The Winner’s Curse, Reserve Prices and Endogenous Entry: Empirical Insights From eBay Auctions.

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Abstract

Internet auctions have recently gained widespread popularity and are one of the most successful forms of electronic commerce. We examine a unique data set of eBay coin auctions to explore the determinants of bidder and seller behavior. We begin by documenting a number of empirical regularities in our data set of eBay auctions. Next, we specify and estimate a structural econometric model of bidding on eBay. Using our parameter estimates from this model, we measure the extent of the winner’s curse and simulate seller revenue under different reserve prices.

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1 Introduction

Auctions have found their way into millions of homes with the recent proliferation of auction sites on the Internet. The web behemoth eBay is by far the most popular online auction site. In 2001, 423 million items in 18,000 unique categories were listed for sale on eBay. Since eBay archives detailed records of completed auctions, it is a source for immense amounts of high quality data and serves as a natural testing ground for existing theories of bidding and market design.

In this paper, we explore the determinants of bidder and seller behavior in eBay auctions. For our study, we collected a unique data set of bids at eBay coin auctions from September 28 to October 2, 1998. Our research strategy has three distinct stages. We first describe the market and document several empirical regularities in bidding and selling behavior that occur in these auctions. Second, motivated by these empirical regularities, we specify and estimate a structural econometric model of bidding on eBay. Finally, we simulate our model at the estimated parameter values to quantify the extent of the winner’s curse and to characterize profit maximizing seller behavior.

The bidding format used on eBay is called “proxy bidding.” Here’s how it works. When a bidder submits a proxy bid, she is asked by the eBay computer to enter the maximum amount she is willing to pay for the item. Suppose that bidder A is the first bidder to submit a proxy bid on an item with a minimum bid of $10 (as set by the seller) and a bid increment of $.50. Let the amount of bidder A’s proxy bid be $25. eBay automatically sets the high bid to $10, just enough to make bidder A the high bidder. Next suppose that bidder B enters the auction with a proxy bid of $13. eBay then raises bidder A’s bid to $13.50. If another bidder submits a proxy bid above $25.50 ($25 plus one bid increment), bidder A is no longer the high bidder and the eBay computer will notify bidder A of this via e-mail. If bidder A wishes, she can submit a new proxy bid. This process continues until the auction ends. The high bidder ends up paying the
second highest proxy bid plus one bid increment. Once the auction has concluded, the winner is notified by e-mail. At this point, eBay’s intermediary role ends and it is up to the winner of the auction to contact the seller to arrange shipment and payment details.

There are a number of clear empirical patterns in our data. First, bidders on eBay frequently engage in a practice called “sniping,” which refers to submitting one’s bid as late as possible in the auction. We find that the median winning bid arrives after 98.3% of the auction time has elapsed. Second, there is significant variation in the number of bidders in an auction. We find that a low minimum bid and a high book value increase entry into an auction. This is consistent with a model in which bidding is a costly activity, as in Harstad (1990) and Levin and Smith (1994) where bidders enter an auction until expected profits are equal to the costs of participating. Finally, we find that sellers on average set minimum bids at levels considerably less than an item’s book value, and that they tend to limit the use of secret reserve prices to high value objects.

In Section 4, using a stylized theoretical model, we demonstrate that sniping can be rationalized as equilibrium behavior in a common value environment. If, in equilibrium, bids are an increasing function of one’s private information, then a bidder would effectively reveal her private signal of the common value by bidding early. However, if a bidder bids at the “last-second,” she does not reveal her private information to other bidders in the auction. As a result, we show that there is no symmetric equilibrium except to bid at the very end of the auction. We also demonstrate that the equilibrium in the eBay auction model is formally equivalent to the equilibrium in a second-price sealed-bid auction.

In Section 5, we specify a structural model of bidding in eBay auctions. Using the explanation for last-minute bidding from Section 4, we argue that, to a first approximation, it is possible to regard the bidding on eBay as equivalent to a second-price sealed-bid auction. Therefore, we choose to model eBay auctions as second-price sealed-bid auctions where there is a common value and the number of bidders is endogenously determined by a zero profit condition.

Estimating a model with a common value is technically challenging. The approach we take is most closely related to the parametric, likelihood-based method of Paarsch (1992). However, as Donald and Paarsch (1993)
point out, the asymptotics for maximum-likelihood estimation of auction models are not straightforward, because the support of the likelihood function depends on the parameter values. We utilize Bayesian methods used in Bajari (1997) to overcome this difficulty.

We use our parameter estimates to measure the effect of the winner’s curse, to infer the entry costs associated with bidding, and to characterize profit maximizing reserve prices. For the average auction, we find that bidders lower their bids by 3.2% per additional competitor. Based on our parameter estimates, and a zero profit condition governing the entry decisions of bidders, we calculate the average implicit cost of submitting a bid in an eBay auction to be $3.20. We also compute a seller’s optimal reserve price decision using our parameter estimates. We find that the observed practices of setting the minimum bid at significantly less than the book value and limiting secret reserve prices to high value items are consistent with profit maximizing behavior.

2 Description of the Market and the Data

eBay is currently the most popular auction site on the Internet boasting 37.6 million registered users. The value of goods sold on eBay exceeded $5 billion in the year 2000. The average sale price of an item on eBay is about $20. On any given day, millions of items are listed on eBay, most of which are one-of-a-kind second-hand goods or collectibles. The listings are organized into thousands of categories and subcategories, such as antiques, books, and consumer electronics. Within any category, buyers can sort the listings to first view the recently listed items or the auctions that will close soon. eBay also provides a search engine that allows buyers to search listings in each category by keywords, price range, or ending time. The search engine allows users to browse completed auctions, a useful tool for buyers and sellers who wish to review recent transactions.

In Figure 1 below, we display a recent listing on eBay for six Morgan Dollars. Listings typically contain detailed descriptions and pictures of the item up for bid. The listing also provides the seller’s name, the current bid for the item, the bid increment, the quantity that is being sold, and the amount of time left in the auction.

FIGURE 1 ABOUT HERE

In this paper, we focus on bidding for mint and proof sets of U.S. coins that occurred on eBay between
Both types of coin sets are prepared directly by the U.S. Treasury, and contain uncirculated specimens of a given year’s coin denominations. These sets are representative of items being traded on eBay, in that they are mundanely priced and are accessible to ordinary collectors.

For each auction in our data set, we collected the “final bids” as reported on the “bid history page.” These final bids correspond to the maximum willingness-to-pay expressed by each bidder through the proxy bidding system (except for the winning bidder, whose maximum willingness-to-pay is not displayed). We found the book values for the auctioned items in the November 1998 issue of *Coins Magazine*, which surveys prices from coin dealers and coin auctions around the U.S. As reported in Table 1, the average book value of the coins is $47, with values ranging from $3 to $3700, reflecting the wide dispersion of prices even within this relatively narrow section of the collectible coin market. We reviewed item descriptions to see if the seller reported anything missing or wrong with the item being sold. This investigation labeled about 10% of the items with self-reported blemishes.

TABLE 1 ABOUT HERE.

Sellers on eBay are differentiated by their feedback rating. For each completed auction, eBay allows both the seller and the winning bidder to rate one another in terms of reliability and timeliness in payment and delivery. The rating is in the form of a positive, negative or neutral response. Next to each buyer or seller’s ID (which is usually a pseudonym or an e-mail address), the number of net positive responses is displayed. By clicking on the seller’s i.d., bidders can view all of the seller’s feedback, including all comments as well as statistics totalling the total number of positive, neutral and negative comments.

In Table 1, we see that the average bidder has 41 overall feedback points and the average seller has 203 overall feedback points. Negative feedback points are very low compared to the positive feedback points.

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5 There were a total of 516 auctions completed in the mint-proof category during the five day sample we considered. We could not find reliable book values for 59 of the auctions and since the book value serves as our primary controlling variable for cross-object heterogeneity, we had to discard those observations from our data set. The book values were recorded by two independent coders. We checked whether this truncation makes any difference in the regressions reported in the next section that do not involve the book value, and did not find a qualitative change in our results. There were 11 Dutch auctions and since these auctions are subject to a different set of rules, they were omitted from our analysis. Detailed bid histories for 39 auctions were lost due to a technical error in data transfer. Aside from those, our data set of 407 auctions represents all mint-proof set auctions conducted on eBay during the dates considered.

6 Mint sets contain uncirculated specimens of each year’s coins for every denomination issued from each mint. Proof sets also contain a specimen of each year’s newly minted coins, but are specially manufactured to have sharper details and a more-than-ordinary brilliance than mint sets.

7 The November issue of the magazine was bought by one of the authors on October 22nd. We confirmed by e-mailing the magazine that the prices quoted were market prices of mid-October.
of the sellers. Users of eBay ascribe this to fear of retaliation, since the identities of feedback givers are displayed. Therefore, the conventional wisdom among eBay users is that the negative rating is a better indicator of a seller’s reliability than her overall rating. The average bidder and seller in this market has a reasonably high number of overall feedback points, indicating that they can be classified as a serious collector, or possibly a coin dealer.

3 Some Empirical Regularities

In this section we report several salient empirical regularities we have found about bidding behavior on eBay. These regularities can be summarized as follows:

1. Bidding activity is concentrated at the end of the auction.
2. Minimum bids are on average set to levels considerably less than book value.
3. Sellers tend to limit the use of secret reserve prices to high value objects.
4. There is significant variation in the number of bidders in an auction. We find that a low minimum bid, a high book value, low negative ratings, and high overall ratings all increase entry into an auction.
5. Auction revenues increase with the number of bidders in the auction, and the overall reputation of the seller. The presence of a blemish decreases revenues, as does the presence of a secret reserve price.
6. Having more or less experienced bidders does not appear to affect auction revenues. The presence of “early bids” does not affect the magnitude of the winning bid in the auction.
7. There appears to be a common value component to bidder valuations.

3.1 Timing of bids

A striking feature of eBay auctions is that bidding activity is concentrated at the end of each auction. Figure 2 is a histogram of final bid submission times. More than 50% of final bids are submitted after 90% of the auction duration has passed and about 32% of the bids are submitted after 97% of the auction has passed (the last 2 hours of a 3 day auction).

FIGURE 2 ABOUT HERE.

Winning bids tend to come even later. The median winning bid arrives after 98.3% of the auction time has elapsed (within the last 73 minutes of a 3 day auction), and 25% of the winning bids arrived after 99.8% of the auction time elapsed (the last 8 minutes of a 3 day auction). In our data, we can also observe how many total bids were submitted throughout the auction. In the set of auctions we consider, an average bidder submits about two proxy bids.

8 The maximum negative rating of 21 corresponds to a seller with 973 overall feedback points.
3.2 Minimum bids and secret reserve prices

A seller on eBay is constrained to use the proxy bidding method, but has a few options to customize the sale mechanism. She can set a minimum bid, a secret reserve price, or both. The minimum bid is observed by all bidders, and the average minimum bid, as can be seen in Table 1, was $16.28. In contrast, in a secret reserve price auction, bidders are only told whether the reserve price is met or not. The reserve price is never announced, even after the auction ends. As reported in Table 1, 16% of the auctions were conducted with a secret reserve price.

As we will discuss in section 5.4, the use of secret reserve prices is somewhat of a puzzle in auction theory. Given this, one might wonder whether there is a systematic difference between auctions with secret reserve prices and those without. Table 2 reports the comparison of the quantiles of book values across both types of auctions. As we see from the table, sellers of high book value items are more likely to use secret reserve prices compared to sellers of low book value items.

We also found that sellers who decide to use a secret reserve price keep their minimum bids low to encourage entry. The average minimum bid in secret reserve price auctions is set to 25% of book value, with several equal to 0. In auctions with no secret reserve price, the average minimum bid is set to 70% of the item’s book value. Since eBay does not divulge the secret reserve prices at the close of the auction, we cannot compare the secret reserve price to minimum bid levels in ordinary auctions; however, the fact that far fewer secret reserve price auctions result in a sale (50% vs. 84%) leads one to conjecture that secret reserve price levels are, on average, higher than the minimum bids in ordinary auctions.

3.3 Determinants of entry

There is wide variation in the number of bidders for auctions of similar objects. We investigate the determinants of entry in Table 3, where we regress the number of bidders in an auction on various covariates. Our regression specification interacts the minimum bid (normalized by dividing through by the book value) with the presence of a secret reserve price to disentangle the effects of these seller strategies. After a specification search, we decided to use a Poisson specification in the regression.

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9 Dutch auctions for the sale of multiple objects are also allowed, but we exclude such auctions from our study.

10 Some conversations we came across in community chat-boards on eBay suggest this as a strategy for sellers.

11 With the Poisson specification, the likelihood of observation $t$, conditional on the regressors $X_t$, depends
The constant term of the regression reveals an average 3.01 bidders per auction. We find, as expected, that the minimum bid has a very significant negative effect on the number of the bidders; setting the minimum bid equal to the book value causes the expected number of bidders to drop to 0.6. The presence of a secret reserve price seems to deter some bidders, but not in a statistically significant manner. We find that items with higher book values tend to attract more bidders on average and that both the negative and overall reputation measures of the seller play an important role in determining entry, with the negative reputation measure having the higher elasticity.

### 3.4 Determinants of auction prices

Perhaps the most interesting empirical question to ask, at least from the perspective of eBay users, is, “What determines prices in eBay auctions?” A seller might modify this question to ask, “What should I set as my minimum bid/secret reserve price to maximize my revenues?” To answer both of these questions consistently, one needs to estimate the primitives of the demand side of the market, explicitly taking into account the effect of the supply side policies chosen by the seller (through minimum bid and secret reserve price levels) on price formation. However, before we attempt to perform this exercise using a structural model of price formation, we first examine what a straightforward regression analysis yields.

In Table 4, we regress the highest observed bid in the auction (normalized by the book value of the item) on auction specific variables from Table 1. Variables that are significant at the 5% level are marked with an asterisk. In the first column, we use data from all auctions, including those in which no bids were posted. The second column does the same regression, but this time including a control for the average experience level of the bidders; hence limiting the sample to those auctions in which there was at least one bidder. The third column is the first regression repeated using a Tobit specification, taking into account that the revenue from the auction is truncated at zero.

In all three of these specifications, we find that an increase in the number of bidders increases revenue.

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12 Since the log of the dependent variable enters the Poisson regression, we calculated \( \exp(1.102) = 3.01 \) for the mean.

13 For auctions with a secret reserve price in which the reserve was not met, we still include the highest bid as the “revenue” of the seller.
The presence of an extra bidder appears to increase auction revenue by about 10%, taking all auctions into account, and by 6.6% if we only consider auctions in which there is at least one bidder.

Not surprisingly, the coefficient in the second column of Table 4 shows that a higher minimum bid results in higher revenues conditional on entry. The first and second columns show that once we account for the truncation caused by the minimum bid, this positive effect diminishes and becomes statistically insignificant with the Tobit correction.

The presence of a secret reserve price appears to reduce expected revenues, both conditional and unconditional on entry by bidders. The presence of a blemish reduces auction revenues by 8.4% conditional on entry and 15% unconditionally. The negative reputation of a seller seems to reduce revenues, but this reduction is not statistically significant. However, the overall reputation of the seller seems to increase revenues quite significantly. On the other hand, bidder experience does not seem to play any role in determining auction revenues.

In the fourth and last column of Table 4, we included a dummy variable for “Early bidding activity,” which was coded to be 1 if the bidders who submitted the losing bids made their submissions earlier than the last 10 minutes of the auction. This means that the winner of the auction had time to revise his bid based on information revealed before the very end of the auction, presumably resulting in a higher bid. Out of the 278 auctions with more than 2 bidders, we found 18 auctions with bids coming in the last 10 minutes of the auction and 260 with at least one bid coming in early enough that the winner could revise his bid. However, we see from Table 4 that the presence of early bidding activity does not appear to change the winning bid in the auction in a statistically significant fashion.

Based on the estimates in Table 4, one might be tempted to think that an increase in the minimum bid will, on average, always increase revenues. However, this reasoning ignores the fact that bidders choose to enter an auction. It is quite likely that there will be a threshold minimum bid level above which expected revenues will start to decline due to lack of entry. Similarly, using a secret reserve price will also affect the entry process, as well as interact with the use of a minimum bid. Therefore the regressions in Table 4 should be interpreted with caution from a policy perspective.

### 3.5 Private values versus common value

As pointed out in the seminal work of Milgrom and Weber (1982), the private values and common value
assumptions can lead to very different bidding patterns and policy prescriptions. Although the private values specification is a useful benchmark, our belief is that coin auctions on eBay possess a common value component. Two types of evidence leads us towards this conclusion. The first is the nature of the object being sold and the second is the relationship between the sale price and the number of bidders.

A first piece of evidence for a common value is resale. We have documented that there is a liquid resale market for coins, from which we obtained our “book values.” Given the existence of this market, it is not surprising that a fair number of coin collectors are driven by speculative motives. In this context, learning about other bidders’ assessments of a coin’s resale value might improve the forecast of an individual bidder.

A second piece of evidence for a common value element is that bidders on eBay cannot inspect the coin first-hand. Coins that are scratched or that display other signs of wear are valued less by collectors. Even though we see sellers on eBay expending quite a bit of effort posting detailed descriptions and pictures of the object being auctioned, bidders will probably still face some uncertainty about the condition of the set being sold.

An empirical test to distinguish between the private values model and the common value model is the “winner’s curse” test suggested by the model of Milgrom and Weber (1982). In a common value auction, the Milgrom and Weber (1982) model predicts that bidders will rationally lower their bids to prevent a winner’s curse from happening. The possibility of a winner’s curse is greater in an $N$ person auction as opposed to a $N - 1$ person auction; therefore, the empirical prediction is that the average bid in an $N$ person second-price or ascending auction is going to be lower than the average bid in an $N - 1$ person auction. However, in a private value ascending or second-price auction setting, the number of bidders should not have an effect on bids since the dominant strategy in these auctions is to bid one’s valuation.

In table 5, we report regressions of the bids normalized by book value ($b$) on the realized number of bidders ($N$). We use linear, quadratic and logarithmic specifications to check the robustness of our results. We see that for each specification there is a significant negative relationship between the number of bidders and the normalized bid. In the appendix we discuss the relationship between the winning bid and the

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14 For a concise proof, see Haile, Hong, and Shum (2000).
15 This idea was first operationalized by Paarsch (1992), in a first-price auction context, to distinguish between a common values vs. a private values setting. See Haile, Hong, and Shum (2000) regarding the qualifications needed in the first-price auction.
number of bidders in a private values auction. 16, 17

TABLE 5 ABOUT HERE.

We take the results of this regression-based test as suggestive, but not conclusive evidence against a pure private value model. We recognize that the correct model of bidding on eBay auctions probably involves both a common value and a private value element. However, the analysis and structural estimation of such a model, which involves two sources of private information, would be much less tractable than a pure common value or pure private value model. Therefore, the rest of the analysis in this paper is based on a pure common value model of bidding.

4 Last-Minute Bidding

An important step in building a structural model of equilibrium bidding in eBay auctions is to understand why last-minute bidding occurs, since this behavior has not previously been described as an equilibrium to an auction game (with the important exception of Roth and Ockenfels (2000) in a private values setting). In this section, we demonstrate that in a common value framework, last-minute bidding is an equilibrium.

The eBay auction format does not fit neatly under the category of “open-exit English auctions” studied by Milgrom and Weber (1982). 18 In the model of Milgrom and Weber (1982), a bidder cannot rejoin the auction once she has dropped out. Therefore, her drop-out decision conveys information to other bidders, who will update their estimates of the item’s true worth and adjust their drop-out points. In contrast, on eBay agents have the opportunity to update their proxy bids anytime before the auction ends. With the

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16 Before interpreting this result as empirical evidence for the presence of common values, we should point out that the dependent variable of the regression does not contain the highest bid submitted in the auction. eBay only reports the payment of the highest bidder, which is not his bid, but essentially the second-highest bid. However, as we show in the appendix, this left-censoring actually causes the coefficient on \( N \) under the null hypothesis of private values to be positive. Therefore, finding a negative coefficient on \( N \) still goes against the null hypothesis of private values. However, we should also point out that the alternative hypothesis of a common value does not yield an unambiguous answer under this left-censoring; there can exist joint distributions of bidder signals under which the (censored) regression coefficient on \( N \) can be positive, as well as those under which it is negative.

17 Another problem with the regressions in Table 5 is that the number of bidders who participate in an auction can be correlated with attributes of the auction that are observable to the bidders, but not to the econometrician. Unfortunately, it is hard to think of a natural instrumental variables strategy in this situation, since factors correlated with the number of bidders should also enter into the distribution of valuations. Also, the regressions above do not take into account the effect of the reserve prices on the number of bidders.

18 Observe that the eBay auction format is also different from traditional oral-ascending auctions, since the proxy-bidding system precludes “jump” bids – a bidder can not unilaterally increase the going price to the amount he desires, the auction price is always determined by the proxy bid of the second-highest bidder.
option to revise one’s proxy bid, bidders might not be able to infer others’ valuations by observing their drop-out decisions, since drop-outs can be insincere.

Assume that bidders are risk neutral, expected utility maximizers. We abstract from any cross-auction considerations that bidders might have, and model strategic behavior in each auction as being independent from other auctions.\textsuperscript{19} Given our observation that there is not a big difference in bid levels across bidders with different feedback points, we assume that bidders are ex-ante symmetric.

The informational setup of the game takes the following familiar form: let \( v_i \) be the utility every bidder gains from winning the auction, and let \( x_i \) be his private information regarding the value of the object. We use the symmetric common value model of Wilson, in which \( v_i = v \) is a random variable whose realization is not observed until after the auction. In this model, private information for the bidder is \( x_i = v + \epsilon_i \), where \( \epsilon_i \) are i.i.d.

Let us view the eBay auction as a two-stage auction. Taking the total auction time to be \( T \), let the first stage auction be an open-exit ascending auction played until \( T - \varepsilon \), where \( \varepsilon \ll T \) is the time frame in which bidders on eBay cannot update their bids in response to others. The drop-out points in this auction are openly observed by all bidders, who will be ordered by their drop-out points in the first stage auction, \( \theta_1 \leq \theta_2 \leq \ldots \leq \theta_n \), with only \( \theta_n \) unobservable (bidders can only infer that it is higher than \( \theta_{n-1} \)). The second stage of the auction transpires from \( T - \varepsilon \) to \( T \) and is conducted as a sealed-bid second-price auction.

In this stage, every bidder, including those who dropped out in the first stage, is given the option to submit a bid, \( b \). The highest bidder in the second stage auction wins the object.

Given the above setup, we make the following claim:

**Proposition 1** \( \text{Bidding } 0 \text{ (or not bidding at all) in the first stage of the auction and participating only in the second stage of the auction is a symmetric Nash equilibrium of the eBay auction. In this case the eBay auction is equivalent to a sealed-bid second-price auction.} \)

This result is based on the following lemma, whose proof is in the appendix:

**Lemma 2** \( \text{The first stage drop-out points } \theta_i, i = 1, \ldots, n \text{ cannot be of the form } \theta(x_i), \text{ a monotonic function in bidder } i \text{'s signal.} \)

\textsuperscript{19} We recognize that similar objects are often being sold side-by-side on eBay and that bidding strategies could be different in this situation. We believe that investigating such strategies is an interesting avenue of future research; however, for the sake of tractability, we have chosen to ignore this aspect of bidding.
This lemma leads to the conclusion that the eBay auction format generates less information revelation during the course of the auction than in the Milgrom and Weber (1982) model of ascending auctions. 20

If we take the extreme case in which nobody bids in the first stage of the auction (or everybody bids 0), the second stage becomes a sealed-bid second-price auction, where each bidder knows only her own signal. In this case, the symmetric equilibrium bid function, as derived by Milgrom and Weber (1982), will be \( b(x) = v(x, x) \), where \( v(x, y) = E[v_i | x_i = x, y_i = y] \), with \( y_i = \max_{j \in \{1, \ldots, n\} \setminus i} \{ x_j \} \). 21

5 A Structural Model of eBay Auctions

In this section, we specify a structural econometric model of eBay auctions. The structural parameters we will estimate are the distribution of bidder’s private information and a set of parameters that govern entry into the auction. Estimates of the structural parameters allow us to better understand the determinants of bidder behavior. Also, the structural parameters can be used to simulate the magnitude of the “winner’s curse” in this market and the impact of different reserve prices on seller revenue.

Motivated by the results of the previous sections, we model eBay auctions as second-price sealed-bid auctions where the number of bidders is endogenously determined by a zero profit condition in a manner similar to Levin and Smith (1994).22 Assume there are \( n \) potential bidders viewing a particular listing on eBay. However, not every potential bidder enters the auction, since each entering bidder has to bear a

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20 Note that one of the main competitors of eBay, Amazon’s auction site, extends its auctions beyond the prespecified closing time until bidding activity ceases. This feature changes the analysis above, since the “sealed-bid second-stage” of the auction no longer exists (\( T \) is no longer set, so bidders can not coordinate on the “blind-bidding” period \( [T - \varepsilon, T] \)). In this case, “drop-outs” are final, since the end of the auction is dependent on bidder activity. Therefore, the open-exit ascending auction framework of Milgrom and Weber (1982) can be applied to Amazon auctions. In fact, comparisons of Roth and Ockenfels (2000) regarding bid timings on eBay and Amazon confirm that on Amazon, “last-minute-bidding” is a rarer phenomenon, as expected from our model.

21 To rule out profitable deviations, observe the following: if bidder \( j \) were to unilaterally deviate from the proposed equilibrium of not bidding at all in the first stage, then, the proxy-bidding system of eBay would indicate that this bidder’s signal \( x_j \), is greater than 0, and not much else, since bidder \( j \)’s bid would advance the “maximum bid” to one increment above 0; therefore bidder \( j \) would not be able to signal or “scare” away competition. Further, since signals are affiliated, \( E[v_i | x_i = x, y_i = x] \geq E[v_i | x_i = x, y_i = x_j] \), so bidder \( j \) would unilaterally decrease her probability of winning.

22 We acknowledge that this equilibrium does not entirely fit the observed bidding behavior on eBay; as Figure 2 displays, not all of the bids arrive during the last 5 to 10 minutes of the auction. However, our finding in section 3.4 that early-bidding activity does not affect the winning bid is consistent with Lemma 1: early bids can not be taken seriously in the “second-stage” if bidders can jump in at the last second. Observe that Lemma 1 does suggest a continuum of symmetric equilibria if bidders with signal \( x \geq x’ \) bid \( \theta’ \) in the first stage, and submit second-stage bids that are conditioned on the cutoff point \( x \geq x’ \). However, our finding that the winning bid in the auction is not affected by early bidding activity points out that imposing the assumption that \( \theta’ = 0 \) for all auctions in the data set (as opposed to letting each auction have a different cut-off point) is not going to have egregious consequences.
bid-preparation cost, $c$. In the context of eBay, $c$ represents the cost of the effort spent on estimating the value of the coin and the opportunity cost of time spent bidding. After paying $c$, a bidder receives her private signal, $x$, of the common value of the object, $v$. Like Levin and Smith (1994), our analysis focuses on the symmetric equilibrium of the endogenous entry game, in which each bidder enters the auction with an identical entry probability, $p$. In equilibrium, entry will occur until each bidder’s ex-ante expected profit from entering the auction is zero.

Let $V_n$ be the ex-ante expected value of the item when it is common knowledge that there are $n$ bidders in the auction. Let $W_n$ be the ex-ante expected payment the winning bidder will make to the seller when there are $n$ bidders. Since bidders are ex-ante symmetric, a bidder’s ex-ante expected profit, conditional on entering an auction with $n$ bidders will be equal to $\frac{V_n - W_n}{n} - c$. The seller can set a minimum bid, which we denote by $r$. Define $T_n(r)$ to be the probability of trade given $n$ and a minimum bid $r$. Given these primitives, the following zero-profit condition will hold in equilibrium:

$$c = \sum_{n=1}^{N} p_n^i T_n(r) \left( \frac{V_n - W_n}{n} \right).$$

(1)

In equation (1), $p_n^i$ is the probability that there are $n$ bidders in the auction, conditional on entry by bidder $i$.

As in Levin and Smith (1994), we assume that the unconditional distribution of bidders within an auction is binomial, with each of the $N$ potential bidders entering the auction with probability $p$. On eBay, the potential number of bidders is likely to be quite large compared to the actual number of bidders. Therefore, we will use the Poisson approximation to the binomial distribution. With the Poisson approximation, the probability that there are $n$ bidders in the auction, conditional on bidder $i$’s presence, is $p_n^i = \frac{\lambda^{n-1}}{(n-1)!} e^{-\lambda}$, $n = 1, \ldots, \infty$, where $\lambda = E[n] = Np$. The mean of the Poisson entry process, $\lambda$, will be determined endogenously by the zero profit condition (1). In equilibrium, $\lambda$ is assumed to be common knowledge.

Conditional on entry, we will assume that the bidders will play the “last-minute bidding” equilibrium of the game derived in the previous section. However, bidders do not observe $n$, the actual number of competitors, when submitting their bids. Therefore, by the results in Section 4, the equilibrium bidding strategies are equivalent to a second-price sealed-bid auction with a stochastic number of bidders.

We also allow the sellers to use a secret reserve price. To abstract from any strategic concerns from the
When there is a minimum bid conditional on bidder bidders, excluding bidder, we assume that the seller sets a secret reserve price using the same bid function as the bidders in the auction. Therefore, from the bidders’ perspective, the seller is just another bidder. With this assumption, the only difference between the secret reserve price auction and a regular auction is that now all potential bidders know that they face at least one competitor – the seller.

Following Milgrom and Weber, define \( y_i^{(n)} = \max_{j \in \{1, \ldots, n\} \setminus \{i\}} \{x_j\} \) to be the maximum estimate of the \( n \) bidders, excluding bidder \( i \). Let \( v(x, y, n) = E[v|x_i = x, y_i^{(n)} = y] \) denote the expected value of \( v \) conditional on bidder \( i \)'s signal and the maximum of the other bidders’ signals. The proposition below characterizes the equilibrium bid functions.

**Proposition 3** Let \( p_i^{(a)}(\lambda) \) be the (Poisson) p.d.f. of the number of bidders in the auction conditional upon entry of bidder \( i \) into the auction.

a) When there is no secret reserve price and no minimum bid, then the equilibrium bid function is

\[
b^*(x, \lambda) = \frac{\sum_{n=2}^{\infty} v(x, x, n) f_{y_i^{(n)}}(x|x) f_{y_i^{(a)}}(n|\lambda)}{\sum_{n=2}^{\infty} f_{y_i^{(n)}}(x|x) p_i^{(a)}(\lambda)} \text{ for } x \geq x^*,
\]

b) The equilibrium bid function with a minimum bid, \( r \), and no secret reserve price is

\[
b(x, \lambda) = \begin{cases} \frac{\sum_{n=2}^{\infty} v(x, x, n) f_{y_i^{(n)}}(x|x) p_i^{(a)}(\lambda)}{\sum_{n=2}^{\infty} f_{y_i^{(n)}}(x|x) p_i^{(a)}(\lambda)} & \text{for } x \geq x^*; \\ 0 & \text{for } x < x^*, \end{cases}
\]

where \( x^* \) is the cut-off signal level, above which bidders participate in the auction, and is given by \( x^*(r, \lambda) = \inf \{E_n E[v|x_i = x, y_i < x, n] \geq r\} \).

c) When there is a minimum bid \( r \) and a secret reserve price, the equilibrium bid function becomes

\[
b^{SR}(x, \lambda) = \frac{\sum_{n=2}^{\infty} v(x, x, n) f_{y_i^{(n)}}(x|x) p_i^{(a)}(\lambda)}{\sum_{n=2}^{\infty} f_{y_i^{(n)}}(x|x) p_i^{(a)}(\lambda)} \text{ for } x \geq x^*,
\]

\[
b(x, \lambda) = \begin{cases} \frac{\sum_{n=2}^{\infty} v(x, x, n) f_{y_i^{(n)}}(x|x) p_i^{(a)}(\lambda)}{\sum_{n=2}^{\infty} f_{y_i^{(n)}}(x|x) p_i^{(a)}(\lambda)} & \text{for } x \geq x^*; \\ 0 & \text{for } x < x^*, \end{cases}
\]

where the screening level, \( x^{SR}(r, \lambda) \), is in this case the solution to

\[
\sum_{n=2}^{\infty} p_i^{(a)}(\lambda) \frac{\int_y^n v f_y^2(x^*|v) F_x^{n-1}(x^*|v) f_v(v) dv}{\int_y^n f_y^2(x^*|v) F_x^{n-1}(x^*|v) f_v(v) dv} = r.
\]

Interested readers can consult the appendix for the derivation of the above expressions. When the number of bidders is known to be \( n \), it is well known (see Milgrom and Weber (1982)) that a bidder with a signal of
In Proposition 3a), we see that when entry is stochastic and there is no minimum bid, the equilibrium is a straightforward modification of the equilibrium with a known number of bidders. The bid function in this case is a weighted average of the bids in the deterministic case, where the weighting factor is given by \( \frac{\int_{0}^{\infty} f(x|\theta) p_0(\theta) \, d\theta}{\sum_{n} \int_{0}^{\infty} f(x|\theta) p_0(\theta) \, d\theta} \). Proposition 3b) covers the case of a positive minimum bid and no secret reserve. The bids are the same as in 3a), except that bidders with signals falling below the screening level \( x^*(r, \lambda) \) choose to bid 0. This result is similar to the analysis in Milgrom and Weber (1982) and McAfee, Quan, and Vincent (2000). Proposition 3c) covers the case when there is a secret reserve and a minimum bid. The bid function is identical to 3b), except for the fact that bidders treat the seller as an additional bidder who always enters. For readers who are interested in further properties of the equilibrium bid functions, we have included a numerical example in the appendix, which also outlines a convenient computational method to evaluate equations (2)-(5).

5.1 Empirical specification of the information structure and entry process

We assume that both \( v \) and \( x \) are normally distributed. The normality assumption is used because it is computationally convenient and the normal distribution seems to do a reasonable job in matching the observed distribution of bids.\(^{23}\) We specify the prior distribution \( f_0(v) \) of the common value in auction \( t \) to be \( N(\mu_t, \sigma_t^2) \), where

\[
\mu_t = \beta_1 \text{BOOKVAL}_t + \beta_2 \text{BLEMISH}_t \text{BOOKVAL}_t - 2.18 \\
\sigma_t = \beta_3 \text{BOOKVAL}_t + \beta_4 \text{BLEMISH}_t \text{BOOKVAL}_t, \tag{6}
\]

This specification is motivated by our previous finding that objects with blemishes are sold at a discount in an auction. Also, since agents must pay shipping and handling fees, we subtract \$2.18 in computing \( \mu_t \), the average shipping and handling fee for the mint/proof sets.\(^{24}\) We assume that the distribution of individual signals \( f(x|v) \) conditional on the common value \( v \) is \( N(v_t, k\sigma_t^2) \), where \( k = \beta_5 \) is a parameter to be estimated.

The zero profit condition (1) suggests that in our estimation procedure, we should treat the entry cost, \( c \), as an exogenous parameter and endogenously derive the equilibrium distribution of entrants as characterized

\(^{23}\) Negative valuations implied by a normal distribution are justified due to the presence of a shipping and handling fee.

\(^{24}\) Our data on shipping and handling has a lot of missing values, so we chose not to use the individual values.
by $\lambda_t$. This is a computationally intractable problem. Solving for $\lambda_t$ requires that we find the solution to a nonlinear equation, which can only be evaluated approximately, since the expectation terms involve multi-
dimensional integrals. Instead, motivated by the regression in Table 4, we use the following reduced form specification for $\lambda_t$:

$$
\log \lambda_t = \beta_6 + \beta_7 \text{SECRET}_t + \beta_8 \ln(\text{BOOK}_t) + \beta_9 \text{NEGATIVE}_t \\
+ \beta_{10} \text{SECRET}_t \frac{\text{MINBID}_t}{\text{BOOKVAL}_t} + \beta_{11} (1 - \text{SECRET}) \frac{\text{MINBID}_t}{\text{BOOKVAL}_t}.
$$

(7)

In equation (7), the entry probability depends on the book value, the seller’s negative feedback, and the reserve price. Our reduced form specification for $\lambda_t$ is consistent with equations (1) and (6) since these equations imply that the distribution of entrants should be a function of the book value, the dummy for a blemish, and the reserve price used by the seller.

5.2 Estimation

The model outlined in the previous subsection generates a likelihood function in a natural way. Let $\Omega_t = \{\text{SECRET}_t, r_t\}, \beta = (\beta_1, ..., \beta_{11})$ and $Z_t = (\text{BOOKVAL}_t, \text{BLEMISH}_t)$. Let $b(x|\Omega_t, \beta, Z_t)$ be the equilibrium bid function conditional on $\Omega_t$, the structural parameters $\beta$ and the set of covariates $Z_t$. Since bids are strictly increasing in $x$, an inverse bid function exists, which we denote as $\phi(b|\Omega_t, \beta, Z_t)$. Let $f_b(b_t|\Omega_t, \beta, Z_t, v)$ denote the p.d.f. for the distribution of bids conditional on $\Omega_t, \beta, Z_t$ and the realization of the common value $v$. By a simple change of variables argument, $f_b$ must satisfy the equation below:

$$
f_b(b_t|\Omega_t, \beta, Z_t, v) = f_v(\phi(b|\Omega_t, \beta, Z_t)|v, k, \sigma_t)\phi'(b_t|\Omega_t, \beta, Z_t).
$$

(8)

It is not possible to directly use equation (8) for estimation since the econometrician does not observe $v$. In equation (9), we derive the likelihood function conditional on the information that is available to the econometrician. To do this, we must integrate out the latent variable $v$. Let $n_t$ be the number of bidders who actually submit a bid in auction $t$ and let $(b_1, ..., b_{n_t})$ be the set of bids observed by the econometrician.

\footnotetext{25}{A possible alternative approach is to use a non-parametric or semi-parametric regression framework to estimate the distribution of bidders in a given auction, conditional on auction specific covariates. This approach would eliminate making functional form assumptions such as the Poisson specification here. The problem with this approach is that the number of bidders “entering” the auction may not be equal to the number of bidders who actually submit a bid. Bidders with a $x_2$ less than the “screening level” do not submit a bid even though they have paid the entry fee $c$. Hence, the parameters of the “entrant” distribution has to be estimated jointly with all other unobservable primitives.}
in auction $t$. Let $f_{b_t}(b_1, b_2, ..., b_{n_t} | \mu_t, \sigma_t, \Omega_t)$ be the p.d.f. for the vector of bids observed in auction $t$. By equation (8), when the minimum bid is zero and there is no secret reserve price, this p.d.f. must satisfy the following equation:

$$
 f_{b_t}(b_1, b_2, ..., b_{n_t} | \Omega_t, \beta, Z_t, v) = p(n_t | \lambda_t) \int_{-\infty}^{\infty} (1 - F_b(\phi(b_1 | \Omega_t, \beta, Z_t, v))) \times \prod_{i=2}^{n_t} f_b(b_i | \Omega_t, \beta, Z_t, v) f_v(v | \mu_t, \sigma_t) dv,
$$

where $\lambda_t$ satisfies (7) and $(\mu_t, \sigma_t)$ satisfy (6). (9)

Equation (9) takes into account the fact that what we observe about the winning bid is the actual payment made. Also, we take into account that $n_t$ is random by multiplying through by $p(n_t | \lambda_t)$.26

In an auction with a minimum bid, we have to take into account that some entrants might have drawn signals that are lower than the screening level, $x^*(\lambda_t, r_t)$, and hence have not submitted a bid. The likelihood function in this case is

$$
\sum_{j=n_t}^{N} p(j | \lambda_t) \times \int_{-\infty}^{\infty} \{F_x(x^* | v, k, \sigma_t)\}^{j-n_t} \times (1 - F_x(\phi(b_1 | \Omega_t, \beta, Z_t) | v, k, \sigma_t))^{1\{n_t \geq 1\}} \times \prod_{i=2}^{n_t} f_b(b_i | \Omega_t, \beta, Z_t, v) f_v(v | \mu_t, \sigma_t) dv .
$$

(10)

where $1\{n_t \geq 1\}$ is an indicator variable for the event that there is at least one bidder in the auction and $N$ is an upper bound on the number of entrants.27 Observe that this specification allows us to assign a positive likelihood to auctions with no bidders. For auctions with a secret reserve price, the likelihood function in (10) is modified to reflect that bidders view the seller as another competitor (who is certain to be there) and as a result the bidders use a different bid function.

When computing these likelihood functions, the following result helps immensely in reducing the computational complexity of the estimation algorithm:

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26 We assume that with a zero reserve price the probability of not bidding conditional upon entering the auction is zero. That is, bidders always draw high enough valuations after they’ve incurred the bid preparation cost to make bidding worthwhile. Correcting for truncation at zero slows down the computation of the likelihood function and does not change the results discernibly.

27 With a Poisson process, $N = \infty$. But this makes it impossible to account for the truncation in bids due to the reserve price, hence we take $N$ to be sufficiently large so that the difference between the truncated Poisson distribution and the full distribution is negligible. We use $N = 30$. 

18
Proposition 4. Let \( b(x|\mu^0,\sigma^0,k,\lambda) \) be the bid function in an auction where the common value, \( v \), is distributed normally with mean \( \mu^0 \) and standard deviation \( \sigma^0 \). If the common value is distributed with mean \( r_1\mu^0 + r_2 \) and standard deviation \( r_1\sigma^0 \), the equilibrium bid function is \( r_1b(r_1x + r_2|\mu^0,\sigma^0,k,\lambda) + r_2 \).

Readers can consult the appendix for a proof of Proposition 4. This is a key result in our estimation procedure because it allows us to compute the bid function only once for a base-case auction. We can then apply an affine transformation to this “pre-computed” function to find the bid function in an auction where the distribution of the common value has a different mean and variance.\(^{28}\)

We use recently developed methods in Bayesian econometrics to conduct the estimation. In Bayesian econometrics, the econometrician first specifies a prior distribution for the parameters and then applies Bayes Theorem to study the properties of the posterior. We use Markov chain Monte Carlo methods described in the appendix to simulate the posterior distribution of the model parameters.

There are four advantages to a Bayesian approach when estimating parametric auction models. First, the Bayesian methods are computationally simple and we have found these methods are much easier to implement than maximum likelihood, where the econometrician must perform numerical optimization in several dimensions. Second, it is not possible to apply standard asymptotic theory in a straightforward manner in many auction models because the support of the distribution of bids depends on the parameters. Third, confidence intervals in a classical framework are most commonly based on an asymptotic approximation. The results that we report will be correct in finite samples and do not require invoking an asymptotic approximation. Fourth, Porter and Hirano (2001) have found that in some parametric auction models, Bayesian methods are asymptotically efficient while some commonly used classical methods are often not efficient. The details of how to compute our estimates can be found in the appendix.\(^{29}\)

\(^{28}\) Observe that the above argument depends only on the linearity of the expectation of the common value given other bidder’s signals, and the existence of monotonic bid functions. Therefore, we can extend this result to any auction where bidder signals are distributed according to a location-scale family, and where the common value or the expectation of the common value is a linear function of the bidders’ signals. It is worthwhile to note that bid functions will be likewise linearly scalable in many commonly studied auctions, such as the first-price sealed-bid auction. Elyakime, Laffont, Loisel, and Vuong (1994) note that efficient evaluation of the bid function is a key problem in the structural estimation of auction models. The simplification we propose can be used for structural estimation in a wide variety of models where the bid function is linearly scalable and the distribution of valuations is characterized by its first two moments.

\(^{29}\) One traditional objection to Bayesian methods is that the specification of the prior is typically ad hoc. In our analysis, there are only 8 parameters and over 1000 bids. As a result, we have found our parameter estimates are quite robust to changes in the prior distribution.
6 Results

Table 6 reports our parameter estimates for the structural model. Since the posterior mean for the coefficient multiplying $BOOKVAL$ in equation (6) is almost 1, we infer that published book values do indeed provide point estimates for how the coins are valued on the market. Our estimates in Table 6 demonstrate that blemishes reduce the value of the object, but the standard deviation of the posterior is quite large. This could stem from the fact that these “blemishes” were coded by “non-collectors” who might have introduced noise into our coding. An increase in $BOOKVAL$ by $1 increases the posterior mean of $\sigma_1$ by $.56$. Therefore, a large amount variation between the values of the different coins in our data set is not captured by the book value and the presence of a blemish alone.

Our estimates imply that the standard deviation of the bidders’ signals, $x_i$, is only $k^{1/2}\sigma_v = 28.25\%$ of the book value. Therefore, the bidders’ signals of $v$ are much more clustered than the unconditional distribution of $v$, suggesting that bidders are quite proficient in acquiring information about the items for sale.

The main determinants of entry are the reserve price mechanism, the minimum bid level, and the book value of the coin. Negative seller reputation was found to reduce entry, while the minimum bid level in a secret reserve price auction does not affect the entry decision significantly.

We attempt to explore the robustness of our results by modifying our likelihood function in the spirit of Haile and Tamer (2000). The final bids we observe might not reflect the maximum amount a bidder was willing to pay. For instance, suppose that bidder A, early in the auction submits a proxy bid that was much lower than his valuation. If a number of very high bids arrive in the middle of the auction, bidder A might not find it worthwhile to respond if the current high bid exceeds his maximum willingness to pay. Following Haile and Tamer (2000), we interpret the final bid of each bidder as a lower bound on $v(x, x)$. Similarly, we interpret the winning bid as an upper bound on a bidder’s conditional valuation. Haile and Tamer (2000) argue that these bounds hold true for a very rich set of bidding strategies, even including some non-equilibrium bidding strategies. We found only very small departures from the parame-

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$^{30}$ All results are from a posterior simulation of 30,000 draws with an initial “burn-in” of 10,000 draws. Convergence to the posterior appeared to be rapid.
ter estimates in Table 6 employing the Haile and Tamer (2000) approach. However, we acknowledge that even the bounding strategy of Haile and Tamer (2000) may not take into account certain complex dynamic strategies that might be underlying the data-generation process on eBay.

In order to gauge the degree of fit our model achieves, we simulated the auctions in our data set using our parameter estimates. That is, given the auction specific variables, we generated $n_t$ signals for each auction $t$, where $n_t$ was generated by the estimated entry process. We then computed the bid functions corresponding to the signals. The top two plots in Figure 3 compare the actual and simulated bid distributions.

**FIGURE 3 ABOUT HERE.**

Although the mean values of the bid distributions appear to match (the mean for the simulated distribution is 82.7% of the book value, compared to 83% for the actual data), Figure 3 tells us that our estimate of the variance of cross-auction heterogeneity ($\beta_3$ in particular) is higher than what seems apparent in the data. The bottom plot in Figure 3 is the simulated distribution of bids when we set $\beta_3 = 0.3$. This seems to match the actual bid distribution better.

In interpreting the above plots, we should bear in mind that Figure 3 alone does not tell us anything about the within-auction dispersion of bids. However, our empirical specification attempts to match the within-auction dispersion of bids alongside the cross-auction dispersion. Figure 4 is a histogram of the simulated and actual dispersion of bids. We find that the mean dispersion (difference between max and min) of the simulated bids is 23% of the book value. This is lower than the actual dispersion of bids, which was found to be 32%. We see that in trying to match all moments of the data, our specification has to make a trade-off between overstating cross-auction heterogeneity and understating within-auction heterogeneity. We believe that adding explicit consideration of unobserved heterogeneity into our econometric specification will help achieve a tighter fit; however, such an exercise would greatly complicate the estimation procedure and is beyond the scope of this paper.

**FIGURE 4 ABOUT HERE.**

In order to evaluate our parameter estimates for the entry process, we compare them with the results in Table 3 in section 3.2. The signs of the coefficients on the independent variables seem to be in accord. However, the structural model accounts for the fact that some bidders who “enter” the auction may choose not to submit a bid because their signal $x_i$ is less than the screening level. When we predict the expected
number of bidders using the structural model, we find that on average 3.3 bidders bear the entry cost $c$. The average number of bids submitted is 3.01.

Controlling for the minimum bid level, we see that bidder entry is adversely affected by the existence of a secret reserve, a result that is present in both Table 3 and Table 6. This is intuitively plausible since holding the bid level fixed, the ex-ante profit of a bidder entering a secret reserve price auction is lower, since he will also have to outbid the seller.

6.1 Quantifying the “winner’s curse”

We define the winner’s curse correction as the percentage decline in bids in response to one additional bidder in the auction. To quantify the magnitude of the winner’s curse correction, we computed the bid function given in equation (3) for a “representative” auction using the estimated means of the parameters in Table 6 and sample averages of the independent variables entering equations (6) and (7). The bid function for this “representative” auction is plotted in Figure 5. In this figure, we also plotted the bid function in an auction where we change $\lambda$ to add one more bidder in expectation (with all other parameters and independent variables kept the same). We find that adding an additional bidder decreases bids by $1.50 in this $47 auction. In other words, the winner’s curse reduces bids by 3.2% for every additional competitor in the auction. Observe that this estimate is very close to the effect of adding an additional bidder implied by our reduced form regressions in Section 4.1.\(^{31}\)

FIGURE 5 ABOUT HERE.

The bid function in the representative auction is very close to being linear and has a slope of 0.9 with an intercept of -0.85. Therefore, a bidder with a signal $x_i$ equal to the book value of $47 should bid $41.5. This is $5.5 less than in a private values auction, which is plotted as the 45 degree line in Figure 7. Hence we see that the winner’s curse correction does play an important role in transactions within this market.

To understand how reducing uncertainty about the value of the object affects the bids, we plot the bid function for the case where the standard deviation of $v$ is only 10% of book value (as opposed to 52%). As expected, the bids increase: a bidder with a signal $x_i$ of $47, bids $45. This is about a 7% gain! Clearly, sellers can increase profits by reducing the bidders’ uncertainty about the value of the coin set.

\(^{31}\) Recall that the regressions in Table 5 suffer from the fact that the highest bid in the auction is not observed. They also treat $N$ as being ex-ante observable. The structural model we have estimated handles both of these complications.
6.2 Quantifying the entry cost to an eBay auction

Our finding that the minimum bid plays an important role in determining entry into an auction naturally leads us to question the magnitude of the implicit entry costs faced by the bidders. We can estimate entry costs using the zero profit condition

\[ c = \sum_{n=1}^{\infty} \frac{e^{-\lambda \lambda^{n-1}}}{(n-1)!} T_n(r) \frac{(V_n - W_n)}{n}. \]  

(11)

Using our parameter estimates, assuming that the item characteristics are from the “representative auction” used in the winner’s curse calculation, we can calculate all of the terms on the right hand side of equation (11). This allows us to infer the value of \( c \). Using the simulation method described in the working paper version of this paper (Bajari and Hortaçsu (2000)), we find that the implied value of \( c \) is $3.20. At a wage rate of $10/hour, this corresponds to 20 minutes of foregone earnings.

In general, we expect this implied entry cost to change with auction characteristics. However, the entry cost as a percentage of the book value seems very stable. For example, for an auction with book value $1000 and a minimum bid of $600, the computed entry cost was found to be $66. Therefore, it is advisable for sellers of high value items to reduce this cost by giving out as much information about the item as possible.

6.3 Optimal minimum bids with endogenous entry

Standard results in auction theory regarding optimal minimum bids in private value environments tell us that the seller should set a minimum bid above her reservation value.\(^{32}\) However, when entry decisions are endogenously determined, the seller has to make a trade-off between guaranteeing a high sale price conditional on entry versus encouraging sufficient participation to sell the object. In Figure 6, we use the zero-profit condition in equation (11) to compute the equilibrium expected number of bidders for different values of the minimum bid. The figure was generated for the “representative” auction of the previous section, using an entry cost of $3.20.\(^{33}\) When the minimum bid is zero, on average 4.5 enter the auction as opposed to only about 2 bidders if the minimum bid is set to 0.8 times the book value.

\(^{32}\) See, for example, Myerson (1981).

\(^{33}\) Since this amounts to finding the zeros of a nonlinear equation, we used the Newton-Raphson algorithm to find the equilibrium expected number of bidders numerically.
Suppose that the seller sets the minimum bid in order to maximize the following objective function:

$$E[\text{Revenue}] = \Pr\{\text{Sale}|r\}E[\text{Winning bid}|r] + (1 - \Pr\{\text{Sale}|r\})\text{Residual value},$$ \hspace{1cm} (12)

where the residual value is the seller’s valuation of the object in dollars if she fails to sell the object to any bidder. In Figure 7, we plot different minimum bid levels on the horizontal axis and on the vertical axis we plot the corresponding expected revenues of the seller. We make these plots for three different residual values. We see that for a $47 book value coin, a seller whose residual value is 100% of book value maximizes revenue by setting her minimum bid about equal to the book value. Similarly, sellers with residual values of 80% or 60% of the book value maximize expected revenue by setting the minimum bid approximately equal to their reservation value. Hence the optimum minimum bid policy appears to be for sellers to set the minimum bid approximately equal to their reservation value.

The results in Figure 7 also demonstrate that setting a minimum bid 10 or 20 percent over the book value will drastically reduce seller revenues. However, setting the minimum bid at 10 to 20 percent less than book value is fairly close to revenue maximizing.

FIGURE 7 ABOUT HERE.

In section 3.2, we reported that the average minimum bid was set at 70% of book value for auctions with no secret reserve price. Since we do not know the sellers’ residual values for the items on auction, we cannot assess whether eBay sellers are setting an optimal minimum bid. However, considering many sellers are coin dealers or coin shop owners who have lower acquisition costs than retail buyers and have to balance inventory, it makes sense that they would have residual values less than the book value of the item. At 70% of book value, the residual value of the seller of a $47 coin set would be $33. It is not implausible to assume that the opportunity cost of time for the typical seller to launch an auction is at least $5 or $10 less than the book value.

6.4 Secret reserve prices

Why are secret reserve prices used on eBay? The use of secret reserve prices as opposed to observable minimum bids has been somewhat of a puzzle in the auction theory literature. For example, Elyakime et al. (1994) show that in an independent private value first-price auction, a seller is strictly worse off using a secret reserve price as opposed to a minimum bid.\(^{34}\) A partial resolution to this puzzle was offered by

\(^{34}\) However, the secret reserve price might be beneficial if bidders are risk averse, as shown by Li and Tan (2000).
Vincent (1995). In a common value second-price sealed-bid auction, keeping a reserve price secret can increase a seller’s revenue for some particular specifications of utilities and information. However, the explanation in Vincent (1995) does not completely characterize the conditions under which secret reserve prices might be desirable and it also fails to account for free entry by bidders.

The absence of a clear-cut, analytic answer to the secret reserve price versus observable minimum bid question compels us to provide an answer based on simulations of our estimated model of bidding. We compare two different seller policies in our simulations. In the first policy, we assume that the seller uses a minimum bid equal to his residual value. In the second policy, we assume that the seller sets a secret reserve price equal to his residual value and sets the minimum bid to zero. These two policies will lead to different levels of bidder entry (due to equation (8)) and different bidding strategies conditional on entry (due to equations (3) and (4)). We simulate the expected revenue of the auctioneers by drawing bidder valuations from the estimated distributions. We vary the book value of the object from 5 to 200 dollars in our simulations and assume that the seller has a residual value that is equal to 80% of the book value of the object.

**FIGURE 8 ABOUT HERE.**

Figure 8 displays the results of this simulation. We find that using a secret reserve price has a discernible revenue advantage over using a minimum bid. From our parameter estimates in Table 6, the expected number of bidders will be larger under the second policy, where a secret reserve price with no minimum bid is used, than under the first policy. This “entry” effect of the secret reserve price is countervailed by a winner’s curse effect. Bidders shade their bids more in the secret reserve price auction since the number of expected bidders is higher and since the seller is always present as an additional bidder. However, Figure 8 seems to suggest that the positive effect of secret reserve prices on entry outweighs the bid-shading effect.

Given our result above, one might wonder why all auctions on eBay do not have secret reserve prices. For current auctions, this is not a mystery. Since August 30, 1999, eBay has been charging sellers a $1 fee for using a secret reserve – but refunding the fee if the auction ends up in a sale. We would expect few sellers to use a secret reserve price for items with low book values, since these auctions attract fewer bidders and would frequently result in sellers paying the $1 fee.

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We assume that the seller sets his secret reserve price equal to his residual value as a simplifying assumption. In practice, the seller might find it optimal to follow an alternative strategies.

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The explanation given above still does not resolve the “too few secret reserves” puzzle in our data set, since there was no $1 fee in the time-frame we studied. When we read about the events surrounding eBay’s decision to institute the $1 fee, we gained a bit more insight. eBay announced the reason for its decision was that their limited support staff could not address the many complaints from bidders who were confused by the secret reserve rule or who claimed that sellers were setting unreasonably high reserves (Wolverton (1999)). The announcement of the reserve price fee, however, resulted in a very vocal dissent from eBay sellers, some of whom interpreted the move as a way for eBay to increase its revenues rather than decrease costs. This is an indication that at least some sellers thought that using a secret reserve was a revenue enhancing mechanism for them. The reluctance of 84% of sellers to use a secret reserve can be attributed to the fact that bidders were frequently confused and frustrated by the reserve. In fact, we observed that sellers using a secret reserve price had (statistically significant) higher negative feedback points than sellers who did not use a secret reserve price. However, we also found that secret reserve prices were more likely to be used for higher book value items – items for which there were higher gains to using the secret reserve. Hence, a potential explanation for the puzzle of too few secret reserves can be that sellers were trading off customer goodwill for higher revenues.36

7 Conclusion

In this paper, we studied the determinants of bidder and seller behavior on eBay using a unique data set of eBay coin auctions. Our research strategy had three parts. First, we described a number of empirical regularities in bidder and seller behavior. Second, we developed a structural econometric model of bidding on eBay auctions. Third, we used this model to quantify the magnitude of the winner’s curse and to simulate seller profit under different reserve prices.

There were a number of clear empirical patterns in our data. First, we found that bidders engage in “sniping,” submitting their bids close to the end of the auction. Second, we found some surprising patterns in how sellers set reserve prices. Sellers tend to set minimum bids at levels considerably less than the items’

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36 Another possible explanation for using secret reserve prices for high ticket items is seller risk aversion. On eBay chat sites and in popular guides to buying and selling on eBay, using a secret reserve price with a low minimum bid is thought to have two advantages. First, the seller is insured against selling the item for far less than his reservation value for the item, as would be the case, for instance with a zero minimum bid and no secret reserve. Second, a low minimum bid encourages entry relative to a high minimum bid. We do not, however, choose to pursue this line of inquiry in this paper.
book values. Also, sellers tend to limit the use of secret reserve prices to high value objects. Finally, we found that there is significant variation in the number of bidders in an auction. A low minimum bid, a high book value, low negative ratings, and high overall ratings all increase entry to an auction.

We demonstrated that last-minute bidding is an equilibrium in a stylized model of eBay auctions with a common value, and that, to a first approximation, bidding in eBay auctions resembles bidding in a second-price sealed-bid auction. Motivated by our empirical findings, we specified and estimated a structural model of bidding on eBay with a common value and endogenous entry.

We then simulated the structural model to study the winner’s curse and to compute seller profit under different reserve prices. Our main findings are, first, the expectation of one additional bidder decreases bids by 3.2% in a representative auction. Also, in this representative auction, bids are 10% lower than bidder’s ex-ante estimate of the value of the coin set. Second, entry into an auction plays a very important role in determining revenues. Since bidding is a costly activity, a high minimum bid can reduce the expected profit of a seller by discouraging bidders to enter. Finally, we found that a seller who uses a secret reserve price with a low minimum bid can in some cases earn higher revenue than sellers who only use a minimum bid. Since bidders are frequently confused and frustrated by secret reserve prices, using a secret reserve can involve a trade-off between customer good-will and higher revenues.

There are several directions in which our empirical specification and estimation approach can be improved upon. First, as we mentioned in Section 3, the bidding environment is likely to possess both a common value as well as private value elements. We chose to focus on the pure common value case in light of the regression based “winner’s curse” test and the nature of the collectible coins market. However, we believe that modifying the empirical specification to allow for both private values and a common value is an important direction for future research.

Second, we did not model the entry decisions of the bidders as a sequential problem and assumed that the entry decision is made simultaneously. Relaxing this assumption could open the way to dynamic, rather than static, bidding strategies that could yield different empirical implications.

Finally, we have not explicitly taken into account the “multi-market” nature of eBay and have treated each auction in isolation. Often, identical or very similar goods are sold side-by-side or back-to-back. Since we also observe significant variation among sellers’ choices of minimum bids and reserve prices,
it might be possible to test theories of competition between sellers as in Peters and Severinov (1997) and McAfee (1993).
Appendix A.

7.1 Proof that the regression coefficient of bids on \( N \) is positive under private values when the highest bid is not observed

Under the private values assumption, bids are going to be equal to the bidders’ signals, \( x_i \). Hence, the left-censored mean bid in an \( N \) person auction can be written as

\[
E(x|x \leq 2\text{nd highest bid among } N) = \int_0^\infty \left( \int_0^t x f(x)dx \right) \frac{f_X^{[N-1:N]}(t)}{1 - F_X^{[N-1:N]}(t)} dt
\]

(A-1)

and the left-censored mean bid in an \( N - 1 \) person auction can be written as

\[
E(x|x \leq 2\text{nd highest bid among } N - 1) = \int_0^\infty \left( \int_0^t x f(x)dx \right) \frac{f_X^{[N-2:N-1]}(t)}{1 - F_X^{[N-2:N-1]}(t)} dt,
\]

(A-2)

where the notation, \( X^{[N-1:N]} \), denotes the \( (N - 1)^{st} \) order-statistic among \( N \).

Using the following formulas for distribution functions of order statistics, we get:

\[
F^{[N-1:N]}(t) = NF(t)^{N-1} - (N - 1)F(t)^N
\]

(A-3)

\[
F^{[N-2:N-1]}(t) = (N - 1)F(t)^{N-1} - (N - 2)F(t)^{N-1}
\]

(A-4)

\[
F^{[N-1:N]}(t) - F^{[N-2:N-1]}(t) = -F(t)^{N-2}(N - 1)[1 - F(t)]^2,
\]

(A-5)

which is negative for \( t \in (0, 1) \). Hence, \( X^{[N-1:N]} \) first-order stochastically dominates \( X^{[N-2:N-1]} \).

Now, for notational brevity, let \( F_1(t) = F^{[N-1:N]}(t) \), \( F_2(t) = F^{[N-2:N-1]}(t) \), and let \( g(t) = \int_0^t x f(x)dx \). Since \( \frac{f(t)}{1 - F(t)} = -\frac{d}{dt} \ln(1 - F(t)) \), the difference between equations (A-1) and (A-2) can be written as

\[
\int_0^\infty g(t) \frac{d}{dt} [\ln(1 - F_2(t)) - \ln(1 - F_1(t))] dt,
\]

(A-6)

which, through integration by parts, becomes

\[
\left[ g(t) \ln \frac{1 - F_2(t)}{1 - F_1(t)} \right]_0^\infty - \int_0^\infty g'(t) [\ln(1 - F_2(t)) - \ln(1 - F_1(t))] dt.
\]

(A-7)

Assuming \( \lim_{t \to \infty} \frac{1 - F_2(t)}{1 - F_1(t)} = 1 \), we get rid of the first term and are left with the second term, which is positive since \( g(t) \) is an increasing function of \( t \), and since \( 1 - F_2(t) < 1 - F_1(t) \) by (A-5), we get the difference between (A-1) and (A-2) to be positive, proving the claim.
7.2 Proof of Lemma 2

Lemma 2 The first stage drop-out points $\theta_i, i = 1, \ldots, n$ cannot be of the form $\theta(x_i)$, a monotonic function in bidder $i$’s signal.

Suppose $\theta_j = \theta(x_j)$ is a monotonic function in $x_j$. Then, since $\theta(\cdot)$ is invertible, at the beginning of the second stage of the auction, the signals of bidders $i = 1, \ldots, n - 1$ become common knowledge. Then, the second-stage bid functions will be the expected value of the object given the information available to the bidders:\footnote{Since bidders are risk neutral, this follows from exactly the same argument behind sincere bidding in private value second-price auctions: if, given the information available, I bid higher than my conditional expectation of the value of the item, then on average, I make a loss. If, on the other hand, I bid lower than my conditional expectation, then I lower the probability I will win, but since my payment does not change (since it is the bid of the second highest bidder), my expected profit is lower.}

$$b_{i \neq n} = \max \{ \theta_i, E[v|x_1 = \theta^{-1}(\theta_1), \ldots, x_{n-1} = \theta^{-1}(\theta_{n-1}), x_n \geq \theta^{-1}(\theta_{n-1})] \}$$

$$b_n = \max \{ \theta_n, E[v|x_1 = \theta^{-1}(\theta_1), \ldots, x_{n-1} = \theta^{-1}(\theta_{n-1}), x_n = x_n] \}.$$ 

Observe that this results in identical bids for bidders $i \neq n$, provided that their signals are high enough, since they all form the same expectation of the common value. The expectation formed by the highest bidder in the first stage is a little different, since she knows her own value with certainty.

In this situation, there is a profitable deviation. If bidder $j$ decreases her drop-out point in the first stage to $\hat{\theta}_j < \theta_j$, then in the second stage

$$b_{i \neq j}(\hat{\theta}_j) \leq b_{i \neq j}(\theta_j),$$

since the conditional expectations are decreasing in $\theta_j$ by Milgrom and Weber (1982)’s Theorem 5. But, because all other bidders are bidding sincerely in the first stage, $j$’s bid will not change and she will unilaterally increase her probability of winning the auction.

7.3 Proof of Proposition 3

a) Without loss of generality, focus on the decision of bidder 1. Suppose the bidders $j \neq i$ adopt strategies $b_j = b^*(x_j, \lambda)$. Then the highest bid among them, conditional on there being $n$ bidders, will be $W^{(n)} = \max_{2 \leq j \leq n} b_j(x_j)$. Since $b^*$ is increasing in $x$, $W^{(n)} = b^*(y_1^{(n)})$ and the expected payoff of bidder 1 from bidding $b$ will be

$$E_n[\mathbb{E}(v - b^*(y_1^{(n)})) \mathbb{1}_{b^*(y_1^{(n)}) < b}] | x_1 = x, n \geq 2] \Pr(n \geq 2) + E[v|x_1 = x] \Pr(n = 1),$$
where \( \chi \) is the indicator variable for the event that bidder 1 wins. This is equal to

\[
E_n \left[ \frac{1}{\mathcal{F}} \int_{b^{-1}(b)} \left( v(x, \alpha, n) - b^*(\alpha, \lambda) f_{y_{i|n}}(\alpha|x) d\alpha \right) | n \geq 2 \right] \Pr(n \geq 2) + E[v|x_1 = x] \Pr(n = 1)
\]

\[
= \int_{b^{-1}(b)} \left\{ \sum_{n=2}^{\hat{N}} p(n, \lambda) v(x, \alpha, n) f_{y_{i|n}}(\alpha|x) - b^*(\alpha, \lambda) \sum_{n=2}^{\hat{N}} p(n, \lambda) f_{y_{i|n}}(\alpha|x) \right\} d\alpha \Pr(n \geq 2)
+ E[v|x_1 = x] \Pr(n = 1)
\]

where,

\[
f_{y_{i|n}}(x|x) = \Pr(y_{i|n} = x|x_i = x)
= \int_{v} f_{y_{i|n}}(x|v)f(v|x)dv
= \int_{v} (n - 1) f_{x}(x|v)F_{x}^{n-2}(x|v)f_{v}(v|x)dv
\]

and \( f_{v}(v|x) \) is the posterior density of the common value given the signal.

The first order condition with respect to \( b \) yields:

\[
0 = \sum_{n=2}^{\hat{N}} p(n, \lambda) v(x, x, n) f_{y_{i|n}}(x|x) - b^*(x, \lambda) \sum_{n=2}^{\hat{N}} p(n, \lambda) f_{y_{i|n}}(x|x);
\]

solving, we get \( b^*(x, \lambda) \) as above.

A sufficient condition for \( b^*(x, \lambda) \) to be non-decreasing is the log-supermodularity condition.

\[
\frac{\partial}{\partial v} \log \left( \frac{f(x|v)}{1 - N \lambda [1 - F(x|v)]} \right) \geq 0.
\]

b) The “screening level” argument in the case where there are a deterministic number of bidders follows from Milgrom and Weber (1982): bidders with signal level below \( x^*(r, n) = \inf \{ E[v|x_i = x, y_i < x, n] \geq r \} \) do not participate in the auction. McAfee, Quan, and Vincent (2000) extend this to the case in which there a stochastic number of bidders. In the case analyzed here, the “expected screening level” becomes

\[
x^*(r, \lambda) = \inf \{ E_n E[v|x_i = x, y_i < x, n] \geq r \}. A sufficient condition for \( b(x, \lambda) \) to be increasing is given by McAfee, Quan, and Vincent (2000) to be

\[
\frac{\partial}{\partial v} \log \left( \frac{f(x|v)}{1 - N \lambda [1 - F(x|v)]} \right) \geq 0 \text{ for all } x \geq x^*(r, \lambda).
\]

c) Follows from 2b).

### 7.4 Proof of Proposition 4

The easiest way to see why this should hold true is to look at the von Neumann-Morgenstern (vNM) utility of a particular bidder \( i \) (the payoff given that other bidders submit bids that are monotonic in their signals,
The objective function that bidder \( i \) maximizes is the expectation of the above vNM utility and the bid he submits is the maximizer

\[
\hat{b}_i = \arg \max_b E_{\hat{\mu}, \hat{\sigma^2}} u_i(b, \hat{\mu}, \hat{\sigma^2}, x_{-i}, n) = \frac{k\mu + \sum_{j=1}^{n} x_{j}}{n+k} - \max_{b \geq \max \{b_{-i}(x_{-i}) \}} \left( \frac{k\mu + x_i + \sum_{j=1}^{n} x_{j}}{n+k} - \max \{b_{-i}(x_{-i}) \} \right).
\]

The bid that maximizes bidder \( i \)'s expected utility in an auction where the signals are scaled linearly as \( r_1 x + r_2 \) and where all other bidders follow linearly scaled strategies \( r_1 b_{-i}(x_{-i}) + r_2 \) is

\[
\hat{b}'_i = \arg \max_b E_{\hat{\mu}, \hat{\sigma^2}} u_i(\hat{b}'_i, r_1 \hat{b}_{-i}(x_{-i}) + r_2, r_1 x_i + r_2, r_1 x_{-i} + r_2, n) = \frac{k\mu + x_i + \sum_{j=1}^{n} x_{j}}{n+k} - \max_{b \geq \max \{b_{-i}(x_{-i}) \}} \left( \frac{k\mu + x_i + \sum_{j=1}^{n} x_{j}}{n+k} - \max \{b_{-i}(x_{-i}) \} \right).
\]

Comparing the last expression with the last line in (15), we see that \( \frac{\hat{b}' - r_2}{r_1} = \hat{b}_i \), i.e. \( \hat{b}'_i = r_1 \hat{b}_i + r_2 \). Hence, the bids follow the same linear transformation that the signals are scaled with.

### 7.5 Properties and computation of the bid function

Although we have characterized the equilibrium bid functions pertaining to the eBay auctions, to use these findings in an econometric estimation procedure, we have to evaluate the bid functions explicitly. For general distributions of \( x \) and \( v \), the expectation terms \( v(x, x, n) \) and the expression defining the screening level have to be evaluated numerically. Fortunately, if we assume that \( v \sim N(\mu_v, \sigma^2_v) \) and \( x | v \sim N(v, k \sigma^2_v) \), we can compute these integrals with very high accuracy using the Gauss-Hermite quadrature method (see Judd (1998) and Stroud and Secrest (1968) for details).

Figure 9 plots the bid functions, \( b(x, \lambda) \), in an auction where \( \mu = 1, \sigma = 0.6 \) and \( k = 0.3 \) for different values of \( \lambda \). We see that the functions look quite linear (with a slope of about 0.85) and are indeed increasing.
in $x$. More interestingly, these plots also show that bids are also decreasing in $\lambda$, the expected number of bidders in the auction. So the “winner’s curse” does increase with the number of competitors.

FIGURE 9 ABOUT HERE.

The expected screening level, $x^*(r, \lambda) = \inf \{ E_N E[v | x_i = x, x_i < x, N = n] \geq r \}$, can be calculated as the solution of

$$p(1, \lambda) E[v | x] + \sum_{n=2}^{\infty} p(n, \lambda) \int_{-\infty}^{x} \frac{v f_2^2(x^* | v) F_2^{n-1}(x^* | v) f_c(v) dv}{\int_{-\infty}^{x} f_2(x^* | v) F_2^{n-1}(x^* | v) f_c(v) dv} = r. \tag{A-9}$$

Once again, normality assumptions allow us to compute the integrals in the above expressions with very high accuracy using the Gaussian quadrature method. However, we also need to solve for $x^*$ in expression (A-9) using a numerical procedure. We use a simple Newton-Raphson algorithm to find this solution.

Figure 10 plots the screening level as a function of the minimum bid for the auction with $\mu = 1, \sigma = 0.6$ and $k = 0.3$. The screening level appears to depend on the minimum bid in a similar, near-linear fashion. From the two cases, $\lambda = 3$ and $\lambda = 5$, that are plotted in this figure, we also see that the screening level is decreasing in $\lambda$ and, as shown by Milgrom and Weber (1982), is above the minimum bid.

FIGURE 10 ABOUT HERE.

7.6 Numerical computation of the inverse bid function

Likelihood function evaluations require that we find the value of the inverse bid function, $\phi$, for each bid in auction $t$. Since a closed form expression for the inverse bid function does not exist, this potentially means that we will have to evaluate the bid function $b(x | \mu_t, \sigma_t, k, \lambda_t)$ at least a few times for each bid to find the inverse of $b(\cdot)$ using a numerical algorithm like Newton-Raphson. However, as we discovered while working out the closed forms for equilibrium bid functions, this requires the numerical evaluation of integrals. This is a costly step since we have to evaluate the full-likelihood about 30,000 times in our posterior simulation and with 1500 bids in the data, the integrals would have to be evaluated at least 45 million times.

To reduce the computational complexity, we first exploit the linearity property of the bid functions. Recall that linearity implies that if $b(x | \mu^o, \sigma^o, k, \lambda)$ is a pure strategy equilibrium bid function to the auction where the common value is distributed normally with mean $\mu^o$ and standard deviation $\sigma^o$, then $r b_1(r_1 x + $
$r_2|\mu^0, \sigma^0, k, \lambda) + r_2$ is the equilibrium bid function in the auction where the common value is distributed with mean $r_1\mu^0 + r_2$ and standard deviation $r_1\sigma^0$. This result holds regardless of whether there is a known or random number of bidders in the auction. Similarly, if there is a posted reserve price in the auction, we can apply the linear transformation to the screening level $x^*(R, \lambda, k)$. That is, if $x^*(R; \mu^0, \sigma^0, \lambda, k)$ is the screening level in the original auction, then $r_1x^*(r_1R + r; |\mu^0, \sigma^0, k, \lambda) + r_2$ will be the screening level in the transformed auction.

How does linearity help us? Suppose at a particular evaluation of the likelihood function, the distributional parameters are $\{\mu_t, \sigma_t\}$ for auction $t$. Then we do not have to evaluate $b(x|\mu_t, \sigma_t, k, \lambda)$. Setting $r_{1t} = \frac{\sigma}{\sigma_t}$ and $r_{2t} = \mu_t - r_{1t}\mu^0$, we interpolate the value of $b(\frac{x - \mu_t}{\sigma_t}|\mu^0, \sigma^0, k, \lambda)$ from a set of pre-computed values and return $r_{1t}b(\frac{x - \mu_t}{\sigma_t}|\mu^0, \sigma^0, k, \lambda) + r_{2t}$. Assuming the interpolation is good enough, this scheme lets us avoid computing the Nash equilibrium in the course of likelihood function evaluations. The equilibrium is solved beforehand for a normalized set of parameter values and adapted to the parameter values generated by the posterior simulation routine.

When inverting the bid function, one can use a simple algorithm like Newton-Raphson with relative ease, since the bid function is monotonic and $b(x) = x$ is usually a good starting value. Alternatively, one can evaluate $b(x|\mu^0, \sigma^0, k, \lambda)$ on the grid used for interpolation and instead of using polynomial interpolation to find $b$ as a function of $x, k, \lambda$, one can use polynomial interpolation to evaluate $x$ as a function of $b, k, \lambda$. This allows one to approximate the inverse bid function directly, instead of having to evaluate the bid function several times until Newton-Raphson converges. In fact, this method is much more efficient in terms of calculating the derivative of the inverse bid function (the Jacobian), since the derivative of the polynomial interpolation can be written out in closed form, and once again, be evaluated in one step.

### 7.7 Posterior simulation

Let

\[ X_t = (\text{SECRET}_{t}, \text{BOOKVAL}_{t}, \text{BLEMISH}_{t}, \text{NEGATIVE}_{t}, \text{MINBID}_{t}) \]

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38 The computational simplification from linearity can be exploited fully only when we have precomputed values for all possible values of $x, k$ and $\lambda$. Since this is not possible in a computational setting, we precompute the bid function for the “base-case” auction (where the CV is distributed $N(\mu^0, \sigma^0)$) at a large number grid points $x \in \{x_0, x_1, \ldots, x_{N_x}\}, k \in \{k_0, k_1, \ldots, k_{N_k}\}, \lambda \in \{\lambda_0, \ldots, \lambda_{N_\lambda}\}$ and perform a polynomial interpolation to calculate $b(\frac{x - \mu^0}{\sigma^0}|\mu^0, \sigma^0, k, \lambda)$ at intermediate values. We took care to restrict the domain of interpolation to a region that is a rather tight cover of the region covered by simulation draws to ensure good performance.
be the auction-specific data vector and let \( X \) denote the vector \( X = (X_1, \ldots, X_T) \), where \( T \) is the total number of auctions. Let \( p(\beta) \) denote the prior distribution of the parameters, \( L(BID|\beta, X) \) denote the likelihood function and \( p(\beta|BID, X) \) denote the posterior distribution of the vector of parameters \( \beta \) conditional upon the observed data. By Bayes theorem:

\[
p(\beta|BID, X) \propto p(\beta)L(BID|\beta, X).
\]

That is, the posterior is proportional to the prior times the likelihood function. In many problems, we are interested in the expected value of some known function of the parameters which we represent as \( g(\beta) \).

\[
E[g(\beta)|BIDS_t, X_i] = \int g(\beta)p(\beta|BID, X)d\beta
\]

\[
= \frac{\int g(\beta)p(\beta)L(BID|\beta, X) d\beta}{\int p(\beta)L(BID|\beta, X) d\beta}
\]

Examples include the posterior mean and standard deviation of \( \beta_1 \) or the expected revenue from an auction with a particular set of characteristics \( X \).

Recent work in Bayesian econometrics has developed methods to simulate the posterior distribution, so that integrals such as \( E[g(\beta)|BIDS_t, X_i] \) can be evaluated using Monte Carlo simulation. Posterior simulators generate a sequence of \( S \) pseudorandom vectors \( \beta^{(1)}, \ldots, \beta^{(S)} \) from a Markov chain with the property that the invariant distribution of the chain is the posterior distribution \( p(\beta|BIDS_t, X_i) \). Under regularity conditions discussed in Geweke (1997) (that are trivially satisfied in our application), it can be shown that:

\[
\lim_{S \to \infty} \frac{\sum_{s=1}^{S} g(\beta^s)}{S} = \int g(\beta)p(\beta|BID, X)d\beta.
\]

Therefore, the sample mean \( \frac{\sum_{s=1}^{S} g(\beta^s)}{S} \) is a consistent estimator of \( E[g(\beta)|BID, X] \).

### 7.7.1 Priors

We assume that the prior is proportional to a constant if \( \{0 < \beta_1 < 3\}, \{0 < \beta_2 < 2\}, \{0 < \beta_3 < 2\}, \{0 < \beta_4 < 2\}, \{0 < \beta_5 < 4\}, \{0 < \beta_6,7,8,9,10 < 5\} \) and is equal to zero otherwise. We impose these flat priors to minimize the effect of prior choice on the shape of the posterior distribution. As for
the appropriateness of the cutoff points, we made sure that these contained the “reasonable bounds” for the possible realizations of the parameters.

7.7.2 Posterior simulation

We used a variant of the Metropolis-Hastings algorithm to simulate the posterior distribution of the parameters. We start out by finding a rough approximation to the posterior mode which we will refer to as \( \hat{\beta} \). The algorithm generates a sequence of \( S \) pseudorandom vectors \( \beta^{(1)}, \ldots, \beta^{(S)} \) from a Markov chain as follows. First set \( \beta^{(1)} = \hat{\beta} \), then generate the other simulations as follows:

1. Given \( \beta^{(n)} \) draw a candidate value \( \tilde{\beta} \) from a normal distribution with mean \( \beta^{(n)} \) and variance-covariance matrix \( -H^{-1} \).
2. Let
   \[ \alpha = \min\{ \frac{p(\tilde{\beta}|BID, X)}{p(\beta^{(n)}|BID, X)}, 1 \} . \]
3. Set \( \beta^{(n+1)} = \tilde{\beta} \) with probability \( \alpha \).
4. Return to 1.

Geweke (1997) provides sufficient conditions for the invariant distribution of this chain to be the posterior distribution. It is easy to check that these conditions are satisfied in this application.
References.


You are bidding on 6 morgan silver dollars, dates are 1921P,D,S, 1884-O, 1885, 1899-O. I'll make this a true auction and start the bid at $1.00 with "NR". $5.00 covers S/H plus $1.10 for insurance if wanted. Coins grade from VF to AU. Happy Bidding.
## Payment Details

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CHECKS ONLY PLEASE

### Bidding

**6 MORGAN SILVER DOLLARS "NR"
Item # 1320292746**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current bid</td>
<td>$59.01</td>
</tr>
<tr>
<td>Bid increment</td>
<td>$1.00</td>
</tr>
<tr>
<td>Your maximum bid</td>
<td></td>
</tr>
<tr>
<td>(Minimum bid: $60.01)</td>
<td></td>
</tr>
</tbody>
</table>

[Review bid](#)

**How to Bid**

1. [Register to bid](#) - if you haven't already. It's free!
2. [Learn about this seller](#) - read feedback comments left by others.
3. [Know the details](#) - read the item description and payment & shipping terms closely.
4. If you have questions - contact the seller geno_coins before you bid.
5. Place your bid!

**eBay purchases are insured.**
Figure 2: Histogram of bid submission times

Histogram of bid submission times

Fraction of auction duration

Frequency
Figure 3: Actual and simulated bid distributions

Figure 4: Actual vs. simulated bid dispersion within an auction
Figure 5: Illustration of the winner’s curse effect

Bid functions for the representative auction

- $b(x)$
- $b(x)$ with an extra bidder
- $b(x)$ with $\sigma = 0.1$
- $b(x) = x$

Figure 6: Expected number of entrants for an entry cost of $3.20

Equilibrium expected number of bidders

Minimum bid/Bookvalue

Equilibrium expected number of bidders

Minimum bid/Bookvalue
Figure 7: Expected revenue from minimum bid policy

Figure 8: Expected revenues from secret reserve vs. minimum bid for different book values
Figure 9: Bid functions computed for different $\lambda$

![Bid functions for different values of lambda (exp. no. of bidders)](image)

Figure 10: The calculated “screening level” in a common value second price auction with a reserve price

![Screening level as a function of minimum bid](image)
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Book value</td>
<td>$47.51</td>
<td>$212.63</td>
<td>$3</td>
<td>$3700</td>
</tr>
<tr>
<td>Highest bid/Book</td>
<td>0.81</td>
<td>0.43</td>
<td>0</td>
<td>1.48</td>
</tr>
<tr>
<td>Winning bid/Book (cond. on sale)</td>
<td>0.96</td>
<td>0.28</td>
<td>0.16</td>
<td>1.48</td>
</tr>
<tr>
<td>% Sold</td>
<td>0.85</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Shipping and Handling</td>
<td>$2.18</td>
<td>$.92</td>
<td>$1</td>
<td>$5</td>
</tr>
<tr>
<td>% Blemished</td>
<td>0.10</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Number of bidders</td>
<td>3.05</td>
<td>2.46</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Minimum bid</td>
<td>$16.28</td>
<td>$28.59</td>
<td>$0.01</td>
<td>$230</td>
</tr>
<tr>
<td>% Secret Reserve</td>
<td>0.16</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Seller’s overall reputation</td>
<td>203.06</td>
<td>208.43</td>
<td>0</td>
<td>973</td>
</tr>
<tr>
<td>Seller’s negative reputation</td>
<td>0.43</td>
<td>1.63</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>Bidders’ average overall rating</td>
<td>41.22</td>
<td>38.40</td>
<td>0</td>
<td>261.5</td>
</tr>
</tbody>
</table>

Table 2: Quantiles of Book Value Across Auction Formats

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Secret Reserve</th>
<th>Posted Reserve</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>4</td>
<td>4.75</td>
</tr>
<tr>
<td>10%</td>
<td>6.75</td>
<td>6</td>
</tr>
<tr>
<td>25%</td>
<td>9</td>
<td>8.5</td>
</tr>
<tr>
<td>50%</td>
<td>42.5</td>
<td>13.5</td>
</tr>
<tr>
<td>75%</td>
<td>148.5</td>
<td>22.25</td>
</tr>
<tr>
<td>90%</td>
<td>620</td>
<td>41.25</td>
</tr>
<tr>
<td>95%</td>
<td>910</td>
<td>56</td>
</tr>
<tr>
<td>99%</td>
<td>3700</td>
<td>200</td>
</tr>
</tbody>
</table>
Table 3: Determinants of bidder entry - Poisson regression

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Number of bidders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum bid*(Secret reserve)</td>
<td>$-0.883^*$</td>
</tr>
<tr>
<td></td>
<td>(0.329)</td>
</tr>
<tr>
<td>Minimum bid*(1-Secret reserve)</td>
<td>$-1.571^*$</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
</tr>
<tr>
<td>Secret Reserve</td>
<td>$-0.162$</td>
</tr>
<tr>
<td></td>
<td>(0.125)</td>
</tr>
<tr>
<td>ln(Book value)</td>
<td>$0.123^*$</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
</tr>
<tr>
<td>Blemish</td>
<td>$-0.062$</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
</tr>
<tr>
<td>ln(Negative reputation)</td>
<td>$-0.248^*$</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
</tr>
<tr>
<td>ln(Overall reputation)</td>
<td>$0.108^*$</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
</tr>
<tr>
<td>Constant</td>
<td>$1.102^*$</td>
</tr>
<tr>
<td></td>
<td>(0.148)</td>
</tr>
<tr>
<td>No. of observations</td>
<td>407</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Table 4: Determinants of auction prices

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Maximum bid in auction/book value</th>
<th>All Auctions (OLS)</th>
<th>≥ 1 bidder (OLS)</th>
<th>All Auctions (Tobit)</th>
<th>≥ 2 bidder (OLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bidders</td>
<td></td>
<td>0.0989*</td>
<td>0.0661*</td>
<td>0.1104*</td>
<td>0.0553*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0098)</td>
<td>(0.0071)</td>
<td>(0.0114)</td>
<td>(0.0085)</td>
</tr>
<tr>
<td>Minimum bid/bookvalue</td>
<td></td>
<td>0.1351*</td>
<td>0.5241*</td>
<td>0.0896</td>
<td>0.4744*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0595)</td>
<td>(0.0482)</td>
<td>(0.0704)</td>
<td>(0.0579)</td>
</tr>
<tr>
<td>Secret Reserve</td>
<td></td>
<td>−0.1135*</td>
<td>−0.3007</td>
<td>−0.1299*</td>
<td>−0.0596</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0554)</td>
<td>(0.3758)</td>
<td>(0.0641)</td>
<td>(0.0407)</td>
</tr>
<tr>
<td>Blemish</td>
<td></td>
<td>−0.1458*</td>
<td>−0.0842</td>
<td>−0.1594*</td>
<td>−0.1054*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0613)</td>
<td>(0.0432)</td>
<td>(0.0713)</td>
<td>(0.0495)</td>
</tr>
<tr>
<td>ln(Negative reputation)</td>
<td></td>
<td>−0.0597</td>
<td>−0.0134</td>
<td>−0.0624</td>
<td>−0.0018</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0421)</td>
<td>(0.0296)</td>
<td>(0.0489)</td>
<td>(0.0353)</td>
</tr>
<tr>
<td>ln(Overall reputation)</td>
<td></td>
<td>0.0893*</td>
<td>0.3262*</td>
<td>0.1033*</td>
<td>0.0383*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0149)</td>
<td>(0.0109)</td>
<td>(0.0175)</td>
<td>(0.0128)</td>
</tr>
<tr>
<td>Avg. bidder experience</td>
<td></td>
<td>−</td>
<td>−0.0001</td>
<td>−</td>
<td>0.0002</td>
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<tr>
<td></td>
<td></td>
<td>(0.0003)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Early bidding activity</td>
<td></td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>0.0777</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.0604)</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>0.0476</td>
<td>0.2914*</td>
<td>−0.0523</td>
<td>0.2702*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0875)</td>
<td>(0.0653)</td>
<td>(0.1028)</td>
<td>(0.0993)</td>
</tr>
<tr>
<td>No. of observations</td>
<td></td>
<td>407</td>
<td>345</td>
<td>407</td>
<td>278</td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td>0.3134</td>
<td>0.3436</td>
<td>0.2427</td>
<td>0.2926</td>
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</tbody>
</table>
Table 5: Regression of bid/bookval on the number of bidders

<table>
<thead>
<tr>
<th>Specification</th>
<th>Linear</th>
<th>Quadratic</th>
<th>Logarithmic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b = \beta_0 + \beta_1 N$</td>
<td>$b = \beta_0 + \beta_1 N + \beta_2 N^2$</td>
<td>$\ln(b) = \beta_0 + \beta_1 \ln N$</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>.9836 (.0202)</td>
<td>1.0932 (.0354)</td>
<td>- .0420 (.033)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-.0342 (.0035)</td>
<td>-.0804 (.0128)</td>
<td>-.0572 (.005)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>N/A</td>
<td>.0038 (.0010)</td>
<td>N/A</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0625</td>
<td>0.072</td>
<td>0.065</td>
</tr>
</tbody>
</table>

Table 6: Posterior Means and Standard Deviations of Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
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<tbody>
<tr>
<td>$\mu$</td>
<td>$BOOKVAL$</td>
<td>0.9911</td>
<td>0.0387</td>
</tr>
<tr>
<td></td>
<td>$BOOKVAL \ast BLEMISH$</td>
<td>-0.1446</td>
<td>0.1628</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$BOOKVAL$</td>
<td>0.5599</td>
<td>0.0260</td>
</tr>
<tr>
<td></td>
<td>$BOOKVAL \ast BLEMISH$</td>
<td>0.0326</td>
<td>0.0260</td>
</tr>
<tr>
<td>$k$</td>
<td>$CONSTANT$</td>
<td>0.2545</td>
<td>0.0259</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$SECRET$</td>
<td>1.4069</td>
<td>0.0897</td>
</tr>
<tr>
<td></td>
<td>$LN(BOOK)$</td>
<td>-0.2857</td>
<td>0.1080</td>
</tr>
<tr>
<td></td>
<td>$NEGATIVE$</td>
<td>0.0883</td>
<td>0.0276</td>
</tr>
<tr>
<td></td>
<td>$SECRET \ast \frac{MINBID}{BOOKVAL}$</td>
<td>-0.0414</td>
<td>0.0228</td>
</tr>
<tr>
<td></td>
<td>$(1 - SECRET) \ast \frac{MINBID}{BOOKVAL}$</td>
<td>-0.2514</td>
<td>0.3268</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.7893</td>
<td>0.0864</td>
</tr>
</tbody>
</table>