2S_{LN}S_{LQ}N) + S_{LN}H + S_{LL}N}{S_{QQ} (S_{NN}H + S_{LN}N)^2 P_Q'(N) + 2 (S_{LQ}N + S_{NQ}H) (S_{NN}H + S_{LN}N) P_Q'(N) + N^2 S_{LL} + 2NS_{LN}H + S_{NQ}H^2} \quad (A-32)
\]