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Uncertainty and the Dynamics of R&D

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Uncertainty and the Dynamics of R&D

Nick Bloom*

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Abstract

Uncertainty varies strongly over time, rising by 50% to 100% in recessions and by up to 200% after major economic and political shocks. This paper shows that higher uncertainty reduces the responsiveness of R&D to changes in business conditions - a "caution-effect" - making it more persistent over time. Thus, uncertainty will play a critical role in shaping the dynamics of R&D through the business cycle, and its response to technology policy. I also show that if firms are increasing their level of R&D then the effect of uncertainty will be negative, while if firms are reducing R&D then the effect of uncertainty will be positive.

Keywords: R&D, uncertainty, real options

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I. Introduction

Uncertainty about future productivity and demand conditions varies strongly over time, rising by 50% to 100% during recessions, and by 100% to 200% after major political and economic shocks.¹ These fluctuations in uncertainty appear to generate fluctuations in investment, hiring and productivity as higher uncertainty generates a temporary slowdown and bounceback as firms postpone activity and wait for uncertainty to resolve (Bloom, 2006).

An omitted factor from this analysis, however, is R&D which has become a focus of recent business cycle models (Diego Comin and Mark Gertler, 2006, and Gadi Barlevy, 2007). R&D may respond differently to uncertainty because of different adjustment costs. Investment in the capital stock typically incurs stock adjustment costs from changing the capital stock, while R&D investment in the knowledge stock typically incurs flow adjustment costs from changing the flow rate of the knowledge stock (see section II). I show that these different adjustment costs lead to different predicted dynamics under uncertainty, including making R&D rates much less responsive to business-conditions and more persistent over time at higher uncertainty.

These adjustment-cost and uncertainty effects can help to explain the high persistence of R&D across time, which at the firm-level is about three times more autocorrelated than investment. They may also help to explain why across-business cycles R&D is highly persistent and responds to recessions with a lag. The higher uncertainty in downturns will reduce the responsiveness of R&D, delaying its response to worsening business conditions. Finally

¹William Schwert (1989) shows that uncertainty over future industrial production, stock and bond prices fluctuate over the business cycle, increasing by 50% to 100% in recessions. Nick Bloom (2006) shows stock-market volatility jumps 100% to 200% after economic and political shocks like the Cuban Missile crisis, the assassination of JFK and the 9/11 attack.
the results imply firms will be much less responsive to technology policies during periods of high uncertainty, for example if the policy change itself increases uncertainty.

II. Time varying uncertainty with stock and flow adjustment costs

The traditional real options models assumed time constant uncertainty in order to derive analytical solutions. They assumed some driving process, for example price ($P$), evolved as a Geometric Brownian motion with a constant drift $\mu$ and constant volatility $\sigma$.

\[
(\text{II.1}) \quad dP_t = P_t(\mu + \sigma dW_t) \quad \text{where } dW_t \sim N(0, 1)
\]

Since volatility is fixed, investigating the impact of time-varying fluctuations in uncertainty is not possible in these models.

A small literature has tried to extend these models to incorporate time varying uncertainty ($\sigma_t$). It finds temporary increases in uncertainty cause a drop and rebound in investment, employment and productivity growth due to a “delay-effect”, which can be summarized as $dI_t/d\sigma_t < 0$. At high levels of uncertainty firms postpone making decisions so aggregate investment and employment activity slows down. Productivity growth also slows down as reallocation of factors of production from low to high productivity firms slows. Higher uncertainty also induces a “caution effect” whereby firms are less responsive to any given shock because higher uncertainty increases the chances of making a costly mistake, so responsiveness is lower (Bloom et al., 2007), which can be summarized as $\partial^2 I_t/\partial \sigma_t \partial P_t < 0$.

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2 See, for example, Robert MacDonald and Daniel Siegel (1986), Giuseppe Bertola and Ricardo Caballero (1994), Avinash Dixit and Robert Pindyck (1994) or Andrew Abel and Janice Eberly (1996).

3 See Ben Bernanke (1983) and John Hassler (1996) for a single agent and factor model, and Bloom 2006 for a micro to macro multi-factor model and empirical evidence.
These extensions, however, have yet to examine the impact of time varying uncertainty on R&D and the knowledge stock. In the productivity and innovation literature the knowledge stock \((G_t)\) is usually modelled as the accumulation of R&D expenditures \((R_t)\) over time, in a similar way that capital stocks \((K_t)\) are modelled as the accumulation of investment expenditures \((I_t)\) over time:

\[
K_{t+1} = (1 - \delta_K)K_t + I_t
\]

\[
G_{t+1} = (1 - \delta_G)G_t + R_t
\]

Although uncertainty and real-options are not modelled, one could speculate that R&D will be affected in the same way by uncertainty as investment.\(^4\) But, this turns out not to be true due to the different adjustment costs for capital and knowledge stocks.

Capital stock adjustment costs are typically assumed to arise from direct changes to their stocks, for example from resale losses for capital goods. This can be written as

\[
C_K(I_t) \approx C_K(\Delta K_t)
\]

Knowledge stocks, however, are intangible and can not typically be bought or sold.\(^5\) Instead, knowledge stocks are adjusted (more slowly) by changing the level of R&D, which changes the growth rate of the knowledge stock. The adjustment costs for R&D - for example resale losses on R&D equipment or scientists hiring/firing costs - are similar, however, to the adjustment costs for capital in that they depend on the change in R&D levels. Given the law of motion

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\(^4\)There is a literature looking at “R&D real-options”, which uses stochastic calculus to value complex multi-stage R&D projects, but these assume no R&D adjustment costs and no change in uncertainty over time, so are focused on R&D project valuation rather than dynamics (see for example, Eduardo Schwartz, 2003).

\(^5\)Patents are one exception to this, although these cover a small fraction of R&D as they are only available on innovative codified knowledge, typically with a few years lag due to delays in the patenting process.
for the knowledge stock (II.3) this implies

\[
C_G(\Delta R_t) = C_G(\Delta \Delta G_t - \delta G_t) 
\approx C_G(\Delta \Delta G_t) 
\]

Comparing (II.4) to (II.5) the adjustment costs for the knowledge stock are one order of difference apart from the adjustment cost for the capital stock. This distinction arises because the costs of adjustment for capital arises directly from changing its stock. The costs of changing knowledge stocks arise not from changing its stock, but from changing the rate of change of its stock (R&D). Thus, adjustment costs arise in changing the level of the capital stock and changing the flow rate of the knowledge stock. This distinction plays a critical role in shaping the response of investment and R&D to uncertainty.

Interestingly, in the macro literature a number of recent papers (for example Lawrence Christiano, Martin Eichenbaum and Charles Evans (2005)) assume a flow cost for changing investment rates between periods, \(C_K(\Delta I_t)\). Under these assumption my results for R&D would extend to capital. This would also be true more generally of the impact of uncertainty if it affected industries producing productive assets, such as the capital goods producing industry (see, for example, Sherwin Rosen and Robert Topel, 1988).

### III. Simulation results for R&D and uncertainty

Firms are uncertain about future business conditions \((X)\), which evolve as a geometric random walk with mean \(\mu\) and stochastic volatility \(\sigma\)

\[
dX_t = X_t(\mu + \sigma_t dZ_t) \quad \text{where } dZ_t \sim N(0, 1) 
\]
The uncertainty process \((\sigma_t)\) is modelled for simplicity as an AR(1) process, consistent with smooth business-cycle fluctuations, noting this could easily be generalized

\[
\sigma_t = \sigma_{t-1} + \rho_s(\sigma^* - \sigma_{t-1}) + \sigma_s S_t \quad \text{where } dS_t \sim N(0, 1)
\]

(III.2)

There are adjustment costs for changing R&D. In the baseline model these are assumed to be linear, reflecting the hiring/firing costs for scientists and buying/selling costs for R&D equipment, \(C(\Delta R_t) = \kappa|\Delta R_t|\), where \(\kappa > 0\). I also present results for quadratic adjustment costs for comparison, \(C(\Delta R_t) = \kappa_Q G_t(\Delta(R_t/G_t))^2\), where \(\kappa_Q > 0\). In the model I assume the adjustment costs for capital and labor are zero for analytical tractability, and focus on the implications of R&D adjustment costs. This should not change the stylized results for R&D, because in a Cobb-Douglas production function with iso-elastic demand each factor responds most to its own adjustment costs, with limited cross-factor response.\(^6\) Analytical results can show a unique solution to the firm’s optimization problem exists (see the Appendix for the full model), with numerical methods used to solve for exact values.\(^7\)

Figure 1 plots the optimal rates of R&D as a function of current business conditions for low uncertainty \((\sigma_t = 5\%)\), medium uncertainty \((\sigma_t = 20\%)\) and high uncertainty \((\sigma_t = 50\%)\). There are two key results from the simulation.

First, the adjustment costs for changing R&D generate a zone of inaction in the response of R&D to changes in business (demand and productivity) conditions. Given the costs of changing R&D rates firms only incur this when the gap between the actual and desired R&D rate is above a certain threshold, generating a central region of inaction. This creates a dynamic link between current and past R&D rates, consistent with the empirical evidence

\(^6\)See the tables of results in Bloom (2006).

\(^7\)The code available on http://www.stanford.edu/~nbloom.
that R&D rates change only slowly over time, and are more persistent than sales growth, employment growth or investment rates.\textsuperscript{8}

Second, the zone of inaction is larger for higher values of uncertainty, and the response is more muted when it does occur. This is the “caution effect” of uncertainty on R&D behavior. When uncertainty is high the probability of business conditions changing are greater, and since it is costly to change R&D rates the option value of waiting is greater.

Figures 2a and 2b plot the optimal rates of R&D expenditures at low, medium and high uncertainty for low prior values of R&D and high prior values of R&D.\textsuperscript{9} The key result is that the direct impact of uncertainty depends on the difference between optimal R&D and lagged R&D. If optimal R&D is higher than lagged R&D (the right side of both figures) - so that firms want to raise R&D - then higher uncertainty reduces R&D, a negative “delay-effect”. If optimal R&D is below lagged R&D (the left side of both figures) - so the firms want to cut R&D - then higher uncertainty increases R&D, a positive “delay-effect”. Thus, the impact of the “delay effect” depends on the relationship between desired R&D and lagged R&D.

Of course, if R&D depreciates over time (due to scientists quitting and equipment wearing out), then temporary increases in uncertainty will reduce R&D at the steady state. This is because the inherited level of R&D will have depreciated below the optimal level. This is very similar to the reasoning behind the negative steady state “delay effect” of uncertainty on investment and hiring which arises because depreciation, attrition and growth mean inherited capital and labor are always below their optimal levels.

\textsuperscript{8}For example, in Compustat data (1990 to 1999, manufacturing) the correlation between current and two-year lagged sales growth rates are 0.082, labor growth rates are 0.095, investment rates are 0.274, and R&D rates are 0.690. The aggregate figures show a similar pattern (Comin and Gertler (2006)).

\textsuperscript{9}These values are 1.875\% and 7.5\%, chosen as half and twice the steady-state rate of R&D expenditure, \( r_t = \frac{R_t}{G_t} \), given the 15\% depreciation in \( G_t \) and quarterly periodicity.
Figure 3 plots the optimal rates of R&D expenditure for low, medium and high uncertainty assuming only quadratic adjustment costs for R&D. The effects of uncertainty almost completely disappear. With quadratic adjustment costs no real options effects arise, and the assumed homogeneity on revenue function in demand conditions and knowledge stocks minimizes any Jensen’s effects from a concave/convex marginal revenue product of R&D in demand conditions. Hence, the impact of uncertainty on R&D depends critically on the adjustment costs for R&D, for which empirical evidence is very limited.

IV. Implications of uncertainty for micro and macro R&D

At the micro level the “caution-effect” of uncertainty on R&D implies much lower responsiveness of firms in periods of high uncertainty. This could explain why, for example, US firms have been so slow to respond to the R&D tax credit, a policy beset by continued uncertainty over its survival (Bloom et al., 2002). This could be investigated by estimating (with appropriate instrumentation) the following type of regression\(^{10}\)

\[
(IV.1) \quad r_{i,t} = \alpha_0 + \beta_1 r_{i,t-1} + \beta_2 \Delta y_{i,t} + \beta_3 \sigma_{i,t} + \beta_4 r_{i,t-1} \sigma_{i,t} + \beta_5 \Delta y_{i,t} \sigma_{i,t} + X_{i,t} + \epsilon_{i,t}
\]

where \(r_{i,t}\) is firm \(i\) year \(t\) (R&D/sales), \(y_{i,t}\) is firm \(i\) year \(t\) log(sales) and \(X_{i,t}\) are a full set of controls including fixed effects and year dummies. The empirical implication from section (III) for R&D is that higher uncertainty should reduce the responsiveness of firms to sales growth (\(\beta_5 < 0\)) and increase the responsiveness to lagged R&D expenditure (\(\beta_4 > 0\)).

\(^{10}\) Micro data is particularly suitable for testing the “caution effect” of because of the large samples of impulses and responses in firm panel data. Fixed effects are included to control for possible differences across firms, such as due to variations in management practices (Bloom and Van Reenen, 2007). Macro data is particularly suitable for testing the “delay-effect” because of the role of re-allocation across firms in driving the productivity component of the “delay-effect” which only arises under aggregation.
At the macro level the “delay-effect” of uncertainty on R&D is highlighted in table 1, with uncertainty effects on investment in table 2 for comparison. The two columns in Table 1 reflect the result that higher uncertainty increases R&D if the current period R&D is a downward adjustment \((R_t < R_{t-1})\), and reduces R&D if the current period R&D is an upward adjustment \((R_t > R_{t-1})\).

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<thead>
<tr>
<th>R&amp;D decreasing*</th>
<th>R&amp;D increasing</th>
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<tr>
<td>Knowledge stock decreasing</td>
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* If R&D rates depreciate at rate \(\delta_R\), then the condition is \(R_t < (1 - \delta_R)R_{t-1}\).

In contrast table 2 shows that lagged investment plays no role in determining current investment. Instead comparing across the two rows shows that uncertainty increases current investment if the capital stock is decreasing after controlling for depreciation \((K_t < (1 - \delta_K)K_{t-1})\), and reduces it if capital stock is increasing \((K_t > K_{t-1})\).

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* After controlling for depreciation, \(K_t < (1 - \delta_K)K_{t-1}\).
Thus, this implies uncertainty would reduce R&D when it is increasing - typically during the recovery from a recession and initial part of a boom. In comparison it would reduce investment when capital is above trend - typically during a boom. In addition higher uncertainty will tend to increase the persistence of R&D changing its dynamics, but reduce the responsiveness of investment changing its amplitude. Thus, uncertainty will have a differential impact on the levels and dynamics of R&D versus investment, due to different flow versus stock adjustment costs.

V. Conclusions

Uncertainty varies strongly over time, persistently rising by 50% to 100% during recessions, and temporarily rising by 100% to 200% after major political and economic shocks. The impact of these changes in uncertainty on investment and hiring appears to be two-fold: first higher uncertainty typically reduces aggregate investment, hiring and productivity growth due to a “delay effect”, and second higher uncertainty makes firms less responsive to any changes in their environment, a “caution effect”. These effects have been shown to be analytically and empirically important in micro and macro investment and employment behavior.

This paper extends these results on time varying uncertainty to R&D by modelling the flow adjustment costs of knowledge stocks and contrasting this to the stock adjustment costs of capital and labor. I show that higher uncertainty reduces the responsiveness of R&D to changes in demand conditions and increases the persistence of R&D over-time, the R&D equivalent to the “caution-effect”. I also show that if firms are increasing R&D then the marginal effect of uncertainty on R&D will be negative, while if firms are reducing R&D
then the marginal effect of uncertainty on R&D will be positive. Thus, the R&D equivalent to the “delay-effect” depends on the desired change in R&D. I then present micro and macro predictions, with the hope that future empirical research will make progress in testing these.
VI. Appendix

The model underlying the simulations assumes firms have a revenue function \( F(X, K, L, G) = X^\phi K^{\alpha(1-\epsilon)} L^{\beta(1-\epsilon)} G^{(1-\alpha-\beta)(1-\epsilon)} \) which nests a Cobb-Douglas production function in capital (\( K \)), labor (\( L \)) and the knowledge stock (\( G \)), and an iso-elastic demand curve with elasticity (\( \epsilon \)). Demand and productivity conditions are combined into an index (\( X \)), henceforth called “business conditions”.

Business conditions (\( X \)) evolve as an augmented geometric random walk with mean \( \mu \) and variance \( \sigma_t \)

\[
(\text{VI.1})\quad dX_t = X_t(\mu + \sigma_t dZ_t) \quad \text{where} \quad dZ_t \sim N(0,1)
\]

The uncertainty process (\( \sigma_t \)) is modelled for simplicity as an AR(1) process, noting this could easily be generalized

\[
(\text{VI.2})\quad \sigma_t = \sigma_{t-1} + \rho_{\sigma}(\sigma^* - \sigma_{t-1}) + \sigma_S S_t \quad \text{where} \quad dS_t \sim N(0,1)
\]

In the model for analytical tractability I assume the adjustment costs for capital and labor are zero and focus on the implications of R&D adjustment costs. This should not change the stylized results for R&D, but facilitates a numerical solution to the model since the state and control spaces are both reduced by two dimensions.

There are no structural estimates of R&D adjustment costs in the literature. But there is a long literature on capital and labor adjustment costs which I use as a starting point for modelling R&D adjustment costs (Bloom (2006), and Russell Cooper and John Haltiwanger (2006)). This literature focuses on three cost terms - linear costs reflecting per unit adjustment costs: \( C(\Delta R_t) = \Delta R_t * (\kappa_{+}[\Delta R_t > 0] - \kappa_{-}[\Delta R_t < 0]) \), quadratic ad-
justment costs reflecting higher costs of rapid changes: \( C(\Delta R_t) = \kappa Q G_t (\frac{\Delta R_t}{G_t})^2 \), and fixed costs reflecting the revenue loss from disruption involved in changing factors of production: \( C(\Delta R_t) = \kappa_F F(X, K, L, G)_t (\Delta R_t \neq 0) \). I report results for linear and quadratic R&D adjustment costs, but also investigated a range of other adjustment costs and found real options effects whenever linear or fixed costs were present.

With fully flexible capital and labor I can optimize these out, and then normalize the business conditions process, to derive the concentrated revenue function \( \tilde{F}(Y, G) = AY^{1-\theta}G^\theta \). The Bellman equation can then be stated as follows

\[
V(Y_t, G_t, R_{t-1}, \sigma_t) = \max_{R_t} \tilde{F}(Y_t, G_t) - C(R_t, R_{t-1}) - wR_t + \frac{1}{1+r} E[V(Y_{t+1}, G_{t+1}, R_t, \sigma_{t+1})]
\]

where \( w \) is the cost of R&D and \( r \) is the discount rate. The problem is jointly homogeneous of degree one in \((Y_t, G_t, R_t, R_{t-1})\) so can be normalized by \( G_t \)

\[
V(y_t, 1, r_{t-1}, \sigma_t) = \max_{r_t} \tilde{F}(y_t, 1) - C(r_t, r_{t-1}) - wr_t + \frac{(1 - \delta_K)(1 - r_t)}{1 + r} E[V(y_{t+1}, 1, r_t, \sigma_{t+1})]
\]

where \( y_t = (Y_t/G_t) \) and \( r_t = (R_t/G_t) \). Analytical results can show a unique solution to the firm’s optimization problem exists, with numerical methods used to solve for exact values.
References


Table 1: The marginal impact of an increase in uncertainty on R&D

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\(^a\) If R&D rates depreciate at rate \(\delta_R\) then the condition is \(R_t < (1 - \delta_R)R_{t-1}\)

Table 2: The marginal impact of an increase in uncertainty on investment

<table>
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\(^a\) After controlling for depreciation, \(K_t < (1 - \delta_K)K_{t-1}\)
Figure 1: Higher uncertainty makes R&D less responsive to current business conditions and more persistent over time.

Figures 2a and 2b: The effect of uncertainty on R&D is negative if R&D is increasing, and positive if R&D is falling.

Figure 3: With only quadratic adjustment costs there are no real options effects of uncertainty on R&D.

Notes: Figures plot the numerical solution to the firm’s optimization problem. The control variable, current R&D ($r_t$), is a function of the three state variables: lagged R&D ($r_{t-1}$), current business conditions ($y_t$), and the uncertainty over future business conditions ($\sigma_t$).