Competition Versus Collusion in Procurement Auctions: Identification and Testing.

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Abstract

In this research, we develop an approach to the problem of identification and testing for bid-rigging in procurement auctions that tightly integrates economic theory and econometric practice. First, we introduce a general auction model with asymmetric bidders. We show how asymmetries can arise because of location, capacity constraints and collusion. Second, we study the problem of identification in our model. We state a set of conditions that are both necessary and sufficient for an observed set of bids to be generated by a model with competitive bidding. Third, we demonstrate how to test the conditions that characterize competitive bidding and apply these tests to a data set of bidding for procurement contracts.

1 Introduction.

Bid-rigging is a serious problem in many procurement auctions. According to Pesendorfer (1996), bid-rigging accounts for 50 percent of the cases filed by the Justice Department’s anti-trust division that result in a criminal conviction. There have been many instances of bid-rigging in procurement auctions. According to Engineering News-Record, criminal bid-rigging cases have recently been filed in New York City and Chicago for school construction, bridge repair, interior remodeling, paving and many other types of construction. In the New York cement industry in the 1980’s, organized crime turncoats alleged that the Mafia designed an elaborate bid-rigging scheme that inflated building costs, making, for instance, the price of poured concrete the highest in the nation.\textsuperscript{2}

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\textsuperscript{2} In his biography, Mafia turncoat Sammy (The Bull) Gravano, a former member of the "Concrete Club" stated, "If one
In this research, we develop an approach to the problem of identification and testing for bid-rigging in procurement auctions that tightly integrates economic theory and econometric practice. Our research contributes to the existing literature in several ways. First, we introduce a general auction model with asymmetric bidders. We show how asymmetries can arise because of location, capacity constraints and collusion. Second, we study the problem of identification in our model. We state a set of conditions that are both necessary and sufficient for a distribution of bids to be generated by a model with competitive bidding. Third, we have collected a unique data set of bidding by construction firms for “seal coating” contracts. Our data set has complete coverage of a three state market over a five year time period. Fourth, we demonstrate how to test the conditions that characterize competitive bidding and apply these tests to our data set.

In this paper, we first present a class of auction models with asymmetric bidders. Since there is relatively little theoretical work on asymmetric auctions compared to their symmetric counterparts, we restate some key results from the literature. First, we show it is possible to characterize the equilibrium to the asymmetric auction model as the solution to a system of differential equations with boundary conditions. Second, we show that the solution to this system exists and is unique.

We then turn to the problem of identification. We state a set of conditions that are both necessary and sufficient for a distribution of bids to arise from a model of competitive bidding. A first condition implied by models of competitive bidding is conditional independence. That is, conditional on all publicly observable cost information and cost parameters, firms’ bids should be independent. A second condition that must hold is that bids are exchangeable. That is, if any subset of firms’ observable costs parameters are permuted then the empirical distribution of bids must also permute. While conditional independence and exchangeability are implied by competitive bidding, we demonstrate that there are models of collusive bidding where these properties fail. Rejection of a test of conditional independence and exchangeability would reject the class of competitive bidding models we study with one of the alternative hypotheses being collusion. Our identification results extend the techniques proposed by Guerre, Perrigne and Vuong (2000) to a context with asymmetric bidders.

of them (contractor) gets a contract for, say, thirteen million, the next thing you know, after he knows he’s got it, he jacks up the whole thing before it’s over to a sixteen-or seventeen-million-dollar job. Now he’s increased the cost thirty-three percent. So our greed (the Mafia) is compounded by the greed of them so-called legitimate guys (contractors).” Maas (1997), p. 271.

As we discuss in the text, conditional independence will also hold in common value models if the common value is a conditioning argument. Since the econometrician has ex-post information on all of the bids, we argue that our analysis would hold in this case as well.
We test for conditional independence and exchangeability in an extensive data set of bidding by construction firms for doing a type of road repair work called “seal coating” in Minnesota, North Dakota and South Dakota. Our data set contains nearly every bid submitted in the entire industry along with detailed project descriptions for nearly all federal, state and local government contracts. Testing for collusion is especially interesting in this industry since the owners of a number of the largest firms in the market have previously been sanctioned for bid-rigging. Furthermore, the owner of the largest firm in the market spent one year in prison in the mid-1980’s for bid-rigging.

There have been a number of recent empirical papers on the subject of bid-rigging. A first set of empirical papers describe the observed bidding patterns of cartels and compare cartel to non-cartel bidding behavior. Porter and Zona (1993, 1999) and Pesendorfer (1996) both analyze data sets where it is known that bid-rigging has taken place. These papers find the following empirical regularities: First, cartel members tend to bid less aggressively than non-cartel members. Second, the bids of cartel members tend to be more correlated with each other than with the bids of non-cartel members. Third, collusion tends to increase prices as compared to a non-collusive control group.

A second set of empirical papers, such as Porter and Zona (1993) and Baldwin, Marshall and Richard (1997) propose econometric tests designed to detect collusive bidding. Baldwin, Marshall and Richard (1997) nest both competition and collusion within a single model to test for collusion. Their model is applicable for oral or second-price auctions with private values. Porter and Zona (1993) propose a procedure where two models of bidding are estimated. The first model is a logistic regression on the identity of the lowest bidder. The second model is an ordered logit regression on the ranking of all the bidders. Under the null hypothesis of no collusion, the parameter values from the two models should be equal. We believe that our work sheds new light on the analysis of Porter and Zona (1993, 1999) and Baldwin, Marshall and Richard (1997) by demonstrating how observable differences across firms such as location and capacity play a key role in the identification of collusion. In fact, we believe that in both of these papers, failures of conditional independence and exchangeability are key components of how the authors identify suspicious patterns of bidding.

In a companion paper, Bajari and Ye (2001), we propose a second, complementary approach to testing for collusion based on using structural estimation. We specify two theoretical models: one of competitive bidding and a second where two firms collude. Based on discussions with leading firms and regulators in the industry, we specify as prior distribution over the structural parameters in the industry. We then compute the posterior probabilities of the competitive model and collusive model conditional on the observed data. We believe that taken together, the tests in this paper and Bajari and Ye (2001) provide a useful diagnostic for detecting suspicious bidding behavior in a variety
of procurement auctions.

The paper is organized as follows. Section 2 to 4 present the general model with asymmetric bidders and characterize the equilibrium. In Section 5 we state and prove the necessary and sufficient conditions for identifications. In Section 6 we describe the salient features of our data set and the tests for conditional independence and exchangeability are conducted. Section 7 concludes.

2 A Model of Procurement Auctions

This section develops a model of competitive bidding for a contract to build a single and indivisible public works project. The framework we develop is appropriate for modeling bidding behavior in our seal coating data set as well as bidding for many other types of procurement contracts. In the model, firms submit sealed bids and the contract is awarded to the lowest bidder. Before bidding begins, each firm forms a cost estimate for completing the project, which is private information. Firms are risk neutral and have private values.

In most auction models, it is assumed that firms are symmetric, that is, their private cost estimates are independently and identically distributed. However, this is not an appropriate assumption for the seal coat industry as well as many other procurement auctions. In the seal coat industry, there are at least four sources of asymmetries. The first form of asymmetry is location. Firms that are closer to a construction project are more likely to submit a bid and all else held constant their bids are lower. This is due, in large part, to transportation costs. The second source of asymmetries is capacity utilization. All of the firms in the seal coat industry are small relative to the market as a whole. Therefore, each firm needs to take into account that winning a contract today will limit its available capacity to complete future projects. The third source of asymmetries is different technologies across firms. Ninety-eight firms bid in our data set. Among these, forty-three firms win no contracts and only eighteen have a market share that exceeds one percent. The largest firm in the industry, on the other hand, has a market share of over twenty-percent. Firms clearly differ in their sizes, most likely this is due to technology and managerial efficiency. The fourth form of asymmetry is that firms differ in their success in winning contracts by state. Many firms in our data set complete the majority of their work in a single state. In order to bid successfully, firms must be familiar with local regulations and local procurement officials.

The model is developed in the following steps. First, the information structure of the model is described. Next, both the von Neumann-Morgenstern and expected utility functions for the firms are defined. Lastly, the equilibrium bidding function is characterized. The model of this section is a simple variant of the well-known independent private
values model where firms are allowed to be asymmetric. (See Milgrom and Weber (1982) or McAfee and McMillan (1987) for a discussion of the private values model.)

2.1 Information.

In the model, \( N \) firms compete for a contract to build a project. Before bidding starts, each firm \( i \) forms an estimate of its cost to complete the project. The cost estimate is firm \( i \)'s private information, that is, firm \( i \) knows its own cost estimate but does not know the cost estimates of other firms. The cost estimate for firm \( i \) is a random variable \( C_i \) with a realization denoted as \( c_i \). The random variable \( C_i \) has a cumulative distribution function \( F_i(\cdot; \theta_i) \) and probability density function \( f_i(\cdot; \mu_i) \) where \( \mu_i \) is a vector of parameters. We will let \( \mu_i = (\mu_1, \ldots, \mu_N) \) denote the vector of all firm specific parameters. The cost distribution is assumed to have support \([c, \bar{c}]\) for all firms.

As a simple example, consider a situation where each firm has a different location and hence different transportation costs. Then one natural specification for firm \( i \)'s private information is:

\[
c_i = \text{constant} + \beta_1 \cdot \text{distance}_i + \varepsilon_i
\]  

(1)

In equation (1), \( c_i \), firm \( i \)'s cost estimate is a function of three terms. The first term is a constant, which could reflect attributes of the project that affect all firms identically, such as how many miles of highway must be paved or the tons of concrete that must be poured for the foundation of a new building. The second term, \( \beta_1 \cdot \text{distance}_i \), is different for each firm since each firm has a unique location. The third term, \( \varepsilon_i \), is an independent random variable, which serves to model private information about some component of firm \( i \)'s cost, such as the cost for materials or labor. If \( \varepsilon_i \) is normally distributed with mean zero and standard deviation \( \sigma \) then \( \theta_i = (\text{constant}, \beta_1, \text{distance}_i, \sigma) \).

2.2 The vNM Utility Function.

Let \( b_i \) denote the bid submitted by firm \( i \). If firm \( i \) submits the lowest bid then firm \( i \)'s vNM utility is \( b_i - c_i \), and if it fails to submit the lowest bid then firm \( i \)'s utility is assumed to be 0. If two or more firms submit the same bid, the contract will be awarded at random among the set of low bidders. But ties will never occur with positive probability in equilibrium. Firm \( i \)'s vNM utility function can be written as:

\[
u_i(b_1, ..., b_n, c_i) = \begin{cases} 
  b_i - c_i & \text{if } b_i < b_j \text{ for all } i \neq j \\
  0 & \text{otherwise.}
\end{cases}
\]  

(2)

Firm \( i \) is said to have private values since its utility depends only on \( c_i \) and not the private information of other firms. Armantier, Florens and Richard (1997) and Porter and Zona (1993) both use the assumption of private values
in their models of procurement auctions. If firm specific factors account for the differences in cost estimates, then the assumption of private values is plausible. In the seal coat industry, firms will have a private value component to their costs because labor and material costs will be firm specific and to some extent private information.

2.3 Expected Profit

In the model, firm \( i \)'s strategy is a function \( b_i = b_i(c_i; \theta) \) which maps firm \( i \)'s cost draw, \( c_i \), to a bid \( b_i \) in the interval \([\underline{c}, \overline{c}]\). LeBrun (1994) and Maskin and Riley (1996a,b) have shown that, in equilibrium, the bid functions \( b_i = b_i(c_i; \theta) \) are strictly increasing and differentiable which implies that the inverse bid functions \( \phi_j(b; \theta) = b_i^{-1}(b; \theta) \) are also strictly increasing and differentiable. To simplify the notation, we will often write firm \( i \)'s bid function as \( b_i(c_i) \), suppressing its dependence on the vector of parameters \( \theta \).

In order to win the contract, firm \( i \) must submit the lowest bid. If firm \( i \) submits a bid of \( b_i \), it will win the contract when \( c_j \geq \phi_j(b_i) \) for all \( j \neq i \), that is, all firms \( j \neq i \) have cost draws which cause them to bid more than \( b_i \). When firm \( i \) has a cost draw of \( c_i \), her expected profit from bidding \( b_i \) will be denoted by \( \pi_i(b_i, c_i; \theta) \). The expected profit satisfies:

\[
\pi_i(b_i, c_i; b_{-i}, \theta) = (b_i - c_i)Q_i(b_i; \theta)
\]

where \( Q_i(b; \theta) = \prod_{j \neq i} 1 - F_j(\phi_j(b; \theta); \theta_j) \) is the probability that firm \( i \) is the lowest bidder. As we can see from equation (3), firm \( i \)'s expected profit is a markup times the probability that firm \( i \) is the low bidder. LeBrun (1995a,b,1996) and Maskin and Riley (1996a,b) have demonstrated that an equilibrium to the asymmetric model exists and that the bid functions are strictly increasing and differentiable. Maskin and Riley (1996a,b), LeBrun (1999) and Bajari (1997,2001) have demonstrated that the equilibrium to the model is unique.

3 More General Models

In many empirical applications, static models with competitive bidders may not be sufficiently general. In this section, we demonstrate that models with non-trivial dynamics and collusion can be viewed as special cases of the model developed in section 2.

3.1 Dynamic Bidding

In much of the construction industry, firms must bid in a sequence of auctions over time. Firms are often capacity constrained due to limited physical capital and a limited pool of workers who possess the necessary specialized skills. In a dynamic equilibrium, a firm must account for the fact that winning a project in an early auction will mean that it
has less free capacity for future jobs.

Capacity constraints have important implications for bidding strategies. A stylized fact in many markets is that firms tend to bid more aggressively early in the construction season when they have more unutilized capacity. Also, according to accounts by industry participants, in order not to lose skilled workers, construction firms may even bid at less than cost to prevent their employees from being idle.

We will let \( s \) be a vector denoting the state of the industry and we assume that the state is publicly observed by all participants in the auction. At any given point in time, this vector might include: the capacities for all firms in the industry, the distances of all the firms to the project locations, the market prices of key materials and so forth.

Assume that \( V_{iW}(s) \) is the continuation value attached to winning at state \( s \), and \( V_{iL,j}(s) \) is the continuation value attached to not winning (and that firm \( j \) is the winning firm) at state \( s \). When \( i \) wins the job, its used capacity increases and all the competitors’ capacities remain unchanged for the next period. When \( j \) (\( j \neq i \)) wins the job, firm \( j \)’s used capacity increases and the capacities of all the other firms (including firm \( i \)) remain unchanged for the next period. Note that in general, \( V_{iL,j}(s) \) may not be the same for all \( j \) (\( j \neq i \)).

To simplify the equilibrium analysis, we assume that the option value attached to not winning is the same regardless of the winner’s identity, i.e., we assume that \( V_{iL,L} = V_{iL} \). With this assumption, the firm’s expected payoff will now have the general form:

\[
\pi_i(b, c_i; \theta) = (b - c_i + V_{iW}(s))Q_i(b; \theta) + V_{iL}(s)(1 - Q_i(b; \theta))
\]

(4)

\[
= (b - c_i + V_{iW}(s) - V_{iL}(s))Q_i(b; \theta) + V_{iL}(s)
\]

(5)

Since constants are irrelevant in a firm’s maximization problem, we may write the firm’s maximization problem as:

\[
\max_b (b - c_i + V_{iW}(s) - V_{iL}(s))Q_i(b; \theta)
\]

(6)

Just as in equation (3), a firm’s optimization problem is the difference between its bid and a cost estimate times the probability of winning. The only difference now is that a firm’s cost includes an estimate of the option value of having free capacity available for future periods. Since both \( V_{iW}(s) \) and \( V_{iL}(s) \) are common knowledge to all participants in the game, the above equation is equivalent to maximizing utility in a static auction model with cost \( c_i + V_{iL}(s) - V_{iW}(s) \).\(^4\)

\(^4\) Once option values are included in the analysis, it might be the case that the distributions of costs no longer have common
The key assumption needed for the validity of this analysis is that $V_{iL,j}(s)$ is independent of $j$. While this assumption may not be strictly true in a dynamic model of bidding, our empirical work suggest that this assumption may not be a bad approximation. If firm $i$ was concerned about which firm $j \neq i$ wins the procurement in the event that $i$ does not, firm $i$ should bid differently depending on the capacity utilization of her competitors. However, while a firm’s own available capacity was significant in reduced form bid functions, the capacity of other firms failed to be significant determinant of firm $i$’s bid. This is consistent with firm $i$ being indifferent between which competing firm $j \neq i$ wins the contract.

3.2 Collusion

In the construction industry, firms have found numerous mechanisms for collusion. For example, firms have followed bid rotation schemes to allocate projects, side payments are sometimes made between firms and geographic territories have been established as parts of cartel arrangements.

In this section, we demonstrate that a simple model of collusive bidding where the cartel behaves efficiently is a special case of the model of section 2. Suppose that before the auction begins, all cartel members make cost draws. The cartel members meet before the auction, compare cost draws and the cartel member with the lowest cost draw submits a real bid, while other cartel members either abstain from bidding or submit phantom bids. Let $C \subseteq \{1, 2, ..., N\}$ denote the cartel. The cost to the cartel which we denote as $c_c$ can be denoted as:

$$c_c = \min_{j \in C} c_j$$

(7)

If other bidders are aware of the identity of the cartel, then it is trivial to adapt our previous analysis to the case of a cartel. The cartel is simply modeled as the order statistic of its members’ cost draws.

4 Properties of Equilibrium

In this section, we summarize some theoretical properties of the asymmetric auction model. This section can be skipped by those readers primarily interested in the empirical implementation of our procedures.

In the asymmetric auction model, we assume that firms play a Bayes-Nash equilibrium in pure strategies. Firm $i$ first of all makes a cost draw $c_i$, then taking the cost draw as given, chooses a bid $b_i$ that maximizes (3).

supports across the firms. As we discuss in the next section, this can cause problems for the existence, uniqueness and other characterizations of the equilibrium. The theoretical problem remains, to the best of our knowledge, unresolved. However, if one makes an arbitrarily small perturbation to the information structure so that both firms have the same support, then the existence, uniqueness and characterization theorems still hold. Also see Griesmer, Levitan and Shubik (1967).
Definition. An Equilibrium in pure strategies is a collection of measurable functions $b^*_1, \ldots, b^*_N$ such that for all $i$ and for all $c_i \in [c_1, c_T]$, $b^*_i(c_i)$ maximizes $\pi(b, c_i; b^*_{-i}, \theta)$ in $b$.

The first order condition for maximizing expected profit in our model is:

$$\frac{\partial}{\partial b} \pi_i(b, c_i; \theta) = (b - c_i)Q'_i(b; \theta) + Q_i(b; \theta) = 0$$ (8)

The cost to firm $i$ of increasing her bid is that the probability of winning the auction decreases. This cost is reflected in the term $(b - c_i)Q'_i(b; \theta)$. The benefit to firm $i$ of increasing her bid is, conditional on winning, that the payment to the firm increases, which is reflected in the term $Q_i(b; \theta)$. Equation (8) implies that at a maximum, the marginal benefit to firm $i$’s of increasing her bid is equal to the marginal cost.

The equilibrium to the model can be characterized as the solution to a system of differential equations with boundary conditions. This is done by rearranging equation (8):

$$\frac{\partial}{\partial b} \pi_i(b, c_i; \theta) = \prod_{j \neq i} [1 - F_j(\phi_j(b))] - (b - c_i) \prod_{j \neq i} f_j(\phi_j(b)) \phi'_j(b) \prod_{k \neq j} (1 - F_k(\phi_k(b))) = 0, \quad i = 1, \ldots, N. \quad (9)$$

Equation (9) involves the inverse bid function $\phi_j(b)$ and its derivative $\phi'_j(b)$. Collecting terms and rewriting (9) we can characterize the equilibrium inverse bid functions as the solution to a system of $N$ ordinary differential equations:

$$\phi'_j(b) = \frac{1 - F_i(\phi_i(b))}{(N - 1)f_i(\phi_i(b))} \left[ -\frac{(N - 2)}{b - \phi_i(b)} + \sum_{j \neq i} \frac{1}{b - \phi_j(b)} \right], \quad i = 1, \ldots, N. \quad (10)$$

Throughout our analysis we will impose the following regularity conditions on our model primitives:

- Assumption 1. For all $i$, the distribution of costs $F_i(c_i; \theta)$ has support $[c_1, c_T]$. The probability density function $f_i(c_i; \theta)$ is continuously differentiable in $c_i$.
- Assumption 2. For all $i$, both $f_i(c_i; \theta)$ is bounded away from zero on $[c_1, c_T]$.

LeBrun (1995a,b,1996) and Maskin and Riley (1996a,b) have demonstrated that a set of equilibrium bidding strategies exist under assumptions 1 and 2 and that these strategies are strictly monotone and differentiable.

**Theorem 1** (LeBrun (1995a,b,1996) and Maskin and Riley (1996a,b)). If Assumptions 1 and 2 hold, then an equilibrium in pure strategies exists. Furthermore, the equilibrium is strictly monotone and differentiable.

Another basic result from the theoretical literature is that the inverse bid functions can be characterized as the solution to a system of $N$ differential equations with $2N$ boundary conditions and that the equilibrium is unique.
Theorem 2 (LeBrun (1995a), Maskin and Riley (1996a)) Suppose that Assumptions 1 and 2 hold. Let \( \phi_1(b), ..., \phi_N(b) \) be inverse equilibrium bidding strategies. Then

(i) for all \( i \), \( \phi_i(\tau) = \tau \),

(ii) there exists a constant \( \beta \) such that for all \( i \), \( \phi_i(\beta) = c \),

(iii) for all \( i \) and for all \( b \in [\beta, \tau] \), equation (10) holds.

Theorem 3 (Maskin and Riley (1996a), Bajari (1997), Bajari(2001), LeBrun (1999)) Suppose that assumptions 1 and 2 hold. Then there is a unique equilibrium.

5 Identification

In this section, we state a set of conditions that are both necessary and sufficient for a distribution of bids to be generated by the asymmetric auction model of the previous sections. We will assume that it is possible to write the cumulative distribution function of firm \( i \)'s cost in the form \( F(c_i | z_i) \) where \( z_i \in Z \) is a vector of parameters and covariates that is publicly observable to all other firms. For instance, in equation (1) the vector \( z_i \) would be the tuple \( z_i = (constant, \beta_i, distance_i, \sigma) \). We will let \( z \) denote the vector \( z = (z_1, ..., z_n) \).

Let \( G_i(b; z) \) be the cumulative distribution of firm \( i \)'s bids and \( g_i(b; z) \) be the associated probability density function. Using the Theorems from the previous section, it is straightforward to show that the following conditions must hold in equilibrium.

A1 Conditional on \( z \), firm \( i \)'s bid and firm \( j \)'s bid are independently distributed.

A2 The support of each distribution \( G_i(b; z) \) is identical for each \( i \).

Conditional on \( z \), each firm's signal \( c_i \) is independently distributed and since bids are function of \( c_i \), this implies that A1 must hold in equilibrium. The condition A2 must hold by the characterization Theorem stated in Section 4.

A third condition that must hold in equilibrium is that the distribution of bids must be exchangeable in \( z_i \). Let \( \pi \) be a permutation, that is, a one-to-one mapping from the set \( \{1, ..., N\} \) onto itself. The definition of exchangeability is that for any permutation \( \pi \) and any index \( i \) the following equality must hold:

\[
G_i(b; z_1, z_2, ..., z_N) = G_{\pi(i)}(b; z_{\pi(1)}, z_{\pi(2)}, z_{\pi(3)}, ..., z_{\pi(N)})
\]

Equation (11) implies if the cost distributions for the bidders are permuted by \( \pi \), then the empirical distribution of bids
must also be permuted by \( \pi \). For instance, if we permute the values of \( z_1 \) and \( z_2 \) holding all else fixed exchangeability implies that \( G_1(b) \) and \( G_2(b) \) also permute.

A3 The equilibrium distribution of bids is exchangeable. That is, for all permutations \( \pi \) and any index \( i \)
\[
G_i(b; z_1, z_2, z_3, ..., z_N) = G_{\pi(i)}(b; z_{\pi(1)}, z_{\pi(2)}, z_{\pi(3)}, ..., z_{\pi(N)}).
\]

Equation (11) provides strong testable restrictions on the empirical distribution of the bids across the auctions. As a simple example, suppose that our model of costs is as in equation (1). Then the distance of each firm from the project leads to asymmetries in the cost distributions between firms. Let there be \( t = 1, ..., T \) auctions and let \( DIST_{i,t} \) be the distance of firm \( i \) from the \( t^{th} \) auction. The vector \( z_t = (constant, \beta_1, DIST_{i,t}, \sigma) \) where the constant captures features of the auction that affect each firm’s cost in an identical and observable way. Our theoretical model of bidding implies that the bid functions should depend on the vector \( z = (z_1, ..., z_N) \) and an idiosyncratic term which reflects firm \( i \)’s private information. Up to a first order approximation then we can write the bid function for firm \( i \) as:
\[
b_{i,t} = \beta_t + \beta_{1,i} \cdot DIST_{i,t} + \sum_{j \neq i} \alpha_{i,j} \cdot DIST_{j,t} + \varepsilon_{i,t}
\]  
\[
(12)
\]
In equation (12), \( \beta_t \) is a project fixed effect, \( \beta_{1,i} \cdot DIST_{i,t} \) is the response of firm \( i \)’s bid to \( i \)'s distance and \( \alpha_{i,j} \cdot DIST_{j,t} \) is the response of firm \( i \) to the distance of firm \( j \)\(^5\).

Using the model of bidding in equation (12) and the specification of costs from equation (1), the assumption of exchangeability implies the following restrictions on observed bidding behavior:
\[
\text{For all } i \neq j, \beta_{1,i} = \beta_{1,j} \quad (13)
\]
\[
\text{For all } i \neq j, k \neq i, \alpha_{i,j} = \alpha_{i,k} \quad (14)
\]
\[
\text{For all } i \neq j \text{ and for all } s, \alpha_{i,s} = \alpha_{j,s} \quad (15)
\]
Equation (13) must hold because the model of costs (1) implies that the marginal cost of transportation is identical for all the firms in the industry and therefore, holding all else fixed, the reaction of all firms to an incremental increase in own distance must be the same. Equation (14) implies that firm \( i \) must react symmetrically to an increase in the distance of any of its competitors. Equation (15) implies that both firm \( i \) and firm \( j \) must have the same reaction to an increase in the distance of firm \( s \), holding all else constant. The analysis above is meant to provide the simple intuition behind the empirical implications of exchangeability. The basic ideas can be easily generalized to more complicated specifications for costs than equation (1).

\(^5\) We must of course normalize one of the fixed effects to zero in order for the model to be identified.
Next, we demonstrate the implications of exchangeability using a simple Monte Carlo experiment. Let $c_i$ have a truncated normal distribution with support $[c; \overline{c}] = [2, 8]$. The probability density function for firm $i$’s private information, $f_i(c_i|z_i)$ will then satisfy:

$$f_i(c_i|z_i) \propto \begin{cases} \frac{1}{(2\pi\sigma^2)^{\frac{D}{2}}} \exp\left(-\frac{(c_i-\mu)^2}{2\sigma^2}\right) & c < c_i < \overline{c} \\ 0 & \text{otherwise} \end{cases}$$

(16)

Using the algorithms developed in Bajari (2001), we compute the equilibrium bid functions to explore how the bid, probability of winning and expected profit depend on the $z = (z_1, ..., z_N)$. We set the number of firms equal to three, let $\mu_i$’s vary between 4 and 6 and assume $\sigma_i = \sigma$ for all $i$. The parameter $\sigma$ is allowed to vary between 1 and 2. The bid functions are computed on a symmetric grid containing these parameter points.

We regress firm 1’s bid, profit and probability of winning on $\mu_1, \mu_2, \mu_3, \sigma$ and $c_1$. The regression coefficients are summarized in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bid</th>
<th>Profit</th>
<th>Probability of Winning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>0.1312</td>
<td>0.1111</td>
<td>0.0853</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>0.2079</td>
<td>0.1440</td>
<td>0.0857</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>0.2079</td>
<td>0.1440</td>
<td>0.0857</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-0.0021</td>
<td>-0.0045</td>
<td>-0.0007</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.5522</td>
<td>-0.3028</td>
<td>-0.1871</td>
</tr>
</tbody>
</table>

It is obvious that $\mu_2$ and $\mu_3$ enter firm 1’s bid function (and profit and probability of winning functions as well) in a symmetric way. In other words, the bid functions are exchangeable.

Next, we study the case of collusive bidding. Unlike the previous model, firms 1 and 2 decide to collude before the bidding begins while firm 3 does not participate in the cartel. Firm 1 and 2 make cost draws, $c_1$ and $c_2$ respectively, before the auction begins. The cartel is assumed to operate efficiently so that the low cost firm will submit a “real” bid for the cartel and the other firm will either refrain from bidding or submit a higher “phony” bid. If firm 3 knows that 1 and 2 are in a cartel then the auction can be modeled as an asymmetric auction where the cartel is a single agent who’s cost is the order statistic of firm 1’s cost and firm 2’s cost, $\min\{c_1, c_2\}$. In the table below we report the results of a regression of the cartel’s bid and firm 3’s bid on the structural parameters.6

---

6 The grid of parameters values is the same as in the competitive case.
Bidding With A Cartel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bid For Cartel</th>
<th>Bid For Firm 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_1 )</td>
<td>0.1093</td>
<td>0.1709</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>0.1093</td>
<td>0.1709</td>
</tr>
<tr>
<td>( \mu_3 )</td>
<td>0.2925</td>
<td>0.1595</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.3074</td>
<td>0.2210</td>
</tr>
<tr>
<td>( c_i )</td>
<td>0.6041</td>
<td>0.6064</td>
</tr>
</tbody>
</table>

Now \( \mu_2 \) and \( \mu_3 \) no longer enter firm 1’s bid function in a symmetric way, which is a violation of exchangeability. Another violation of exchangeability is that firm 1’s bid increases 0.1093 units given a unit increase in \( \mu_1 \) whereas firm 3’s bid increases by 0.1595 given a unit increase in \( \mu_3 \).

Using exchangeability to test for collusion relies heavily on the assumption that we can write the probability distribution function for the firm’s cost in the form \( f(c_i|z_i) \) with \( z_i \in Z \). If this is true, then there is a one-to-one mapping from \( z_i \) to the probability distribution function of firm i’s cost. If the econometrician does not observe \( z_i \) or it is not possible to construct a one-to-one mapping from \( Z \) to probability distribution functions then using exchangeability to test for collusion will not be correct.

Even if a cartel is present in the industry, the bid functions of firms who do not collude must still satisfy a certain type of exchangeability. Suppose that the cost structure for all firms in the industry is generated as in equation (1), then the value of \( z_i \) for all the firms who do not collude must be \( z_i = (constant, \beta_1, DIST_{i,t}, \sigma) \). If the first \( m \) firms do collude, then the cost distribution for the cartel, if it colludes efficiently as in the model of section 3, will have cost parameters \( z_c = (constant, \beta_1, DIST_{1,t}, DIST_{2,t}, \ldots, DIST_{m,t}, \sigma) \). The bid functions of the non-colluding firms must be exchangeable in \( z_i \) holding \( z_c \) fixed. This is important in our empirical analysis because it offers a criteria that will hold for firms that do not collude but which will fail for firms who do collude.

The fourth condition that must hold in equilibrium is that the bid functions must be strictly monotone. First of all note that we can write the first order conditions for equilibrium as:

\[
c_i = b - \frac{1}{\sum_{j \neq i} \frac{f(\phi_j(b;z)|z_i)}{F(\phi_j(b;z)|z_i)}}. \tag{17}
\]

Since equilibrium bid functions are strictly monotone it follows using a simple change of variables argument that \( G_i(b; z) \) and \( g_i(b; z) \) must satisfy:

\[
G_i(b; z) = F(\phi_i(b;z)|z_i) \tag{18}
\]

\[
g_i(b; z) = f(\phi_i(b;z)|z_i)\phi_i'(b;z) \tag{19}
\]
The first order conditions for equilibrium can therefore be written as:

\[ \phi_i(b, z) = b - \frac{1}{\sum_{j \neq i} g_i(b; z)} \]  

(20)

In equilibrium, the bid functions must be strictly monotone. An equivalent condition to the monotonicity of the bid functions is the monotonicity of the function \( \xi_i(b, z) \) in \( b \) where \( \xi_i(b, z) \) is defined as:

\[ \xi_i(b, z) = b - \frac{1}{\sum_{j \neq i} g_i(b; z)} \]  

(21)

A4 For all \( i \) and \( b \) in the support of the \( G_i(b, z) \) the function \( \xi_i(b, z) \) is strictly monotone.

Finally, from our characterization theorem, the following boundary conditions should also hold:

A5 \( \xi_i(\bar{b}, z) = \bar{c}, \xi_i(b, z) = c \) for \( i = 1, 2, \ldots, N \).

We formalize the above observations into theorem 4.

**Theorem 4** Suppose that the distribution of bids \( G_i(b, z), i = 1, \ldots, N \) is generated from a Bayes-Nash equilibrium. Then conditions A1-A5 must hold.

The next set of results show that if the conditions A1-A5 hold then it will be possible to construct a distribution of costs \( F(c|z_i) \) that uniquely rationalizes the observed bids \( G_i(b, z) \) as an equilibrium. In other words, the conditions A1-A5 are not only necessary for an equilibrium, but also sufficient.

**Theorem 5** Suppose that the distribution of bids \( G_i(b, z) \) satisfies conditions A1-A5. Then it is possible to construct a unique set of \( F(c|z_i) \) such that \( G_i(b, \theta) \) is generated from an equilibrium to the game where costs are distributed as \( F(c|z_i) \).

**Proof:** To construct the distribution of costs that rationalizes the distribution of bids, note that by A4 the function \( \xi_i(b, z) \) is strictly increasing, and thus we can define the distribution of costs \( F(c|z_i) \) as follows:

\[ F(c|z_i = \Pr(\xi_i(b, z) \leq c) = G_i(\xi_i^{-1}(c, z); z) \]  

(22)

By A1 and (21), all of the \( c_i \)'s are distributed independently of each other conditional upon the variables \( z \). By A2, A5 and (21), the cumulative distribution function \( F(c|z_i) \) all have the same support. By the existence theorem and uniqueness Theorems of the previous section, a unique equilibrium exists when firm \( i = 1, \ldots, N \) have costs distributed according to the construction in equation (21).
Let \( \phi_i(b, z) \) denote the equilibrium bidding strategies. By our uniqueness theorem there is one and only one set of inverse bid functions that satisfies the equation:

\[
\phi_i(b, z) = b - \frac{1}{\sum_{j \neq i} f(\phi_j(b, z)|z_i)\phi'_j(b, z)}
\]

By our construction it must be the case that:

\[
\phi_i(b, z) = \xi_i(b, z) = b - \frac{1}{\sum_{j \neq i} g_i(b, z)}
\]

By equations (17) and (21) it follows for all \( i \) that:

\[
\sum_{j \neq i} g_j(b, z) = \sum_{j \neq i} \frac{f(\phi_i(b, z)|z_i)\phi'_j(b, z)}{1 - F(\phi_j(b, z)|z_i)}
\]

For every value of \( b \), the system of equations defined by (25) can be viewed as a system of \( N \) equations in the \( N \) unknowns \( \frac{f(\phi_i(b, z)|z_i)\phi'_i(b, z)}{1 - F(\phi_i(b, z)|z_i)} \). It is easily shown that this system has a unique solution for all \( N \geq 2 \). It follows immediately that:

\[
\frac{f(\phi_i(b, z)|z_i)\phi'_i(b, z)}{1 - F(\phi_i(b, z)|z_i)} = \frac{g_i(b, z)}{1 - G_i(b, z)}
\]

Since the left hand side is the derivative of \(-\log(1 - F(\phi_i(b, z)|z_i))\) and the right hand side is the derivative of \(-\log(1 - G_i(b, z))\), it follows, combining the boundary conditions \( \phi_i(b; z) = \xi_i \), that:

\[
1 - F(\phi_i(b; z)|z_i) = 1 - G_i(b; z)
\]

\[
F(\phi_i(b; z)|z_i) = G_i(b; z)
\]

Therefore, the equilibrium bidding distribution generated when costs are defined as in equation (21) corresponds to the observed distribution of bids \( G_i(b; z) \). Furthermore, the set of distribution functions \( F(\cdot|z_i) \) is uniquely determined.

Q.E.D.

If there is no variation in \( z \) which is observable to the econometrician, it will typically not be possible to determine whether collusion has occurred. This can be proved using an approach similar to Theorem 5. Suppose that there are \( N \geq 3 \) firms who always submit bids in the auction. We shall show it is possible to construct a cost distribution \( F_c \) and \( F_3 \) such that firm 1 and 2’s bids arise as the behavior of a profit maximizing cartel and firm 3’s bids are those of a non-colluding firm. If firm 1 and 2 are behaving as an efficient cartel, then the real bid of the cartel is \( \min\{b_1, b_2\} \). Equation (24) implies that there is a one-to-one mapping from bids to costs. Using a strategy analogous to Theorem
5, it is then possible to construct latent cost distributions $F_c$ and $F_3$ to rationalize the observed bid distributions as the result of collusive behavior.

**Theorem 6** If there is no variation in $z$ and A1-A5 hold, then competition is observationally equivalent to a cartel $C$ that is not all inclusive.

Even if there is variation in $z$, it may still not be possible to empirically distinguish collusion from competition. A sophisticated cartel that includes all $N$ firms may be able to construct a mechanism for collusion that satisfies conditions A1-A5. For instance, suppose that the cartel operates by first having each firm compute its competitive bid and then submit a bid of 1.1 times its competitive bid. It is straightforward to show that conditions A1-A5 are satisfied if the cartel colludes in this fashion. In Figure 1 below, we present a diagram that summarizes the relationship between A1-A5 and the hypothesis of competition and collusion.

As we can see from Figure 1, it is in some sense never possible to reject the hypothesis of collusion based on observing only $z$ and the distribution of bids. It is always possible to construct a collusive model that satisfies A1-A5. However, if we see that A1-A5 are violated, then we know that the observed distribution of bids could not arise from a competitive model. In previous empirical studies of cartel behavior such as Porter and Zona (1993,1999) the cartels did violate assumptions A1-A5. This was true even in Porter and Zona (1993) where it appears that organized crime effectively controlled who won the contracts for the entire industry.

It is also worth noting that simple rules of thumb that are inconsistent with rational behavior can also satisfy A1-A5. For instance, suppose that all the firms in the industry bid using a polynomial bidding function that is symmetric in $z$. This rule will satisfy assumptions A1-A5 but is not generated by a rational competitive bidding model. However, Theorem 5 states that if we only know the distribution of bids conditional on $z$ we will not be able to distinguish this rule of thumb from rational behavior. Therefore, in Figure 1 we make the competitive models a strict subset of the set of models that generate bids that satisfy A1-A5.

### 6 Competitive Bidding For Seal Coat Contracts

In the next section, we will describe a unique data set we have compiled in order to apply our test for collusive bidding. Our data set was purchased from Construction Market Data (CMD) and contains detailed bidding information for nearly all of the public and private road construction projects in Minnesota, North Dakota and South Dakota. We purchased all archived records of road construction projects for these states awarded during the years 1994-1998. This data set contains nearly 18,000 unique procurement contracts and was a whopping 48 MB in size. Some of the
data fields for projects owned by city and county governments were not complete. We phoned hundreds of county and city governments throughout the Midwest to fill in missing fields in the CMD database.7

For each project, the data set contained a wealth of information. We were able to observe the project location, the deadline for bid submission, bonding requirements, the identities of all of the bidders, an extremely detailed project description and many other variables.

We decided to focus on a particular submarket in road construction called seal coating which is a maintenance process designed to extend the life of a road. Seal coating adds oil and aggregate (sand, crushed rock, gravel or pea rock) to the surface of a road. This gives the road a new surface to wear and also adds oil that will soak into the underlying pavement to slow the development of cracks in the highway. Seal coating is a low cost alternative to resurfacing a highway.8

There are numerous advantages to focusing on seal coating. The first advantage is that it is easier to measure the total work done by any particular firm on a seal-coating project. Other types of construction, such as paving, are often bundled into large, multi-faceted projects that may involve bridge repair, landscaping, installing sidewalks and many other work items that will not be performed by the paving contractor. From our conversations with DOT officials, we found that it is not unusual for a large road repair project to include ten or more subcontractors who account for up to half of the contract cost. This makes it difficult, if not impossible, to directly measure the total work done by any particular firm on a project. As a result, defining and measuring the capacity utilization of paving firms is not possible with the available data. Seal coating contracts, on the other hand, are almost never bundled with other construction activity.

A second advantage to focusing on seal coating is that the technology is relatively simple compared to other forms of construction. The process of seal coating involves a few simple steps. First, a high powered broom sweeps the existing highway to remove dirt and other debris. Second, a truck called a “distributor” shoots from two to three

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7 Construction Market Data sells information to general contractors about upcoming construction projects. Many of the general contractors we have spoken with subscribe to Construction Bulletin, a weekly periodical published by CMD, to search for work. Construction Bulletin also reports bids for contracts that were awarded in previous weeks. From conversations with DOT officials, general contractors, and CMD, we believe that almost all public and private road construction projects exceeding $10,000 are contained in our data set.

8 In our data set, almost all of the seal coating takes places from late May to mid-September since standard engineering specifications require a temperature of at least 60 degrees before seal coating starts. In fact, most State Departments of Transportation have seasonal limitations on seal-coating that do not allow any sealing to be done before the first week of June or after mid-September.

A typical crew for a seal coat company consists of two workers on the chip spreader, one distributor operator, four roller operators, four flag persons, one person to drive a pilot car, one to drive the broom and one to set temporary pavement markings. On a typical project there can be between five to fifteen trucks hauling the aggregate to the project site and a loader operator to fill the trucks with aggregate. According to one company in the industry, who primarily works in the Dakotas, a typical crew (excluding trucks) costs $1,500 per day in labor and $1,000 per day in the implicit rental price for machinery. A crew can typically expect to seal coat seven to fifteen miles of highway per day depending on conditions. The cost of trucking, according to the firm, is $35 per hour (including the driver).
tenths of a gallon of seal coat oil per square yard on the highway. A chip spreader will spread aggregate (crushed rock or sand) onto the surface of the road. The aggregate binds to the seal coat oil to form a new surface to the road. The road is then rolled with a machine called a rubber tire roller (a roller with 4 car tires at the front and back). Finally, the road is swept again to remove any excess aggregate and the highway markings are repainted.

6.1 Contract Award Procedures

All public sector seal coat contracts are awarded through an open competitive bidding process. In seal coat projects, contractors do not submit a single bid; rather they submit a vector of bids. This is known as a unit price contract and has the form:

<table>
<thead>
<tr>
<th>contract item #1</th>
<th>estimated quantity for item #1</th>
<th>unit price for item #1</th>
</tr>
</thead>
<tbody>
<tr>
<td>contract item #2</td>
<td>estimated quantity for item #2</td>
<td>unit price for item #2</td>
</tr>
<tr>
<td>contract item #3</td>
<td>estimated quantity for item #3</td>
<td>unit price for item #3</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

The contract items might include gallons of oil, tons of aggregate and mobilization. Both the contract items and the estimated quantities are established by the owner of the contract (typically a city government or State DOT) and the unit prices are chosen by the contractor.

---

9 The seal coat contract documents contain 6 major parts: bidding documents, general conditions of the contract, supplementary conditions of the contract, specifications, drawings and report of investigations of physical site conditions.

The bidding documents begin with an advertisement for bids in an industry periodical such as Construction Bulletin. The advertisement for bids describes the location of the project, the estimated quantities of materials to be used in the project, the bonding requirements for the project and other contract conditions. In the proposal form, the contractor submits unit prices for each contract item. The proposal form also contains an affidavit that the contractor has not colluded with other firms and bonds from a bonding company as required by the contract.

The general conditions of the contract define in general terms the participants in the contract (i.e. owner, general contractor, engineer, subcontractors, etc...) and their roles, the process for amending the contract with change orders, the contractor’s liability for on time completion of the contract and procedures for extending the completion date, terms describing how payments will be made, and conditions under which the contract may be terminated. In many cases, the general conditions are a “boilerplate” that are similar from contract to contract.

The supplementary conditions of the contract include a set of site or project specific clauses that are additions and/or corrections to the general contract conditions. For instance, in some projects where part of the funding comes from the federal government, the wages for each type of work (laborer, truck driver, crew foreman, etc...) must be at least as high as the lower bounds set in accordance with the Davis-Bacon Act. The supplementary conditions may also contain so-called disadvantaged business enterprise requirements that state how much of the contract total must be awarded to minority owned businesses.

The specifications and drawings contain detailed engineering information about exactly how the project is to be completed such as the materials to be used, the temperature at which the work can proceed and other technical specs. The specifications and drawings are meant to be a sufficiently clear description of how the project is to be built so that the contractor may estimate costs in order to bid. Substantial deviations from the specifications and drawings will result in change orders to the project. The Standard Specifications for Roads and Bridges for each state also contains design parameters and technical specifications for a large number of contract items. The report on physical site conditions often contains geotechnical descriptions of subsurface soil and rock conditions. This often includes soil descriptions of soil borings from the project site or the gravel pit used to supply aggregate.

10 In our analysis, we abstract away from the fact that the firms make a vector of bids instead of a single bid. As in Athey and Levin (2000), it would be possible to model bidding as a two-stage procedure. In the first stage bidders choose a total bid and in the second stage they choose unit prices optimally.

11 The contractor is compensated according to the quantities that are actually used on the job. DOT personnel monitor the firm while the work occurs and are responsible for verifying measurements of quantities of material put in place. If actual quantities are 20% less...
If the contract is awarded, it must by law be given to the lowest responsible bidder. Public officials have the right to reject all bids, but this rarely occurs in practice. Firms also have strong financial incentives to honor their contractual obligations if they are the low bidders. Contractors usually must submit a bid bond of 5 to 10 percent of their total bid guaranteeing that they will not withdraw their bid after the public reading of all bids. After the contract is awarded, the low bidder must submit a performance bond and pay bond to guarantee the completion of the contract and that all subcontractors will be paid. For a more complete discussion of contract procedures see Minnesota Department of Transportation (1995), Bartholomew (1998), Clough and Sears (1994), and Hinze (1993).

6.2 The Data Set

In our data set, we observe all public sector seal coat contracts awarded from January 1994 through October 1998. There are four owner types in our data set: City, County, State and Federal. Most of contracts are owned by City or State. Among all jobs, 230 (46.5%) are owned by Cities, 195 (39.3%) are owned by States, 68 (13.7%) are owned by Counties, and only 2 owned by the Federal Government. The total value of contracts awarded in our data set is $92.8 million.

The size of contracts varied greatly. Of the 495 contracts in our data set, 7 contracts were awarded for more than $1 million, 256 contracts were awarded for less than $1 million but more than $100 thousand, and 232 contracts were awarded for less than $100 thousand. A total of 98 firms bid on at least one of these 495 contracts, with 43 firms never winning any contract for the period reported.

The market concentration can be seen from Table 2, which summarizes the firms’ bidding activities, while the firms’ identities are listed in Table 1. Among those 55 firms winning at least one job, only 18 firms have a market share exceeding 1%.\textsuperscript{12} The largest 7 firms in the market have a market share of 65.6% led by Firm 2 (Astech Paving) who alone accounts for 21% of the market shares, attending 66.9% of the auctions conducted.

The owner of the largest firm in the market, Astech received a one year prison sentence for bid-rigging in the late 1980’s. The owners of two other firms, McLaughlin & Schulz Inc. and Allied Paving were also fined for bid-rigging with Astech in the seal coat industry. The owners of all three firms were, at one time, banned from bidding for public sector seal coat contracts. Whether these or any other firms in the industry are still engaged in anti-competitive behavior is an interesting question.

\textsuperscript{12} A firm’s market share is defined as the ratio of the amount of the firm’s total winning bids over the amount of total winning bids for all the contracts.
In Table 3 we study the concentration of bidders who attend any given auction. The average number of bids in an auction is 3.3 with 29 contracts receiving only one bid. We conjecture that the low participation has to do with bid preparation costs on the part of contractors. The firms we spoke with suggested that significant managerial resources are required to prepare a bid.\footnote{First, firms must gather information about materials prices and find subcontractors for the project which is a time consuming activity. Also, firms must carefully study the project plans and specifications to calculate expected costs. Many of the projects we have studied are spread out over 10 to 100 miles from endpoint to endpoint. Firms must make a careful (and often tedious) calculation of anticipated transportation costs for the project. Finally, firms will need to get a bond for the project which is also time consuming. Standard references about construction bidding such as Park and Chapin (1992) suggest that bid preparation costs are on average one to two percent of total project costs.}

Table 4 summarizes values for the 1st lowest bid (BID1), the 2nd lowest bid (BID2), and “money on the table” (BID2=\(BID2-BID1\)), when the total number of bids is at least 2. Money left on the table averages $15,724 and has a maximum of $352,174. All of the firms in our market are owner operated so whatever profits are made accrue directly to the firm’s manager. If firms had complete information about competitors’ costs, the amount of money that should be left on the table in equilibrium would be near zero. The amount of money left on the table is consistent with the presence of non-negligible private information about costs.

Based on the winning bids and bidding dates, we construct a new variable “CAP”, which is meant to measure each firm’s capacity utilization level. A firm’s capacity at a particular bidding time is defined as the ratio of the firm’s used capacity (measured by the firm’s total winning bids’ amount up to that time) over the firm’s total winning bids’ amount in the entire season.\footnote{Note that as mentioned earlier, the season during which seal coating can take place lasts from late May to mid-September. So a season mentioned in the above definition starts on September 1 and ends on August 3 of the following calendar year. This measure of capacity was computed using the entire data base of bidding information even though in our econometric analysis we will focus on a subset of these projects.}

Another generated variable is distance, which we construct using information about both the location of the firms and the location of the project.\footnote{The calculation is facilitated by using Yahoo’s map searching engine http://maps.yahoo.com/py/ddResults.py. Using city and state’s names as input for both locations, the map searching engine gives distances automatically. Doing this manually would be too time consuming. We would like to thank Hehui Jin for providing an “electronic spider” which greatly facilitated our job.} For jobs covering several locations, we use the midpoints of the jobs to do the calculation. Table 5 summarizes the distance in miles between the project site and the winning firm (DIST1) up to the distance between the job and the firm submitting the 7th lowest bid (DIST7). Firms with shorter distances from project locations are more likely to win the job.\footnote{The small mean distance for the firms with the 7th lowest bid is mainly due to the problem of too many missing observations. If those missing observations are recovered, we expect that DIST7 would have much higher mean.} The average distance of the closest firm is 122.3 miles whereas the distances of firms who fail to win projects tends to be considerably higher.\footnote{In our complete data set, it was not possible to find the locations of all projects or all the firms that bid in these projects. Furthermore, some projects in our original data set had missing information that we could not supplement with information from the relevant branch of City, County, State or the Federal Government. However, we used the available information}
Another important control variable for our analysis will be an engineer’s estimate. This is the estimate formed either by branches of the government or by consulting engineering firms. In speaking with engineers at Minnesota, North Dakota and South Dakota’s Departments of Transportation, the engineers claimed that they formed the estimates by gathering information on materials prices, prevailing wage rates and other relevant cost information. The engineer’s estimate is supposed to represent a “fair market value” for completion of the project. We found that estimates were available for 139 out of the 441 projects in the data set. Table 6 shows that the engineer’s estimate is a useful control for project costs. The normalized winning bid is almost exactly 1 and has standard deviation of 0.1573.

Table 6 suggests that both location and capacity play an important role in bidding. Firms will bid higher the greater the distance to a project and greater capacity utilization implies a higher bid. On the other hand, Probit estimates also suggest that firms with greater capacity utilization and greater distance to a project are less likely to bid all else held constant.

Another important determinant of firm \( i \)’s success in winning contracts is familiarity with local regulators and local material suppliers. In Table 7, we calculate, for each state and for each firm, the percentage of the firm’s total dollar volume done in that state. Our results suggest that the majority of the firms in our data set work primarily in one state. This effect is present even after controlling for distance. For instance, firm 3 is located near the boundaries of Minnesota, North Dakota and South Dakota. Yet it does over 70 percent of its dollar volume of seal coating in South Dakota. Also, firms 6 is located near the Minnesota and South Dakota border. Yet firm 6 has won no contracts in South Dakota.

### 6.3 Reduced Form Bid Functions

Next, we estimate a set of reduced form bid functions to measure the relationship between a number of variables and the firms’ observed bidding behavior. The variables we will use in these regression are as follows:

- **BID\(_{i,t}\)**: The amount bid by firm \( i \) on project \( t \).
- **EST\(_t\)**: The estimated value of project \( t \).
- **DIST\(_{i,t}\)**: Distance between the location of the firm and the project.
- **LDIST\(_{i,t}\)**: \( \log(\text{DIST}\(_{i,t}\)+1.0) \).
- **CAP\(_{i,t}\)**: Used capacity measure of firm \( i \) on project \( t \).
- **MAXP\(_{i,t}\)**: Maximum percentage free capacity of all firms on project \( t \), excluding \( i \).
- **MDIST\(_{i,t}\)**: Minimum of distances of all firms on project \( t \), excluding \( i \).

in calculating firms’ capacities even though other parts of the observation might be incomplete.
- LMDIST<sub>it</sub>: $\log(MDIST_{it}+1.0)$.
- CON<sub>it</sub>: The proportion of work done (by dollar volume) by firm $i$ in the State where project $t$ is located prior to the auction.

We hypothesize that firm $i$'s cost estimate for project $t$ satisfies the following structural relationship:

$$
\frac{c_{it}}{EST_{it}} = c(DIST_{it}, CAP_{it}, CON_{it}, \omega_i, \delta_i, \varepsilon_{it})
$$

Equation (29) implies that firm $i$'s cost in auction $t$ can be written as a function of its distance to the project, its backlog, the previous experience that firm $i$ has in this market which we proxy for using $CON_{it}$, a firm $i$ productivity shock $\omega_i$, an auction $t$ specific effect $\delta_i$, and $\varepsilon_{it}$, an idiosyncratic shock to firm $i$ that reflects private information she will have about her own costs. The results of Section 3 demonstrate that under certain simplifying assumptions about dynamic competition, a dynamic model with capacity constrained bidders is formally equivalent to a static model where the firm’s cost is $c_t + V_{iL}(s) - V_{iW}(s)$, a sum of current project costs, $c_t$, plus a term $V_{iL}(s) - V_{iW}(s)$ that captures the option value of keeping free capacity. In practice, the measure of backlog $CAP_{it}$ will be a good proxy for $V_{iL}(s) - V_{iW}(s)$. Mapping the structural cost function back to the framework of Section 5 implies that $z_i = (DIST_{it}, CAP_{it}, CON_{it}, \omega_i, \delta_i)$.

According to our theoretical model, firm $i$’s bid function should depend on the entire parameter vector $z = (z_1, ..., z_N)$. However, given the limited number of data points in our sample, it will not be possible to model the bid functions in a completely flexible fashion because $z$ is a vector with $5 \times N$ elements. We choose to include a firm’s own distance, capacity and concentration. From our conversations with firms who actually bid in these auctions, we believe that the most important characteristics of the other firms to include in the reduced form bid function are the location of the closest competitor and the backlog of the competitor that has the most free capacity. To control for $\delta_t$, we use fixed effects for the auction and to control for $\omega_i$ we use firm fixed effects for the largest 11 firms in the market. We are able to identify both our auction fixed effect and firm fixed effects because we do not use fixed effects for all of the firms. This implies that firms that are not the 11 largest have an identical productivity shock $\omega_1$ which is probably not a bad assumption in this industry.

Since there are 138 auctions, 11 main firms and one pooled group of non-main firms in our restricted data set, we have 137 auction dummies and 11 firm dummies. The set of regressors thus contains a constant ($C$), 148 dummy variables, own distance($DIST_{it}$), own capacity ($CAP_{it}$), maximal free capacity among competitors ($MAXP_{it}$), minimal distance among competitors ($MINDIST_{it}$), and the job concentration variable ($CON_{it}$). To take care of
the heteroscedasticity problem, we take the ratio of the bid and the value (the engineer’s estimate) as the dependent variable \( \frac{BID_{i,t}}{EST_t} \).

\[
\frac{BID_{i,t}}{EST_t} = \beta_0 + \beta_1 LDIST_{i,t} + \beta_2 CAP_{i,t} + \beta_3 MAXP_{i,t} + \beta_4 LMDIST_{i,t} + \beta_5 CON_{i,t} + \varepsilon_{i,t} \tag{30}
\]

The results from the regression, estimated using ordinary least squares, are (with t-statistics in parentheses):

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS</th>
<th>(t-stat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (Constant)</td>
<td>.68</td>
<td>(5.95)</td>
</tr>
<tr>
<td>( LDIST_{i,t} ) (Own distance)</td>
<td>.040</td>
<td>(3.45)</td>
</tr>
<tr>
<td>( CAP_{i,t} ) (Own used capacity)</td>
<td>.17</td>
<td>(8.51)</td>
</tr>
<tr>
<td>( MAXP_{i,t} ) (Maximal free capacity among rivals)</td>
<td>.026</td>
<td>(.71)</td>
</tr>
<tr>
<td>( LMDIST_{i,t} ) (Minimal distance among rivals)</td>
<td>.024</td>
<td>(1.81)</td>
</tr>
<tr>
<td>( CON_{i,t} ) (Job concentration)</td>
<td>-.059</td>
<td>(-1.87)</td>
</tr>
</tbody>
</table>

Sample Size: 450

\[ R^2 = .85 \]

The regression also includes a fixed effect for each project \( t \) and fixed effect for each of the 11 largest firms in the market.

The results from our reduced form bid function are consistent with basic economic intuition. Firm \( i \)'s bid is an increasing function of firm \( i \)'s distance from the project site and firm \( i \)'s capacity utilization. As firm \( i \)'s distance increases so does \( i \)'s cost. Our theory would lead us to expect a positive coefficient on own distance.\(^\text{18}\) The coefficient on \( CAP_{i,t} \) is also positive and significant. As firm \( i \)'s backlog increases, all else held constant, the option value of free capacity will increase because once firm \( i \) becomes completely capacity constrained, it will no longer have a chance to bid on future projects. The coefficient on \( CON_{i,t} \) is negative, indicating that if firm \( i \) has more prior experience in the state, firm \( i \) will tend to bid more aggressively.

Our reduced form bid function also produces results that are consistent with the strategic interactions implied by the asymmetric auction model. As the distance of firm \( j \neq i \) increases or as the capacity utilization of firm \( j \neq i \) increases competition will soften and firm \( i \) raises her bid. However, the reaction to \( MAXPER_{i,t} \) is not significant at conventional levels.

\(^{18}\) See the results in Section 5 that numerically study the comparative statics of the bid function.
6.4 Test for conditional independence

In this section, we test the conditional independence assumption $A1$ in Section 5. We use a reduced form bid function as in the previous subsection, however, we will allow the model to be more flexible. If firm $i$ is one of the largest 11 firms in the industry we use equation (31) with firm varying coefficients to its bid function. If firm $i$ is not one of the 11 largest firms in the industry we use equation (32) to model its bid function. We pool equations (31) and (32) in the estimation and include auction fixed effects.

\[
\frac{BID_{i,t} \text{ EST}_t}{E_{i,t}} = \beta_{0,i} + \beta_{1,i}LDIST_{i,t} + \beta_{2,i}CAP_{i,t} + \beta_{3,i}MAXP_{i,t} + \beta_{4,i}LMDIST_{i,t} + \beta_{5,i}CON_{i,t} + \varepsilon_{it} \quad (31)
\]

\[
\frac{BID_{i,t} \text{ EST}_t}{E_{i,t}} = \alpha_{0} + \alpha_{1}LDIST_{i,t} + \alpha_{2}CAP_{i,t} + \alpha_{3}MAXP_{i,t} + \alpha_{4}LMDIST_{i,t} + \alpha_{5}CON_{i,t} + \varepsilon_{it} \quad (32)
\]

Suppose the coefficient of correlation between the residual to firm $i$’s bid function and firm $j$’s bid function, $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$, is $\rho_{ij}$. The test of conditional independence is then equivalent to testing the following null hypothesis:

\[H_0 : \rho_{ij} = 0\] (33)

We first report the number of pairwise simultaneous bids and the correlation coefficients (computed when the number of simultaneous bids is no less than 4) in the following matrix:

<table>
<thead>
<tr>
<th>Firms</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>11</th>
<th>14</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.744</td>
<td>-0.5897</td>
<td>-0.2547</td>
<td>-0.1512</td>
<td>0.1330</td>
<td>-0.3010</td>
<td>-0.0909</td>
<td>-0.4260</td>
<td>-0.1304</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>8.6374</td>
<td>-0.439</td>
<td>-0.1910</td>
<td>-0.3365</td>
<td>-0.5742</td>
<td>-0.8854</td>
<td>-0.6963</td>
<td>-0.3588</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>67</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>76</td>
<td>4</td>
<td>0</td>
<td>0</td>
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</tr>
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<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
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<td>2</td>
<td>9</td>
<td>3</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>15</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>18</td>
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<td>0</td>
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<td>0</td>
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</tr>
<tr>
<td>19</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

We use the Fisher test to test the hypothesis (33). Suppose the correlation coefficient between two firms’ bids is $\rho$. Let $r$ be the correlation coefficient calculated from sample data (as reported above), then the Fisher $Z$ transformation is given by

\[
Z = \frac{1}{2} \ln \frac{1 + r}{1 - r}\] (34)

24
Let \( n \) be the number of samples, then the distribution of \( Z \) is approximately normal with:

\[
\mu_Z = \frac{1}{2} \ln \frac{1 + \rho}{1 - \rho} \quad \text{and} \quad \sigma_Z = \frac{1}{\sqrt{n - 3}}
\]  

(35)

Hence \( z = (Z - \mu_Z)\sqrt{n - 3} \) has approximately the standard normal distribution. In our case, under null hypothesis, \( \rho = 0, \mu_Z = 0 \). It remains to calculate the test statistic \( Z\sqrt{n - 3} \) for each pair of firms whenever \( n > 3 \). The results are as follows:

<table>
<thead>
<tr>
<th>Firms</th>
<th>( n )</th>
<th>( r )</th>
<th>( z )</th>
<th>Firms</th>
<th>( n )</th>
<th>( r )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,2)</td>
<td>15</td>
<td>-.744</td>
<td>-3.3234</td>
<td>(4,5)</td>
<td>63</td>
<td>-.1910</td>
<td>-1.4979</td>
</tr>
<tr>
<td>(2,3)</td>
<td>9</td>
<td>-.5897</td>
<td>-1.6588</td>
<td>(4,14)</td>
<td>8</td>
<td>-.3197</td>
<td>-.7408</td>
</tr>
<tr>
<td>(2,4)</td>
<td>67</td>
<td>-.5247</td>
<td>-4.6624</td>
<td>(5,6)</td>
<td>8</td>
<td>-.3365</td>
<td>-.7829</td>
</tr>
<tr>
<td>(2,5)</td>
<td>76</td>
<td>-.1512</td>
<td>-1.3018</td>
<td>(5,8)</td>
<td>5</td>
<td>.5742</td>
<td>.9246</td>
</tr>
<tr>
<td>(2,6)</td>
<td>17</td>
<td>.1330</td>
<td>.5006</td>
<td>(5,11)</td>
<td>4</td>
<td>.8854</td>
<td>1.4002</td>
</tr>
<tr>
<td>(2,7)</td>
<td>9</td>
<td>-.3010</td>
<td>-.7609</td>
<td>(5,14)</td>
<td>10</td>
<td>-.6963</td>
<td>-2.2756</td>
</tr>
<tr>
<td>(2,8)</td>
<td>12</td>
<td>.0909</td>
<td>.2734</td>
<td>(5,20)</td>
<td>5</td>
<td>.3588</td>
<td>.5310</td>
</tr>
<tr>
<td>(2,14)</td>
<td>9</td>
<td>.4260</td>
<td>1.1145</td>
<td>(6,7)</td>
<td>7</td>
<td>-.7850</td>
<td>-2.1165</td>
</tr>
<tr>
<td>(2,20)</td>
<td>5</td>
<td>.1304</td>
<td>.1855</td>
<td>(6,8)</td>
<td>12</td>
<td>.2327</td>
<td>.7111</td>
</tr>
<tr>
<td>(3,4)</td>
<td>4</td>
<td>-.6374</td>
<td>-.7538</td>
<td>(7,8)</td>
<td>6</td>
<td>-.2711</td>
<td>-.4816</td>
</tr>
<tr>
<td>(3,5)</td>
<td>8</td>
<td>.2439</td>
<td>.5566</td>
<td>(14,20)</td>
<td>6</td>
<td>.5768</td>
<td>1.1391</td>
</tr>
<tr>
<td>(3,11)</td>
<td>7</td>
<td>-.2345</td>
<td>-.4779</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Among all 23 pairs which have at least 4 simultaneous bids, the null hypothesis cannot be rejected except for four pairs of firms at 5% significance level. These four pairs are: (Firm 1, Firm 2), (Firm 2, Firm 4), (Firm 5, Firm 14), and (Firm 6, Firm 7). However, of these pairs, only the pair (Firm 2, Firm 4) bid against each other more than a handful of times. The Pairs (Firm 1, Firm 2), (Firm 5, Firm 14) and (Firm 6, Firm 7) bid against each other on average no more than two or three times a year in the data set.

### 6.5 Test for Exchangeability

In this section we use our regression model (31) and (32) to test whether the empirical distribution of bids is exchangeable. Exchangeability implies that capacities and distances should enter the firm’s bid-value function in a “symmetric” way. Formally, in the reduced form bid function, let \( \beta_{11}, \beta_{12}, \beta_{13}, \beta_{14} \) be the coefficients of \( LDIST_1, CAP_1, MAXP, LMDIST \) for firm \( i \), one of the largest 11 firms. Then exchangeability is equivalent to the following hypothesis:

\[
H_0: \beta_{ik} = \beta_{jk} \quad \text{for all } i, j, i \neq j, \text{ and for all } k = 1, \ldots, 4.
\]  

(36)

We use the F-test to test for exchangeability. Let \( SSR_U \) and \( SSR_C \) be the sum of squared errors in the unconstrained and constrained models, respectively. Also let \( T \) be the number of observations (\( T = 450 \) in our data set), \( m \) be the
number of regressors, and \( n \) be the number of constraints implied by \( H_0 \). Then the statistic

\[
F = \frac{(SSR_C - SSR_U)/n}{SSR_U/(T - m)}
\]  

(37)

is distributed as an \( F \) distribution with parameters \((n, T - m)\) under null hypothesis. Note that the \( F \)-test is also a variation of the quasi-likelihood ratio test (QLR) on non-linear two- and three-stage least squares.

We conduct two tests of exchangeability in this section. The first set is to test exchangeability for the whole market, i.e., the constrained regression that pools all the 11 main firms together. The second set is to test the exchangeability on pairwise basis, i.e., the constrained regression pools two of the main firms together at each test (hence the number of constraints is 4). We perform this set of tests for each pair of firms with at least 4 simultaneous bids. The following tables summarize the test results:

<table>
<thead>
<tr>
<th>Firm Pair</th>
<th>( n )</th>
<th>( m )</th>
<th>( F ) Statistics</th>
<th>Upper Tail Area</th>
<th>Firm Pair</th>
<th>( n )</th>
<th>( m )</th>
<th>( F ) Statistics</th>
<th>Upper Tail Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 2)</td>
<td>4</td>
<td>194</td>
<td>1.2188</td>
<td>.3033</td>
<td>(4, 5)</td>
<td>4</td>
<td>194</td>
<td>1.0799</td>
<td>.3669</td>
</tr>
<tr>
<td>(2, 3)</td>
<td>4</td>
<td>194</td>
<td>2.1080</td>
<td>.0803</td>
<td>(4, 14)</td>
<td>4</td>
<td>194</td>
<td>.9756</td>
<td>.4214</td>
</tr>
<tr>
<td>(2, 4)</td>
<td>4</td>
<td>194</td>
<td>1.0187</td>
<td>.3982</td>
<td>(5, 6)</td>
<td>4</td>
<td>194</td>
<td>1.2014</td>
<td>.3107</td>
</tr>
<tr>
<td>(2, 5)</td>
<td>4</td>
<td>194</td>
<td>3.9254</td>
<td>.0041</td>
<td>(5, 8)</td>
<td>4</td>
<td>194</td>
<td>1.2209</td>
<td>.3024</td>
</tr>
<tr>
<td>(2, 6)</td>
<td>4</td>
<td>194</td>
<td>.7856</td>
<td>.5354</td>
<td>(5, 11)</td>
<td>4</td>
<td>194</td>
<td>.2643</td>
<td>.9007</td>
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<td>(2, 7)</td>
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<td>2.3709</td>
<td>.0530</td>
<td>(5, 14)</td>
<td>4</td>
<td>194</td>
<td>2.3162</td>
<td>.0578</td>
</tr>
<tr>
<td>(2, 8)</td>
<td>4</td>
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<td>.6211</td>
<td>.6478</td>
<td>(5, 20)</td>
<td>4</td>
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<td>1.2151</td>
<td>.3048</td>
</tr>
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<td>4</td>
<td>194</td>
<td>2.1288</td>
<td>.0777</td>
<td>(6, 7)</td>
<td>4</td>
<td>194</td>
<td>2.2728</td>
<td>.0619</td>
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<td>4</td>
<td>194</td>
<td>1.6844</td>
<td>.1541</td>
<td>(6, 8)</td>
<td>4</td>
<td>194</td>
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<td>.9781</td>
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<td>(3, 4)</td>
<td>4</td>
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<td>(7, 8)</td>
<td>4</td>
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<td>2.0983</td>
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<td>1.5582</td>
<td>.1860</td>
<td>(14, 20)</td>
<td>4</td>
<td>194</td>
<td>1.1022</td>
<td>.3560</td>
</tr>
<tr>
<td>(3, 11)</td>
<td>4</td>
<td>194</td>
<td>1.1202</td>
<td>.3474</td>
<td>All Pooled</td>
<td>40</td>
<td>158</td>
<td>1.4506</td>
<td>.0474</td>
</tr>
</tbody>
</table>

The above tables show that for almost all the tests, we just fail to reject the null hypothesis at 5% significance level. The \( p \) value for this test is 0.0474. In fact, we only reject the null when we pool all the 11 main firms and when we pool Firm 2 and Firm 5.

### 6.6 Discussion

The results of our tests of exchangeability and conditional independence imply that there are five pairs of firms who exhibit bidding patterns that are not consistent with our characterization of competitive bidding. As we discussed in Section 5, even if a cartel is present in the industry, firms who are not colluding must satisfy conditional independence and exchangeability in a pairwise sense. The four pairs (Firm 1, Firm 2), (Firm 2, Firm 4), (Firm 5, Firm 14), and (Firm 6, Firm 7) fail the conditional independence test and the pair (Firm 2, Firm 5) fails the exchangeability test.
However, of these pairs, only the pairs (Firm 2, Firm 4) and (Firm 2, Firm 5) bid against each other more than a handful of times. The three pairs (Firm 1, Firm 2), (Firm 5, Firm 14) and (Firm 6, Firm 7) bid against each other on average no more than two or three times a year in the data set. Also, according to industry participants, these firms function in different sub-markets and would have no reason to view each other as principal competitors. Therefore, the two pairs of firms that might be of concern to regulators are (Firm 2, Firm 5) and (Firm 2, Firm 4).

By and large, bidding in the industry appears to conform to the axioms A1-A5. This observation is important to policy makers since there is a history of bid-rigging in the seal coat industry. Several of the largest firms in the industry were colluding in the early to mid-1980’s and paid damages for bid-rigging. Our analysis suggests that currently most bidding behavior in the industry is consistent with our model of competitive bidding.

There are at least three limitations to our approach in practice. First, in both our tests for conditional independence and exchangeability, we need to use a correct functional form for the reduced form bid functions. In our empirical analysis, we used a number of different functional forms for the reduced form bid function and the results in the tests for conditional independence and exchangeability were robust across these alternative specifications. The arguments of firm $i$’s bid function is the vector $z$ which has $4 \times N$ arguments. Given that we have only 138 auctions in our data set, we can never be certain that our independence and exchangeability results were not influenced by a poor choice of functional form.

A second limitation is that our results might be incorrect because of omitted variables. If there are elements of $z$ that the firms see but which are not present in our data set our regression coefficients will be biased. In our analysis, we use fixed effects for each contract and for the largest 11 firms. Therefore, we should be most worried about omitted variables that are elements of $z$ but which are not co-linear with the firm or contract fixed effects. This could happen when there are 3 firms and firm 1 and firm 2 always use a quote from a particular subcontractor for computing their cost estimates while firm 3 does not. If this quote is not controlled for in our regression, it will then induce positive correlation between the residuals to the bid functions of firms 1 and 2.

However, in our test for conditional independence, we found that the residual between firm 2 and 4 are negatively correlated. If this is due to omitted cost variables, it must be the case that the omitted variable must induce negative correlation between the costs of these two firms. So far, we have not been able to come up with a scenario that would generate this type of cost shock. However, if firm 2 and firm 4 engaged in a scheme of submitting phony bids, this might induce a negative correlation in the residuals since phony bidding implies that when firm 2 bids high firm 4 must bid low. A similar critique could be made of our test of exchangeability. Omitted cost variables could lead us
to falsely conclude that firm 2 and firm 5 fail to have an exchangeable distribution of bids.

An advantage to focusing on the seal-coat industry is that the cost structure is rather straightforward compared to other parts of the construction industry. In many building and paving projects there can be hundreds of contract items and multiple subcontractors who give quotes only to a subset of firms. Often, large paving projects will be bundled with bridge repair, grading and many other types of construction work. A general contractor will typically not be able to complete all the work herself and hence numerous subcontractors will be hired. As we described above, this will induce a positive correlation between the residuals of certain firms bid functions. However, in the seal coat industry, contracts tend to be rather simple. Most of the contracts have less than 10 contract items and there is relatively little subcontracting compared to paving and building. Seal coat projects are not typically bundled with other types of work.

Third, if a sophisticated cartel is operating in this market, then, as we mentioned in Section 5, the cartel could satisfy assumptions A1-A5 by generating phony bids in a clever fashion. Therefore, from our tests, we will not be able to identify whether those firms who passed the conditional independence and exchangeability tests are competitive or are smart colluders. In recent empirical papers that document cartel behavior such as Porter and Zona (1993,1999) and Pesendorfer (1996) the authors know from court records and investigations the identities of the cartel members. In all these papers, both exchangeability and conditional independence fail. To the best of our knowledge, there is no documented case of cartel bidding where the cartel intentionally submitted phony bids that were both conditionally independent and exchangeable.

While there are a number of limitations to our analysis, we believe that the techniques we have developed will likely apply to an even broader class of models than those studied in this research. First, conditional independence will hold in many common value models. For instance, in the standard mineral rights model, nature first draws a common value $v$ and then, conditional on $v$, each bidder $i$ receives an independent signal $x_i$. In auction data sets, the economist typically observes all of the bids submitted and therefore it is possible to use a fixed effects estimator to control for the realized common value $v$. Since the bids are functions of independent signals, the residuals to correctly specified reduced form bid functions should be independent in this case. The property of exchangeability is likely to generalize to a much broader class of auction models and indeed to non-auction settings such as price competition. The proof of exchangeability relied only on the fact that the equilibrium was unique and that there was nothing special about a firm’s identity. We believe that extending results in these directions is a fruitful direction for future research.

We believe an economist can never be absolutely certain that she has detected bid-rigging using our tests. However,
we do believe that our tests, in many circumstances may be a useful diagnostic in determining whether or not it is worthwhile to investigate a particular subset of firms and collect further information. If fact, in writing this paper, we have been pleasantly surprised by how well the basic theory of competitive bidding appears to work given the limitations of our models and of our data set. Using the implications of competitive bidding, we have generated 46 testable restrictions from the theory. Of these tests, 41 were satisfied. Since these tests were conducted at the 5 percent significance level, we would expect at least two of the tests to fail even if there was no collusion because of randomness in the data.

In our companion paper, Bajari and Ye (2001), we propose a second, complementary approach to testing for collusion based on using structural estimation. The analysis in this paper indicates that two pairs of firms (Firm 2, Firm 4) and (Firm2, Firm5) generate bidding behavior that might be of concern to anti-trust authorities. In Bajari and Ye (2001), we spoke to leading firms and regulators in the industry who claimed that markups over the cost estimate are seldom over 15 percent. Using this bound on markups, we construct a set of bounds on the parameters of the cost function $\tilde{c}_i(t) = c(DIST_i(t), CAP_i(t), CON_i(t), \theta_i, \delta_i, \varepsilon_i)$. We interpret this set of bounds as a prior distribution over structural cost parameters. We then use both a competitive and a collusive equilibrium model of bidding to form likelihood functions for the observed bids. Using this prior distribution and the likelihood functions we compute posterior probabilities for the competitive and the collusive models. Taken together, the approach in this paper and in Bajari and Ye (2001) provide a useful battery of statistical tests that can be of use in detecting suspicious bidding patterns in procurement auctions.

7 Conclusion

In this research, we have analyzed a model of competitive bidding in procurement auctions. We began by building a model of competitive bidding with asymmetric bidders. The modeling framework allows for certain types of collusion and non-trivial dynamics due to capacity constraints as special cases. We stated a set of conditions that are both necessary and sufficient for a distribution of bids to arise from competitive bidding. Two of these conditions, conditional independence and exchangeability, can be tested in a straightforward fashion.

We then presented a unique data set of bidding by construction firms in the Midwest. The data set contained nearly every bid submitted for seal coating contracts in Minnesota, North Dakota and South Dakota over a 5 year period. In the data, four types of asymmetries were present. Firms are asymmetric because of location, capacity utilization, productivity and previous experience with local market conditions and regulations. We estimated reduced
form bid functions that are consistent with our theoretical model. Using these empirical bid functions, we tested for exchangeability and conditional independence. Our results indicated that for most pairs of firms, it is not possible to reject the implications of competitive bidding in our model. Using the implications of competitive bidding, we generated 46 testable restrictions from the theory. Of these tests, 41 were satisfied at the 5 percent level.

No empirical techniques for detecting collusion are likely to be flawless. However, we believe that the tests we propose, when taken together with the tests in our companion paper, Bajari and Ye (2001), can be a useful first step in detecting suspicious bidding patterns.
## Tables.

### Table 1: Identities of main firms

<table>
<thead>
<tr>
<th>Firm ID</th>
<th>Name of the Company</th>
<th>Firm ID</th>
<th>Name of the Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Allied Blacktop Co.</td>
<td>11</td>
<td>Asphalt Surfacing Co.</td>
</tr>
<tr>
<td>2</td>
<td>Astech</td>
<td>12</td>
<td>Bechtold Paving</td>
</tr>
<tr>
<td>3</td>
<td>Bituminous Paving Inc.</td>
<td>14</td>
<td>Border States Paving Inc.</td>
</tr>
<tr>
<td>4</td>
<td>Lindteigen Constr Co. Inc.</td>
<td>17</td>
<td>Mayo Constr Co. Inc.</td>
</tr>
<tr>
<td>5</td>
<td>McLaughlin &amp; Schulz Inc.</td>
<td>20</td>
<td>Northern Improvement</td>
</tr>
<tr>
<td>6</td>
<td>Morris Sealcoat &amp; Trucking Inc.</td>
<td>21</td>
<td>Camas Minndak Inc.</td>
</tr>
<tr>
<td>7</td>
<td>Pearson Bros Inc.</td>
<td>22</td>
<td>Central Specialty</td>
</tr>
<tr>
<td>8</td>
<td>Caldwell Asphalt Co.</td>
<td>23</td>
<td>Flickertail Paving &amp; Supply</td>
</tr>
<tr>
<td>9</td>
<td>Hills Materials Co.</td>
<td>25</td>
<td>Topkote Inc.</td>
</tr>
</tbody>
</table>

### Table 2: Bidding activities of main firms

<table>
<thead>
<tr>
<th>Firm ID</th>
<th>No.of wins</th>
<th>Avg. bid</th>
<th>% mkt. share</th>
<th>No. Participation</th>
<th>% of participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>92</td>
<td>82790</td>
<td>8.2</td>
<td>145</td>
<td>29.3</td>
</tr>
<tr>
<td>2</td>
<td>102</td>
<td>191953</td>
<td>21.1</td>
<td>331</td>
<td>66.9</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>363565</td>
<td>7.8</td>
<td>69</td>
<td>14.0</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>241872</td>
<td>9.1</td>
<td>114</td>
<td>23.0</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
<td>283323</td>
<td>8.9</td>
<td>170</td>
<td>34.3</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>77423</td>
<td>3.3</td>
<td>84</td>
<td>17.0</td>
</tr>
<tr>
<td>7</td>
<td>45</td>
<td>62085</td>
<td>3.0</td>
<td>121</td>
<td>24.4</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>87231</td>
<td>1.5</td>
<td>134</td>
<td>27.1</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>237408</td>
<td>2.6</td>
<td>14</td>
<td>2.8</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>328224</td>
<td>1.4</td>
<td>28</td>
<td>5.7</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>317788</td>
<td>1.0</td>
<td>8</td>
<td>1.6</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>754019</td>
<td>3.2</td>
<td>25</td>
<td>5.1</td>
</tr>
<tr>
<td>17</td>
<td>5</td>
<td>1018578</td>
<td>5.5</td>
<td>8</td>
<td>1.6</td>
</tr>
<tr>
<td>20</td>
<td>13</td>
<td>355455</td>
<td>5.0</td>
<td>38</td>
<td>7.7</td>
</tr>
<tr>
<td>21</td>
<td>2</td>
<td>903918</td>
<td>1.9</td>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>22</td>
<td>2</td>
<td>903953</td>
<td>2.0</td>
<td>8</td>
<td>1.6</td>
</tr>
<tr>
<td>23</td>
<td>2</td>
<td>439619</td>
<td>1.0</td>
<td>4</td>
<td>0.8</td>
</tr>
<tr>
<td>25</td>
<td>3</td>
<td>382012</td>
<td>1.2</td>
<td>13</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Average bid is the average of all bids that a particular firm submitted, No. Participation is the total number of bids that a particular firm submitted and % participation is the fraction of seal coat contracts a particular firm bid for.

### Table 3: Bid concentration

<table>
<thead>
<tr>
<th>Number of bids</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of contracts</td>
<td>29</td>
<td>87</td>
<td>190</td>
<td>118</td>
<td>44</td>
<td>22</td>
<td>5</td>
</tr>
</tbody>
</table>

31
Table 4: First and second lowest bids

<table>
<thead>
<tr>
<th>Observation</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>BID1</td>
<td>466</td>
<td>19135</td>
<td>227427</td>
<td>3893</td>
</tr>
<tr>
<td>BID2</td>
<td>466</td>
<td>207079</td>
<td>244897</td>
<td>4679</td>
</tr>
<tr>
<td>BID21</td>
<td>466</td>
<td>15724</td>
<td>29918</td>
<td>33</td>
</tr>
</tbody>
</table>

Table 5: Distances (in miles)

<table>
<thead>
<tr>
<th>DIST</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIST1</td>
<td>122.3</td>
<td>0</td>
<td>584.2</td>
</tr>
<tr>
<td>DIST2</td>
<td>151.9</td>
<td>0</td>
<td>585.2</td>
</tr>
<tr>
<td>DIST3</td>
<td>177.9</td>
<td>0</td>
<td>637.6</td>
</tr>
<tr>
<td>DIST4</td>
<td>166.4</td>
<td>11.2</td>
<td>608.6</td>
</tr>
<tr>
<td>DIST5</td>
<td>160.3</td>
<td>13</td>
<td>555.2</td>
</tr>
<tr>
<td>DIST6</td>
<td>177.9</td>
<td>63</td>
<td>484.4</td>
</tr>
<tr>
<td>DIST7</td>
<td>91</td>
<td>44</td>
<td>128.9</td>
</tr>
</tbody>
</table>

Table 6: Summary statistics for restricted data set

<table>
<thead>
<tr>
<th>Variable</th>
<th>No. Obs</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winning Bid</td>
<td>441</td>
<td>175,000</td>
<td>210,000</td>
<td>3893</td>
<td>1,732,500</td>
</tr>
<tr>
<td>Markup: (Winning Bid-Estimate)/Estimate</td>
<td>139</td>
<td>0.0031</td>
<td>0.1573</td>
<td>-0.3338</td>
<td>0.5421</td>
</tr>
<tr>
<td>Normalized Bid:Winning Bid/Estimate</td>
<td>139</td>
<td>1.0031</td>
<td>0.1573</td>
<td>0.6662</td>
<td>1.5421</td>
</tr>
<tr>
<td>Money on the Table: 2nd Bid-1st bid</td>
<td>134</td>
<td>15,748</td>
<td>19,241</td>
<td>209</td>
<td>103,481</td>
</tr>
<tr>
<td>Normalized Money on the Table: (1st Bid-2nd Bid)/Est</td>
<td>134</td>
<td>0.0776</td>
<td>0.0888</td>
<td>0.0014</td>
<td>0.5099</td>
</tr>
<tr>
<td>Number of Bidders</td>
<td>139</td>
<td>3.280</td>
<td>1.0357</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Distance of Winning Firm</td>
<td>134</td>
<td>188.67</td>
<td>141.51</td>
<td>0</td>
<td>584.2</td>
</tr>
<tr>
<td>Distance of Second Highest Bidder</td>
<td>134</td>
<td>213.75</td>
<td>152.01</td>
<td>0</td>
<td>555</td>
</tr>
<tr>
<td>Capacity of Winning Bidder</td>
<td>131</td>
<td>0.3376</td>
<td>0.3160</td>
<td>0</td>
<td>0.9597</td>
</tr>
<tr>
<td>Capacity of Second Bidder</td>
<td>131</td>
<td>0.4326</td>
<td>0.3435</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7: Concentration of Firm Activity by State.

<table>
<thead>
<tr>
<th>Firm</th>
<th>MN. Concentration</th>
<th>ND. Concentration</th>
<th>SD Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.2781</td>
<td>0.7218</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.2377</td>
<td>0.7623</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.1246</td>
<td>0.5338</td>
<td>0.3414</td>
</tr>
<tr>
<td>6</td>
<td>0.8195</td>
<td>0.1804</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.9572</td>
<td>0.0427</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0.7290</td>
<td>0.2709</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 8: Reduced form bid function

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>Coefficient</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>.046923 (3.64072)</td>
<td>$\alpha_1$</td>
<td>.031346 (1.99253)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>.174253 (8.42785)</td>
<td>$\alpha_2$</td>
<td>.153158 (3.29708)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>.040483 (.555484)</td>
<td>$\alpha_3$</td>
<td>.036645 (.503462)</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>.020369 (1.45323)</td>
<td>$\alpha_5$</td>
<td>.034269 (2.12646)</td>
</tr>
<tr>
<td>R Square</td>
<td>.849138</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix A. Dynamic Bidding with Continuation Values

In the text, we assume that when bidder $i$ does not win the current job, its continuation value (or option value attached to losing a job) is the same regardless of the identity of the winner for the current job. In this appendix, we will investigate what happens when this assumption fails. To simplify the analysis, without loss of generality, we assume the following: 1) bidder $i$ participates in all the auctions. Auctions are indexed by $t$, $t = 1, \ldots, T$. 2) The state variable is just the used capacity, denoted as $u_{it}$. Suppose the job workload implied by project $i$ is $\Delta u_t$.

We look for (Markov-Perfect) equilibrium in which each bidder bids according to a strictly increasing bid function $B_i(.)$ (in its private cost $c_i$). Let $\phi_i(.)$ be the inverse function of $B_i(.)$. When bidder $i$ wins the current job, its capacity increases by $\Delta u_t$ in the next period, while the capacities of its competitors all remain the same. The continuation function in this case is $V_i(u_{it} + \Delta u_t, u_{-it})$, abbreviated as $V_{i,W}(t)$. When bidder $j$ wins the current job, where $j \neq i$, only $j$’s capacity will increase by $\Delta u_t$ in the next period, and the continuation function in this case for $i$ is $V_i(u_{it}, u_{jt} + \Delta u_t, u_{-(i,j)t})$, abbreviated as $V_{i,j,L}(t)$. Suppose all rival firms bid according to $B_j(.)$. Given $c_i$, when $i$ bids $b$, its expected profit is

$$E\pi(b|c_i) = E[1_{b<B_i(c_i)}(b - c_i) + \sum_{j \neq i} 1_{b>B_i(c_i)}(b - c_i)]$$

$$+ \sum_{j \neq i} \int_0^{\phi_j(b)} \prod_{k \neq j, k \neq i} (1 - F_k(\phi_k(b)))dF_j(c) \cdot V_{i,j,L}(t)$$

Differentiating with respect to $b$, and collecting terms, we have

$$\frac{\partial E\pi}{\partial b} = \prod_{j \neq i} (1 - F_j(\phi_j(b))) - \sum_{j \neq i} f_j(\phi_j(b))\phi_j'(b) \prod_{k \neq j, k \neq i} (1 - F_k(\phi_k(b)))[b - (c_i + V_{i,L,j}(t) - V_{i,W}(t))]$$

It is now clear that if $V_{i,L,j}(t)$ depends on $j$, the identity of the winner firm, then the ensuing equilibrium analysis quickly becomes intractable. However, if the option value of not winning, $V_{i,L,j}(t) - V_{i,W}(t)$, does not depend on $j$ and furthermore, if this option value is the same across all $i$’s, then the equilibrium analysis in Section 5 still applies here. Note that even in that case, the equilibrium bid functions $\phi_j(b)$ depend on the time period $t$, the state
of the industry and project $t$ specific variables such as the distances of all firms to the project, observable project cost information and so forth.
Bibliography.


