What’s in a Name?
Reputation as a Tradeable Asset

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Abstract

A firm’s reputation is considered an important asset. I develop a model in which a firm’s only asset is its name – which is associated with its reputation – and study the economic forces which cause names to be valuable, tradeable assets. A simple adverse selection model together with an assumption on the non-observability of shifts of ownership guarantees that in equilibrium the market for names is active. This result is robust to both finite and infinite horizons, in contrast to standard results in the reputation literature. I also show that situations in which only good types buy names with a good reputation cannot be sustained in equilibrium.

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Our names are labels, plainly printed on the bottled essence of our past behavior.

— Logan P. Smith

Afterthoughts

1 Introduction

What is a name? It is exactly the label that summarizes the list of physical attributes, past behavior, and any other characteristics that the carrier of the name has. In our language-based society this is our way of representing a large amount of information in usually one or two words. Anything we can perceive or recognize is automatically labeled by a unique name in order to distinguish it from anything else in our world. This is also true for firms: Once a firm is established, it will be recognized by its name, which is uniquely associated with its characteristics and past performance.

A large part of the modern theory of the firm is devoted to investigating the consequences of asset ownership, and deals with well defined tangible assets that can be bought and sold (see, e.g., Hart 1995). It is well known, however, that many firms have intangible assets, one of the more important of which is the firm’s name, or actually its reputation that is conveyed by its name. This paper concentrates on the firm as a bearer of reputation (see Milgrom and Roberts (1992, pp 331-2) and Kreps (1990)) and attempts to answer two theoretical questions: First, what forces can create value for names that will guarantee trade of names in equilibrium, and

1 I have taken the liberty of borrowing this quotation from Fombrun (1996).

2 A firm’s intangible assets are usually hidden in its balance sheet, and their value is calculated only when the firm is sold, by subtracting the value of the tangible net assets from the firm’s sale price. This “goodwill”, as accountants refer to it, is meant to capture the value of these intangibles, including the firm’s reputation. In fact, between 1980 and 1990, the value of intangible assets in the United States increased nearly tenfold from $45 billion to an estimated $400 billion (see Fombrun 1996 p. 86).
second, what reputational effects will characterize the market for names. I will then argue that this may shed some light on the reality of name purchases.

An attempt to develop a theory of the firm as a bearer of reputation was first made by Kreps (1990). He gives a simple example which demonstrates, using the ideas of the folk theorem in repeated games, how reputation can become a tradeable asset. Kreps lays out a model in which there is a sequence of short lived suppliers of a service, and short lived buyers of that service. For there to be gains from trade each buyer must, at his turn, trust the supplier, who can either honor or abuse the buyer’s trust – this is an infinitely repeated, one-sided sequential Prisoner’s dilemma. Kreps constructs an equilibrium in which a “firm” is created, and buyers will only trust a supplier who owns this firm – i.e., who is represented by the firm’s name – as long as the firm never abused trust in the past. In the proposed equilibrium a potential supplier will buy the firm (which is just a name) from his predecessor, he will then honor the trust given to him, and will finally sell the firm to a new supplier. In this equilibrium the price of the firm is set so that a supplier will buy the firm only if he intends to maintain its reputation. That is, the short run gains from abusing trust will be outweighed by the inability to resell the firm to a new supplier. Kreps’s example captures an important point: There is a way in which an intangible asset such as reputation can be assigned transferable value, and this enhances welfare for both buyers and suppliers by supporting a trust-honor equilibrium.

As Kreps himself admits, there are two problems with his story. The first is that of multiple equilibria. There are many other equilibria in his example in which the firm is not bought, its name has no value, and thus no intangible asset is preserved. That is, there are many equilibria in which names are not traded, and the forces which should lead the economy to the equilibrium in which names are valuable are not determined. The second problem, which can be corrected at a cost of a more complicated model, is that the horizon must be infinite. However, moving to a finite
horizon with incomplete information (as Kreps suggests) will still require that the horizon be “long enough” for there to be a value of maintaining a reputation. In contrast, this paper presents a model in which names are traded in all equilibria, and this result is robust to very short horizons (in fact, two periods are enough).

In the model presented here, as in Kreps’s story, there are suppliers and buyers of a service, and the only asset a firm has is its name. Thus, selling the firm’s name amounts to selling its reputation. Yet, unlike the standard game-theoretic models that deal with reputation the model presented here is one of adverse selection (different types of agents with different intrinsic abilities) alone; agents do not have a choice of actions that might affect their reputation. This allows to abstract from the problems created by the “end game” effects of shirking which can only be remedied by very long horizons.

Another point in which the model differs from standard economic theory is reflected in the central assumption of the paper. It is assumed that the transactions carried out in the market for names (the shifts of ownership of firms) are hidden from the eyes of the potential clients of these firms. That is, when a client decides to hire a firm’s service, she will not know if the agent who currently owns and runs the firm is himself responsible for the firm’s past performance. This extreme assumption tries to capture the more realistic idea that not all buyers know who owns and runs the firms they buy from. The assumption, and ways to weaken it so as to meet reality are thoroughly discussed in section 2.

The first main result is that names must be actively traded in all equilibria. This

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3When the degree of incomplete information goes to zero, the necessary length of the game needed to support these reputational equilibria goes to infinity (see Kreps, Milgrom, Roberts and Wilson (1982)). Also, the problem of multiple equilibria still persists (see Fudenberg and Maskin (1986)).

4See Fudenberg and Tirole (1990), Chapter 9, for a summary of the standard game-theoretic models. Examples of other reputational models which have both Adverse Selection and possible actions are Diamond (1989) and Tirole (1996).
result relies heavily on the non-observability assumption which is the source of value for names in my model. If names were not traded, a good past should generate expectations for good future performance. Thus, a new agent can secretly buy a name with a good history which creates expectations for good future performance, and he would gain a higher revenue than he would with a name with no history. In equilibrium, clients are aware of this trade and will update their expectations accordingly, but nonetheless names will retain some value and will be actively traded.

The second result is that there cannot be an equilibrium in which good agents fully separate themselves from the population by buying good names; every equilibrium must have some bad types buying names as well. This is where the paper sheds light on an effect that has so far been ignored. In Kreps’s model, people will buy a good name only if they intend to maintain it. Interpreting this into an adverse selection framework would mean that good types value good names more than bad types because it is easier for them to maintain the name. I call this the “Reputation Maintenance Effect”. Another effect, however, is present: it is easier for good types to build their own name, which causes them to value an existing good name less than bad types who cannot easily build a name for themselves. I call this the “Reputation Start-up Effect”. It is shown that if only good types buy names then market expectations cause the Start-Up effect to overcome the Maintenance effect, which in turn causes bad types to value names more than good types do.

An important distinction between my model and Kreps’s model is the fact that it does not rely on bootstrap equilibria. Kreps’s model, as any other repeated game model, relies on the existence of multiple equilibria to support the “desired” equilibrium in which reputation matters and names are traded. This is also true for the “Brand Name” reputation model of Klein and Leffler (1981). That is, punishment to a “bad” equilibrium is what supports the desired equilibrium. Here, in contrast, any equilibrium in which names are traded is not supported by a threat to move to
another equilibrium, but rather by the (correct) updated beliefs of the clients.\textsuperscript{5} An implication of the bootstrap equilibrium in the repeated game approach is its failure to determine the dynamics that cause names with no initial value to become valuable, a process that is well documented in reality. In any repeated game model all reputational equilibria will have some reputation value that is set ad hoc at the beginning of the game and is thereafter fixed.\textsuperscript{6} Here, in contrast, these dynamics will emerge in a natural way. The adverse selection model employed means that fluctuations in value are due to the market correctly changing its beliefs with respect to who is running the firm. I believe that this is a more reasonable behavioral description of what causes the value of names to fluctuate in reality.

The paper is organized as follows: Section 2 describes the model, Section 3 establishes the first main result that trade of names will occur in all equilibria, while the second main result on the composition of name buyers is presented in section 4. Section 5 describes the analysis for the infinite horizon economy, and section 6 provides some concluding remarks and directions for future research.

\section{The Economy}

Consider a simple model of economic activity where in each period a client (or buyer of a service) employs an agent (or seller of a service) for that period only. The model

\textsuperscript{5}Note that when equilibria are restricted to be Markov Perfect Equilibria (MPE), then in Kreps’s model, as in any other repeated game treatment of reputation, no reputational equilibria are MPE. Here, in contrast, all equilibria are MPE because clients change their willingness to pay due to updated beliefs, and not as a punishment. See Maskin and Tirole (1988 p.592) for a similar discussion. For an analysis of reputational MPE see Mailath and Samuelson (1997).

\textsuperscript{6}As Kreps alludes to in his paper, his model can be modified so that names lose value in equilibrium. This can be done by adding some noise to the firm’s performance, and equilibria similar to those in Green and Porter (1984) can be sustained in which names will lose value with probability one. Note, however, that value cannot be enhanced in these equilibria, and the initial value of a reputation in any equilibrium will be set ad hoc.
is one of adverse selection in which there are different types of agents who differ by the probability of succeeding in the task they are employed for (e.g., delivering a “quality” good). It is assumed that there is a continuum of clients and agents, and the price of supplying a service is determined competitively. To simplify, assume that the clients are on the long side of the market. That is, the measure of the continuum of clients is larger than that of the agents so that competition causes each client to pay her full surplus when transacting with an agent. This also implies that there will be full employment of the agents in the economy.

Following the classic adverse selection literature (see, e.g., Akerlof 1970) the following assumption is made,

**Assumption A0:** *Compensation cannot be based on the transaction’s outcome.*

That is, problems of verifiability prevent the parties from writing outcome-contingent contracts because courts cannot distinguish success from failure. This implies that each client who employs an agent will pay up-front for the expected value of the service supplied. For simplicity assume that all clients value success equally, as they do failure: If the outcome is successful it yields a return of 1, while if failure occurs it yields a return of 0.

Each agent in this economy runs his own firm, which is represented by a *name*, and it is assumed that no two firms can share the same name. An agent, at the beginning of his lifetime, has two alternative choices: He can either *choose* a name to represent his firm, which implies that he will have no initial track record, or he can *buy* a name from an agent who is about to exit the active economy, inheriting the track record associated with that name. It will become clear later how the expected value of a firm’s service is determined by the perfect observation of that firm’s past performance.

It is common in economic theory to assume that all transactions that take place in
the market are readily observable to all. Instead, I wish to claim that this is not true in reality, at least not entirely. Consider, for example, a restaurant you enjoyed going to. It might have been recommended by some critic, say, in the dining section of last month’s magazine, or by a friend. That is, you have “observed” the restaurant’s past performance in this indirect way. Now, before going to the restaurant, will you check whether the chef/owner is the same person responsible for the critic’s review or for your friend’s satisfying experience? I believe the answer is no, which comes to the heart of the point I wish to make: It is not true that shifts of ownership are readily observable by all clients. Of course, at some cost almost everything is observable, but to make my point I will go to the extreme case of infinite costs of observation:

**Assumption A1:** Shifts of name-ownership are not observable by clients.

Assumption A1 implies that when a client employs an agent who has a name with a history she cannot determine whether the agent himself is responsible for that history or whether he has just bought it. This extreme assumption can be easily weakened. For example, if only a proportion of the population does not observe shifts of ownership then the qualitative results of the paper will continue to hold. The analysis, however, will become more cumbersome and will not add to understanding of the forces which drive the results.\(^7\) Thus, in the eyes of the clients the firm is represented only by its name, and the identity of the actual agent or group of agents who produce the good and own the firm are left unknown.\(^8\) The point is that the impact of the current owner on the firm’s past performance remains uncertain.

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\(^7\) Another way to weaken the assumption would be with a positive cost of observing shifts of ownership, distributed randomly accross clients with full support over the positive real numbers. This is analogous to a probabilistic observation of shifts of ownership.

\(^8\) I believe that the idea of reputation as an asset will have significance in supporting the formation of partnerships, where reputation will be the “glue” that keeps partners together, and alleviates the problems of free riding and shirking. See the discussion in section 6.3.
Since clients cannot observe how names shift from one agent to another, a natural assumption in the same flavor is that clients cannot observe how agents shift from one name to another. More precisely,

**Assumption A2:** *At the beginning of each period every active agent can either choose to retain his past name or unobservably change it.*

Assumption A2 says that as long as an agent retains his name, the history of his past performance and any previous past performance under this name is perfectly observable. However, once the agent chooses a new name, then the past record of this agent is erased and he can just as well be an agent that has now arrived into the economy with a clean record.\(^9\) It turns out that all the qualitative results of this paper will hold even when a history “sticks” to an agent as long as he is active. However, there is more than a grain of truth to the fact that some people manage to hide their unlucky past by shifting from one area to another. In particular, it is known that many restaurant owners had some failures before they manage to establish a good restaurant, if ever. Moreover, their bad history will not necessarily follow them when they switch the restaurant they own and run. In fact, it has been observed that name changing is common practice among firms and it affects the stock prices in a positive way which is consistent with the model presented here.\(^10\)

A final assumption is made regarding the process of changing one’s name. Clearly, changing a name requires some ingenuity, otherwise one can be revealed as a failure

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\(^9\)An agent can also abandon his past name and then buy a name from another agent. In all the equilibria of the model presented it turns out that agents who wish to abandon their past are indifferent between choosing a new name or buying a name. Therefore, we will assume that agents who wish to erase their past will just choose a new name.

\(^10\)See Horsky and Swyngedouw (1987) who show that name changes are associated with improved performance. It seems plausible to argue that if all clients observe this name-change then there should be no changes in expectations. However, if some clients do not observe this change, and will therefore ignore past poor performance, then the informed agents in the economy should form expectations of higher stock prices.
who tried to erase his past. This can be naturally captured by some distribution of the costs of changing names across the agents. If this distribution will have full support over the positive real line (some agents are infinitely clumsy) then these costs can be captured by the following assumption,

**Assumption A3:** There exists a proportion \( \varepsilon > 0 \) (which can be arbitrarily small) of agents who cannot change their name, and all others can costlessly change their name.

As I will demonstrate later, this assumption is needed to weed out some “unreasonable” equilibria that can arise when all agents can change their name costlessly. Note that if both A2 and A3 are dropped then all the results in the paper will still hold.

The adverse selection is captured by the existence of two types of agents: “good” agents, or \( G \)-types, in proportion \( \gamma \), and “bad” agents, or \( B \)-types, in proportion \( 1 - \gamma \). \( G \)-types succeed with probability \( P_G \in (0,1) \), and \( B \)-types succeed with probability \( P_B < P_G \). Without loss of generality assume that \( P_B = 0.11 \).

Finally, assume that agents are active in the economy for two periods, after which they leave the active economy for “retirement”. Agents will enter and exit the economy in an overlapping-generations (OLG) fashion, in which the total size of the population, and the distribution of types of agents is constant over time. In contrast, clients live for only one period and can observe the firms’ (names’) track records for assessment of their types. Furthermore, clients are anonymous and cannot contract among themselves.12

\[\text{11} \text{It might seem that this assumption may have strong effects. However, as will become clear from the analysis, having } P_B > 0 \text{ will not change the qualitative results. It will also become clear from the analysis that having more than two types will not change the qualitative results of the paper.} \]

\[\text{12} \text{The reason clients need to disappear is to prevent long term contracts between individual clients and agents which can eliminate some of the adverse selection. Also, clients need to be anonymous so that “old” principals cannot inform “new” ones of the agents behind the names.} \]
3 Names as Tradeable Assets

This section describes the forces causing names to be traded in equilibrium, and investigates some characteristics of the market for names. Since an OLG model is employed, it would be natural to follow the standard OLG literature and analyze the steady states of this economy. However, it is well known from the macroeconomic literature on the value of money that many features of equilibria in the steady state would unravel and break down in a finite horizon model. For this reason I begin by investigating a finite version of the OLG model to show that the results of this paper are independent of the length of the economy’s horizon, as long as the flavor of overlapping generations is maintained. To see this in the most transparent way I will start with a two period model.

Consider a two period model in which the size of the population of agents is constant and the features of an OLG model are still captured. To accomplish this, one generation of agents lives in both periods, while two other generations will live in only one period. Furthermore, the size of these one period generations are equal to the size of the two period generation. Thus, this economy will always consist of a proportion $\gamma$ of $G$-types and a proportion $1 - \gamma$ of $B$-types as described in section 2 above. To simplify notation assume that each generation of agents is of measure one, so that the total measure of agents is 2. This convention will be adopted throughout the paper. The time line of this two period economy is described in Figure 2 below:
At date $t = 0$ the economy starts with agents from generations 0 and 1 choosing names for their firms, and then clients paying firms up-front for their services. Clearly, given that no prior information is available to the clients, they will pay the same wage to all firms, which equals the expected benefit from hiring a firm,

$$w_0 = \gamma[P_G \cdot 1 + (1 - P_G) \cdot 0] + (1 - \gamma) \cdot 0 = \gamma P_G$$

This follows because only $G$-types (in proportion $\gamma$) will succeed with probability $P_G$, the client’s value from success is 1, and her value from failure is 0.

At date $t = 1$ the analysis is less straightforward. When a client decides to hire a firm there will be two kinds of firms: some firms will have a past history while others will not. In turn, firms with a past history of success will fall into two categories: they can either be operated by a good type who succeeded and lived on to the second period (recall that $P_B = 0$), or by a new agent who bought the name from a good agent who retired. An equilibrium of the two-period economy will be characterized by two markets at $t = 1$; the wages clients will pay for hiring firms with different track records and the prices agents will pay for names with different track records. Before proceeding with the two-period model, the following additional notation will be used: Since only past histories matter then two distinct names with the same history should generate the same expectations for future success at $t = 1$. For this reason let $S$ denote any name at $t = 1$ with a past success, $F$ with a past failure, and
a name with no past. The equilibrium wages of firms at \( t = 1 \) will be denoted by \( w_1(h) \), \( h \in \{S, F, \emptyset\} \), and the equilibrium prices of names at \( t = 1 \) will be denoted \( v(S) \) and \( v(F) \) respectively. Also, let \( \Pr\{G|h\} \) denote the conditional probability of a firm's agent being a \( G \)-type when its history is \( h \in \{S, F, \emptyset\} \). The first main result of the paper can now be established:

**Proposition 1:** \( S \) names will be traded in all equilibria.

**proof:** Assume in negation that there exists an equilibrium in which no names are traded at \( t = 1 \). This implies that the price of a name with a past success must be zero since the supply of \( S \) names is positive and is equal to the measure \( \gamma P_G \) (the good types of generation 0 exit the economy, of which a proportion \( P_G \) succeeded). From Assumption A3 it must be that \( \Pr\{G|S\} = 1 \), and since \( 1 - \gamma > 0 \) it is always true that \( \Pr\{G|\emptyset\} < 1 \). This implies that \( w_1(S) > w_1(\emptyset) \), which in turn implies that any agent who has no past will be willing to pay a positive price for a \( S \) name, a contradiction.\(^{13}\)

This result is driven by the nature of the adverse selection and by assumptions A1 and A3. Since good names would create expectations of success, it must be that new agents will be willing to buy them, and "hide" behind the name. With full observability of ownership shifts this result is no longer true – assigning beliefs to clients saying that only bad types buy names will support equilibria with no trade of names. This intuition illuminates a central point of this paper: non-observability of

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\(^{13}\)Assumption A3 guarantees that \( \Pr\{G|S\} = 1 \) when no names are traded. This assumption helps rule out "bad" equilibria of the following form: All agents attempt to abandon their name after the first period, and in the second period clients believe that a firm with any history is worse than the average firm with no track record. Since a proportion \( \varepsilon \) of the agents will not be able to abandon their name, these beliefs cannot be sustained in equilibrium for the following reason: No new agent would choose to buy any name, and all agents would attempt to loose their name. But, a proportion \( \varepsilon \) of the population will have their name stick to them, which in turn implies by Bayes rule that \( \Pr\{G|S\} = 1 \).
ownership shifts is a necessary condition for the market for names to be active in all equilibria. Note that if $0 < P_B < P_G$ then the critical part of the proof, that Bayes updating implies $w_1(S) > w_1(\emptyset)$ under no trade, continues to hold and the result goes through.

Before characterizing the equilibria of the two-period economy, note that the end of the economy happening at $t = 2$ has two effects. First, both good and bad types who enter the economy at $t = 1$ will have the same benefit from buying a name. Second, agents who failed and continue to live on to the second period, can change their names and thus will value an $S$ name exactly as new agents will. This yields the following result:

**Lemma 1:** In any equilibrium, new agents and agents who failed and actively continue, will be indifferent between buying a $S$ name and not buying one.

**proof:** We will concentrate on new agents (the analysis readily extends to agents who failed and actively continue). Assume first that some agents strictly prefer buying a $S$ name to not buying one. Observe, however, that the only effect a name has for agents at $t = 1$ is to increase their wages relative to the wage of an agent without a name, and this effect is identical for both $G$-types and $B$-types. So, if some agents prefer buying a $S$ name then all agents entering the economy at $t = 1$ would share these preferences. But the measure of new agents is 1 and the measure of supplied $S$ names is $\gamma P_G < 1$ which creates excess demand, and this in turn will cause the price of a $S$ name to rise. Therefore, in equilibrium no type of agent can strictly prefer to buy an $S$ name. From Proposition 1 we know that trade of $S$ names must occur which concludes the proof of this lemma.\[\]\[\]

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14Note that with symmetric uncertainty in which the agents do not know their types (as in Holmstrom 1982), Proposition 1 will still hold. The intuition is exactly the same.
For completeness, I will characterize the equilibria of the two period model below. Proposition 1 establishes that $S$ names must be traded, and in the characterization of equilibria below I will assume that $F$ names will not be traded. This immediately implies that all agents who fail in their first period will change their name. For ease of the equilibria calculations I will take $\varepsilon$ (from A3) to be zero.\footnote{One can construct equilibria in which both $F$ and $S$ names are traded, but they require "unreasonable" beliefs of clients with respect to agents with no past history. By adding a similar assumption to A3 so that some proportion of agents "lose" their name, these equilibria will not exist. I will refrain from providing the formal analysis which will be a straightforward exercise similar to the rest of the analysis in this section. Also, the calculations at $\varepsilon = 0$ do not change any of the qualitative results.} Also, since "old" agents will be indifferent between buying names (after erasing their past) and just erasing their past and starting fresh, I will also assume that $S$ names are only bought by new agents.

In any equilibrium clients must have correct beliefs as to the composition of new good and bad types who will buy names at $t = 1$. Thus, one must take account of this composition to define an equilibrium. Let $\delta$ ($\rho$) be the proportion of new good (bad) types who buy $S$ names at $t = 1$. In equilibrium $\delta$ and $\rho$ must satisfy the market clearing condition which guarantees that the supply of $S$ names is equal to the demand,

$$\gamma P_G = \delta \gamma + \rho (1 - \gamma). \tag{1}$$

Recall that clients will pay their full expected surplus up-front, so that in equilibrium it must be that $w_1(h) = \Pr\{\text{success}|S\} = \Pr\{G|S\} \cdot P_G$. Given $\delta$ and $\rho$ that satisfy (1) above, the probabilities are determined by Bayes Rule as follows,

$$\Pr\{G|S\} = \frac{\gamma P_G + \delta \gamma}{\gamma P_G + \delta \gamma + \rho (1 - \gamma)} \tag{2}$$

$$= \frac{\gamma P_G + \delta \gamma}{2 \gamma P_G}, \tag{3}$$
and,

\[
\Pr\{G|G\} = \frac{\gamma(1 - P_G) + (1 - \delta)\gamma}{\gamma(1 - P_G) + (1 - \gamma) + (1 - \delta)\gamma + (1 - \rho)(1 - \gamma)}
\]

\[
= \frac{2\gamma - \gamma P_G - \delta\gamma}{2 - 2\gamma P_G},
\]

(4)

(5)

where the second equality in both equations follows from market clearing and some simple algebra. Therefore, the correct beliefs about \( \delta \) and \( \rho \) will determine \( w_1(h) \) for all \( h \) according to Bayes updating as described in (2) and (4) above. Also note that in equilibrium we must have \( v(S) = w_1(S) - w_1(\emptyset) \) which follows immediately from Lemma 1 and Proposition 1. If it were the case that \( v(S) < w_1(S) - w_1(\emptyset) \) then agents would not be indifferent between buying a \( S \) name or not, which contradicts Lemma 1. If it were the case that \( v(S) > w_1(S) - w_1(\emptyset) \) then agents would strictly prefer not to buy a \( S \) name at \( t = 1 \) which contradicts Proposition 1. It turns out that the observations above are all one needs to characterize the equilibria of the two-period model, and an equilibrium will be a six-tuple \( (\delta, \rho, w_1(S), w_1(F), w_1(\emptyset), v(S)) \). Note, however, that wages and prices will be generated by the correct beliefs about \( (\delta, \rho) \), so that a pair \( (\delta, \rho) \) will in fact represent the equilibria parameters.

**Lemma 2:** There exists \( \delta^* < P_G \) so that \( (\delta, \rho) \) is an equilibrium if and only if \( \delta \in [\delta^*, P_G] \) and \( (\delta, \rho) \) satisfy (1) above.

**proof:** Market clearing must be satisfied in any equilibrium, thus \( (\delta, \rho) \) must satisfy (1). Also, it must be the case that in equilibrium \( v(S) \geq 0 \), or equivalently, \( w_1(S) - w_1(\emptyset) \geq 0 \) (otherwise no agent would buy a \( S \) name). Since \( w_1(h) = \Pr\{G|h\} \cdot P_G \), then using (2) and (4) above this inequality can be written as,

\[
\frac{\gamma P_G + \delta\gamma}{2\gamma P_G} \geq \frac{2\gamma - \gamma P_G - \delta\gamma}{2 - 2\gamma P_G},
\]

which after rearranging becomes, \( \delta \geq (2\gamma - 1)P_G \). Define \( \delta^* = \text{Max}\{0, (2\gamma - 1)P_G\} \). Since \( 0 < \gamma < 1 \) it must be the case that \( \delta^* < P_G \). On the other hand
we must have $\delta \leq P_G$ or else $\rho$ must be negative to satisfy market clearing. Thus, if $(\delta, \rho)$ is an equilibrium then it must be that $\delta \in [\delta^*, P_G]$ and $(\delta, \rho)$ satisfy market clearing. Furthermore, if $\delta \in [\delta^*, P_G]$ and $(\delta, \rho)$ satisfy market clearing then the wages and prices generated by correct beliefs will constitute an equilibrium. ■

From Lemma 2 it is clear that there are multiple equilibria in the two-period economy. One extreme equilibrium is where $\delta = \delta^*$, $w_1(S) = w_1(\emptyset)$ and $v(S) = 0$. In other words, one extreme is a pooling equilibrium where having a $S$ name has no meaning. Any other equilibrium will have some degree of separation, up to the other extreme in which $\delta = P_G$ and where strong separation will occur.\textsuperscript{16} In this other extreme equilibrium a proportion $P_G$ of the good types entering the economy at date $t = 1$ will purchase the $S$ names, and no bad types will buy any name. None of the agents who failed in the first period and continue to the second will purchase names, and all of them will change their name. This, of course, implies that $\Pr\{G|S\} = 1$, and,

$$\Pr\{G|\emptyset\} = \frac{2\gamma - 2\gamma P_G}{2 - 2\gamma P_G} < 1.$$  

The wages are determined as described earlier, $w_1(S) = P_G$, and, $w_1(\emptyset) = \frac{\gamma(1-P_G)}{\gamma(1-P_G)+1-\gamma} \cdot P_G$. Finally, the price of a $S$ name must be, $v(S) = w_1(S) - w_1(\emptyset) = \frac{1-\gamma}{\gamma(1-P_G)+1-\gamma} \cdot P_G$.

Proposition 1, the main result of this section demonstrates the common feature of all these equilibria: Trade of $S$ names must always occur in any equilibrium. One can argue, however, that the strongly-separating equilibrium in which only good types own $S$ names is appealing, and it seems almost natural to find some way of selecting it (or equilibria close to it) from the continuum of equilibria which the model offers.

\textsuperscript{16}The separation is “strong” in the sense that $S$ names are owned only by good types, yet there is some mixture of types who own new names due to the scarcity of $S$ names.
This reasoning would very much appeal to the theories of Klein and Leffler (1981) and of Kreps (1990), if one were to envision them in an adverse selection framework. The intuition which would lead to selection of this equilibrium as follows: If good types have an easier time maintaining a good reputation, then they should be able to outbid the bad types who are more likely to ruin a reputation. However, due to the indifference result of Lemma 1 it will not be possible to get such an effect in the two-period model. The characterization of equilibria is different, however, if the economy lasts for more than two periods. In fact, as the following section shows, if only one period is added then the situation in which only good types buy $S$ names is no longer an equilibrium. Thus the simple intuition offered above misses an important point of the model which will be demonstrated below.

4 Long Term Reputational Effects

Assume now that the economy lasts for three periods as follows:

\[
\begin{align*}
& t = 0 & t = 1 & t = 2 & t = 3 \\
\text{Generation 0:} & \quad \quad \quad \quad \quad \quad \\
\text{Generation 1:} & \quad \quad \quad \quad \quad \quad \\
\text{Generation 2:} & \quad \quad \quad \quad \quad \quad \\
\text{Generation 3:} & \quad \quad \quad \quad \quad \\
\end{align*}
\]

\textbf{Figure 3: A three period economy.}

The analysis at dates $t = 0$ and $t = 2$ in this three period model is identical to that of dates $t = 0$ and $t = 1$ respectively in the two-period model. Building on the notation of the previous section, let $w_t(h_t)$ denote the wage at time $t$ to a firm with
history $h_t$, where $h_1 \in \{S,F,\emptyset\}$ and $h_2 \in \{\emptyset,\emptyset S,\emptyset F, SF, SS\}$.
Similarly define $\Pr_t \{G|h_t\}$. For expositional convenience I will refer to names that end with a success and had no failures as a *successful name*. The following proposition shows that in this three-period model it is no longer true that an equilibrium can exist where only good agents buy successful names at $t = 1$ and at $t = 2$. Note that the proposition is stated for the case where only successful names are traded. The equilibrium can be more elaborate with $SF$ names traded at $t = 2$ and a careful analysis which verifies Proposition 2 for the most general case appears in Appendix A.

**Proposition 2:** For the three period model there is no equilibrium in which only successful names are traded, and they are bought only by good types.

Proposition 2 is proven by the following two Lemmas:

**Lemma 3:** If only $G$-types buy successful names and no names ending with $F$ are traded then $\Pr_1 \{G|S\} = \Pr_2 \{G|h_2\} = 1$, and $w_1(S) = w_2(h_2) = P_G$ for all $h_2 \in \{\emptyset S, SF, SS\}$.

**proof:** If only $G$-types buy $S$ names at $t = 1$ then $\Pr_1 \{G|S\} = 1$ and $w_1(S) = P_G$. If only $G$-types buy $\emptyset S$ or $SS$ names at $t = 2$ then $\Pr_2 \{G|SS\} = \Pr_2 \{G|\emptyset S\} = 1$ and $w_2(SS) = w_2(\emptyset S) = P_G$. Also, since no $SF$ names are traded at $t = 2$ then only good types who bought $S$ names at $t = 1$ will continue to the third period with a first period history of $S$ followed by a $F$, which in turn implies that $\Pr_2 \{G|SF\} = 1$ and $w_2(SF) = P_G$. 

The main point of Lemma 3 is that if only good types buy $S$ names at $t = 1$, and no one buys names ending with a $F$ at $t = 2$, then when a client sees a firm at $t = 2$

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17 Using the same reasoning as in the previous section, no name that had a $F$ in the first period will survive at $t = 2$. 

19
with a history of $SF$ which continues to be active, it must be run by a good type. That is, a failure coming after a success cannot cause clients to update their beliefs because it must be a good type who is running the firm. This is a familiar theme of fully separating equilibria in an adverse selection framework.

**Lemma 4:** If $w_2(SF) = P_G$ then $B$-types value $S$ names at $t = 1$ more than $G$-types do.

**Proof:** By starting a new firm at $t = 1$ $B$-types receive a utility of $u_B(\emptyset) = w_1(\emptyset) + w_2(\emptyset)$ since they will fail and change their name before $t = 2$. Buying a name gives them two benefits: getting $w_1(S)$ instead of $w_1(\emptyset)$ at $t = 1$, and getting $w_2(SF)$ instead of $w_2(\emptyset)$ at $t = 2$. Thus, the difference in utilities (net of the cost of a $S$ name) is $\Delta u_B = w_1(S) - w_1(\emptyset) + w_2(SF) - w_2(\emptyset)$. $G$-types, however, have the same benefit at $t = 1$, but they gain less at $t = 2$: if they don’t buy a name then they don’t get $w_2(\emptyset)$ for sure at $t = 2$, but rather they get this low wage only with probability $1 - P_G$, and get $w_2(\emptyset S) = P_G$ with probability $P_G$. Therefore, the difference in utilities for the good types (net of the cost of a $S$ name) is $\Delta u_G = w_1(S) - w_1(\emptyset) + (1 - P_G)[w_2(SF) - w_2(\emptyset)]$ and they are willing to pay less than $B$-types. ■

The intuition behind these two Lemmas is simple. If a dynamic equilibrium exists where only good types buy $S$ names then a name with a history that starts with a success, no matter what that history is, will be attributed to a good type. Thus, failures won’t be “punished” via low wages due to updated beliefs, nor will they reduce the value of the name. This, clearly, will give the bad types an incentive to buy names, and actually benefit from them even more than good types, who might still have a good future ahead of them even without buying a name. This clearly points to the nature of any equilibrium: It must be that both $B$ and $G$ types buy $S$
names at $t = 1$. Again, this result is not sensitive to $P_B = 1$. If $P_B > 0$ then the algebraic expression of Bayes Rule will be more cumbersome but the driving forces would remain in place – good names would depreciate too slowly if only good types bought them.

This analysis shows that different types will have different benefits from buying a name. Generally, two effects are responsible for this difference. The first effect comes from the fact that good types are more likely to maintain a good name than bad types are. This allows good types to reap benefits over a longer period of time (on average), which in turn gives them a higher willingness to pay for a good name than bad types would have. As mentioned in the introduction, I call this the **Reputation Maintenance Effect**. This effect is strongly related to the result obtained by Kreps: Reinterpret a good type as an agent who intends to honor trust, while a bad type cannot honor trust (it is extremely costly for him). Then, in Kreps’s model, an agent will buy the firm if and only if he intends to honor trust. This is the reputation maintenance effect at work. The second effect which arises in the present model is that good types can build a good name of their own while bad types cannot. Therefore, if a firm’s reputation is hard to depreciate (i.e., failures do not cause a strong enough depreciation of the reputation) then good types will have a lower willingness to pay than bad types –bad types gain more until the name is depreciated. I call this the **Reputation Start-Up Effect**. This effect is not present in Kreps’s model (or Klein and Leffler’s) because first, people cannot build their own name, and second, it is not a model of belief updating by the clients but rather a strategic model of cooperation. Proposition 2 can now be intuitively explained using these two effects: If only good types buy successful names then there is no depreciation of reputation after a failure.

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18Klein and Leffler (1981) have a similar force at work; A firm will invest in firm-specific sunk costs only if it intends to produce a high quality good and reap a stream of future profits, which will not be realized if the firm “cheats” and produces a low quality good.
This is the strongest case for the Reputation Start-Up Effect to kick in, and in fact it overcomes the Reputation Maintenance Effect, causing the bad types to value $S$ names more than good types will. In other words, the value of a name for each of the two types is endogenous to the proportions of good and bad types who buy it and good types cannot “outbid” bad types in equilibrium.

The intuition of this result seems to extend beyond the simple adverse selection model employed here. Mailath and Samuelson (1997) explore under what conditions reputational equilibria survive Markov perfection. They employ a model that combines actions (moral hazard) as well as adverse selection and conclude that (continuous) reputational MPE can exist only if there are unobservable exogenous shifts of types along the infinite horizon game. This is analogous to assumption A1 in this paper. Then, when they endogenize the shift of types and introduce an auction for reputations they get a result similar to Proposition 2: very high reputations are more likely to be bought by bad types, whereas average reputations are more likely to be bought by good types.

5 The Infinite Horizon Case

The previous sections demonstrated that the model employed in this paper generates reputation-relevant equilibria that are robust to very short horizons of the finite economy. For the sake of theoretical completeness it is natural to consider longer horizons, and in particular the infinite horizon economy. Note, however, that the analysis of Proposition 1 and Proposition 2 respectively readily extend as follows (the proofs are a straightforward adaptation of the proofs provided earlier),

**Proposition 3:** For any finite or infinite horizon economy names must be traded in all periods of any dynamic equilibrium.

and,
Proposition 4: For any finite or infinite horizon economy successful names must be bought by some bad types in all periods (but the last one) of any dynamic equilibrium.

The forces that drive Proposition 1 (Assumptions A1 and A3) continue to work in any finite or infinite horizon economy – if no names are traded then it must be that at any time $t$, $Pr_t\{G|\emptyset S\} > Pr_t\{G|\emptyset\}$, which would create demand for these names. Also, the forces that drive Proposition 2 (the Reputation Maintenance and Reputation Start-Up effects) will be present in any dynamic equilibrium – if only good types buy a name ending with $S$ at some time $t$, then bad types will value that name more for the next two periods.

It turns out that if reputations are identified with histories then the formal analysis of equilibria in the infinite horizon model is very tricky and quite cumbersome. Moreover, even when concentrating only on Steady State Equilibria (SSE), the analysis becomes intractable. For example, $\emptyset S$ names will clearly be traded in every period (where $\emptyset S$ means that the name was created last period and had a success then). This implies that $\emptyset SS$ names must also be traded, for if not then Assumption 3 implies that $Pr\{G|\emptyset SS\} = 1$. Such an iterative argument implies that any SSE must have all names with any consecutive number of successes (with no failures) traded. This implies that a SSE must satisfy a countable number of market clearing conditions, and must satisfy a countable number of inequalities to ensure trade.

The reason there is a countable number of market clearing conditions is because the supply of each name is endogenous to the proportions of good and bad types who buy other names. That is, even though it is common knowledge that current agents cannot be responsible for performance generated more than one period ago (they live for 2 periods), the proportions of good and bad types who buy names can (artificially) be history dependent. For example, assuming that only agents who exit the economy
sell their name, then the supply of $\emptyset S$ names is generated by agents who failed, then change there name, and finally succeed before exiting the economy. Let $\Sigma_{\emptyset S}$ denote the measure of this supply. Then, letting $\delta_1$ ($\rho_1$) be the proportion of good (bad) types who buy $\emptyset S$ names, the market clearing condition for $\emptyset S$ names is

$$\delta_1 \gamma + \rho_1 (1 - \gamma) = \Sigma_{\emptyset S}.$$ 

This, in turn, directly affects the supply of $\emptyset SSS$ names because the measure of the supply of $\emptyset SSS$ names is generated only by the good types who bought a $\emptyset S$ name, and succeeded twice. Therefore, the market clearing condition for $\emptyset SSS$ names is

$$\delta_3 \gamma + \rho_3 (1 - \gamma) = \delta_1 \gamma P_G^2,$$

and similarly for any odd number $n$ of $S$ repetitions the market clearing condition is

$$\delta_n \gamma + \rho_n (1 - \gamma) = \delta_{n-2} \gamma P_G^2 \quad \forall n \geq 3 \text{ odd}.$$ 

Then we need to make sure that each of these histories generates a wage that is at least as good as having no name, which leads to a countable number of inequalities using Bayes rule. That is,

$$\Pr\{G|\emptyset S \cdots S\} \geq \Pr\{G|\emptyset\} \quad \forall n \geq 1.$$ 

But the analysis is even more complex. One must consider whether failures will cause agents to change their name or to hang on to it. The latter implies that names ending with $SF$ will be traded. This in turn raises the question of whether names ending with two consecutive failures will be changed, and if not then they two must be traded. This iterative procedure can potentially go on \textit{ad infinitum}.

A way to simplify the analysis in order to establish a family of SSE would be to differentiate histories from reputation. One way of doing this is by using a “reputation mapping” so that clients do not observe exact histories, but rather some summary
of histories which is called reputation. Let \( r : H \rightarrow R \) be a mapping from the set of all possible histories to a reputation range which can be arbitrarily defined. For example, \( R = \{g, b\} \) would mean that the countable number of histories \( h \in H \) are mapped into two possible reputations, \( g \) (good) or \( b \) (bad), and these reputations are observed by the clients. This example is fully analyzed in Appendix B. The analysis of more complex reputations ranges (in particular, \( R = H \)) is left for future research.

6 Concluding Remarks

This paper begins with the observation that name-trading seems to be a rule, rather than an exception, and demonstrates how a natural assumption on the non-observability of ownership shifts is the necessary force needed to impose name-trading in all equilibria. The first point the paper makes is that a very simple adverse selection model together with this non-observability assumption yields reputational equilibria that are robust to very short horizons, unlike most of the standard reputation literature. In particular, this is in stark contrast to the only other model of names as an asset introduced by Kreps (1990), in which there are many equilibria where names are not traded. As Kreps writes,

“The reputation construction is decidedly fragile: If reputation works only because it works, then it could fall apart without much difficulty. In real life, these risks will appear as substantial costs of undertaking transactions in this way.” (p. 111)

Indeed, if real life situations were strictly ones of moral hazard alone then this conclusion would be true. However, introducing adverse selection can allow reputation to “build by itself” since past performance contains information about who is likely to be behind a name.
The second point the paper makes sheds light on the effects which cause different types to value reputations differently. Good types value good names because of their future prospects to maintain it, whereas bad types value good names because they lack the ability to build one themselves. It was shown that the appealing sought after situation in which good types fully distinguish themselves from bad types cannot be sustained in equilibrium because then the reputation start-up effect overcomes the reputation maintenance effect. This result is also due to the fact that an adverse selection model is employed, which implies that clients update their beliefs in equilibrium – if only good types buy good names then beliefs cannot be updated.

In support of the model presented here, I believe that adverse selection comes closer in spirit to the actual behavior of clients and markets than does moral hazard. When potential passengers stopped buying Valujet tickets because of their unfortunate accidents in the recent past, were they punishing Valujet for not cooperating? Or, were they rather updating their beliefs about the ability of Valujet’s managers and employees to provide them with adequately safe flight service? It might just be that the very recent merger plan between AirTran Airways and ValuJet, Inc. is a response to these unfavorable beliefs. In fact, the name ValuJet will be replaced and the newly created holding company will operate as AirTran Holdings, Inc. Similarly, is IBM no longer the well regarded “Big Blue” leader because the PC market is punishing IBM for not being at the frontier? Or, has the market concluded that the current generation of IBM managers and employees are not the “good” types who can lead the market. Of course, in reality moral hazard and adverse selection are both present. Diamond (1989) employs a moral hazard and adverse selection model to study reputation formation and the evolution of incentive effects of reputation on borrowers. An interesting exercise would be to include a moral hazard parameter into this paper’s model and to see if the incentive effects of reputation affect the composition of name buyers, and which names provide better work incentives to build a
reputation and maintain it. The work of Mailath and Samuelson (1997) suggests that both the reputational effects discussed earlier will carry over to a model which combines both moral hazard and adverse selection. I will conclude with a brief discussion of three interesting extensions and avenues for future research.

6.1 More Complex Organizations

The model analyzed here has single agent-owner firms providing services for the clients. In reality, firms are complex organizations made up of many individuals. Furthermore, in public firms the (often dispersed) owners do not actually perform the services themselves. I believe, however, that the analysis of the stylized model presented here can be extended to include the more general case of larger organizations.

First, changes in ownership of large organizations (shifts of share-blocks) are commonly associated with replacement of current management. If one interprets good owners as having the ability to screen potentially bad managers, while bad owners cannot, then both the results of name trading and of the reputational effects should continue to hold. Second, when considering dispersedly owned firms, one can interpret the competition for names not from the point of view of the owners, but rather from the managers who want to belong to the successful firms in order to increase their own reputation. This competition can be over the compensation package, that is, the level of their wages and the composition of monetary payments, payments by stock ownership, and payments by options ownership. If managers’ actions have long term effects, then immediate payments can cause the reputation start-up effect to overcome the reputation maintenance effect, which implies that bad managers would fiercely compete for these positions. If, however, payment is postponed to rely somehow on the future performance (thus, overcoming the problem of non-contractibility) then this situation will be remedied. This is a further justification of using stock options
as a form of payment to high ranking managers in firms.

6.2 Names as a Solution to Inefficiencies

In the model of this paper there were no inefficiencies with or without markets for names. The fact that on average it is worth hiring firms, and that the clients are on the long side of the market implies that there will be full employment of all firms (agents) in equilibrium. It would be interesting to modify the model in a way that causes unemployment, and second-best efficiency can only be attained through some signalling of types. Then, it would be natural to compare name-trading to other signalling devices (like bonds or “burning” money) both in the effectiveness of getting to a second-best equilibrium, and in the cost of doing this.

6.3 Collective Reputations and Partnerships

Tirole (1996) develops an interesting model of collective reputation as an aggregate of individual reputations. In his model, agents’ track records are observed with noise, and thus, the aggregate reputation of the group the agent belongs to affects the agents incentives. As in Diamond (1989), adverse selection is combined with moral hazard to extract a clear prediction from the model.

A very interesting avenue of future research would be to extend the model of this paper to try and make an attempt at considering how groups, or partnerships can be formed. First, the reputation of the group should be an aggregate of the individual reputations, and second, the groups should find ways to self-select their members. That is, it would be interesting to see if the fact that names are worth money, would give agents the incentives to form homogeneous groups of good types and bad types, or if some mixing is necessary.

Another aspect that needs to be considered is whether different partner-selection systems can substitute actual trade in names. For example, in situations where shifts
of ownership are observable to clients, such as the case for law and accounting firms\textsuperscript{19}, then the reputation maintenance effect might be overcome by the reputation start-up effect because clients can be locked in for some periods and not be able to leave their current service supplier. In this case, it might be that the up-or-out promotion system can ensure correct self selection, and that the value of good names can be reaped through competition of junior employees to enter good firms. This idea needs to be carefully formulated in order to fully understand these effects.

\textsuperscript{19}I would argue that the stakes of their services are high enough so that agents have an incentive to investigate who is actually behind the name (their identity, not their type). This is clearly true for large accounting and law firms which provide services to large organizations.
Appendix A

This appendix shows that the claim made in Proposition 2 is true for all equilibria of the three-period model discussed in section 4.

Proposition A.1: For the three period model there is no equilibrium in which $S$, $\emptyset S$, and $SS$ names are bought only by good types.

proof: As for the previous sections, it is assumed that only agents who exit the economy sell off their name, and only new agents entering the economy buy names. At $t = 1$, the only names sold will be $S$ names, since those agents who failed in the first period and continue to be active will clearly change their names. At $t = 2$, three different markets for names can exist: $SS$ names, $\emptyset S$ names, and $SF$ names. Assume in negation to the proposition that only $G$-types buy $S$, $SS$, and $\emptyset S$ names. If $SF$ names are not traded at $t = 2$ then the proof of Proposition 2 applies. Now assume that $SF$ names are traded at $t = 2$, and some proportion $\delta$ of $G$-types and $\rho$ of $B$-types buy these names. The utility that a good type entering the economy at $t = 1$ gets out of owning a $S$ name is,

$$u_G(S) = w_1(S) + P_Gw_2(SS) + (1 - P_G)w_2(SF),$$

while his utility from not owning a name is,

$$u_G(\emptyset) = w_1(\emptyset) + P_Gw_2(\emptyset S) + (1 - P_G)w_2(\emptyset),$$

because after failing in his first period such an agent will wish to change his name. The benefit from owning a name is therefore,

$$u_G(S) - u_G(\emptyset) = w_1(S) - w_1(\emptyset) + (1 - P_G)[w_2(SF) - w_2(\emptyset)],$$

which follows because $w_2(\emptyset S) = w_2(SS)$ is implied from the fact that only $G$-types buy $S$, $SS$, and $\emptyset S$ names. Similarly for $B$-types,

$$u_B(S) = w_1(S) + w_2(SF),$$

$$u_B(\emptyset) = w_1(\emptyset) + w_2(\emptyset),$$

30
and,
\[ u_B(S) - u_B(\emptyset) = w_1(S) - w_1(\emptyset) + w_2(SF) - w_2(\emptyset). \]

But since SF names are traded it must be that \( w_2(SF) > w_2(\emptyset) \), which in turn implies that \( u_B(S) - u_B(\emptyset) > u_G(S) - u_G(\emptyset) \). That is, \( B \)-types have a larger benefit from owning a \( S \) name at \( t = 1 \), which contradicts the assumption that only \( G \)-types buy \( S \) names at \( t = 1 \). The conclusion is that there must be some \( B \)-types who buy \( S \) names at \( t = 1 \).

\[ \square \]

**Appendix B**

This appendix characterizes the family of SSE. Let \( H \) be the set of all possible histories (including \( \emptyset \), no history) and define the reputation mapping \( r : H \rightarrow \{g, b, \emptyset\} \) as follows:

**Definition A.2.1:** \( r(\emptyset) = \emptyset \); \( r(h) = g \) for all \( h \in H_S \equiv \{\emptyset S, \emptyset SS, \emptyset SSS, \ldots\} \), and \( r(h) = b \) for all \( h \in H \setminus (\emptyset \cup H_S) \).

The family of SSE will be characterized as follows: Names will be changed after the first failure occurs (\( b \) names will be worse than no history). Thus, only names ending with any consecutive number of successes, i.e., with history \( h \in H_S \) will be traded. Since only \( b \) and \( g \) are observed by clients, all names with reputation record \( g \) will be traded at the same price \( v \), and all other names will not be traded (\( b \) names are not worth buying). Also, the equilibrium wages will be \( w(\emptyset) \), \( w(g) \) and \( w(b) \).

In any such an equilibrium, both old and new agents can potentially buy \( g \) names (old agents who succeeded will stick to their name). As before, I will assume that old agents do not buy names, and let \( \delta \) denote the proportion of new good types who buy \( g \) names (\( \rho \) for bad types). We can therefore characterize such equilibria by \( \{\delta, \rho, v, w(g), w(\emptyset), w(b)\} \). The market clearing condition for the market for \( g \) names becomes,

\[ \gamma \delta + \beta \rho = \gamma P_G. \]  

(6)

This implies that,

\[ \Pr\{G|g\} = \frac{\gamma P_G + \gamma \delta}{\gamma P_G + \gamma \delta + \beta \rho} = \frac{P_G + \delta}{2P_G}, \]

31
and Assumption 3 guarantees that \( \Pr\{G|b\} \) is no larger than,

\[
\Pr\{G|b\} \leq \Pr\{G|g \text{ followed by } F\} = \frac{\gamma \delta (1 - P_G)}{\gamma \delta (1 - P_G) + \beta \rho} = \frac{\delta (1 - P_G)}{P_G(1 - \delta)}. \tag{7}
\]

To verify that an agent who failed will prefer to change names over sticking to a \( b \) name it must be that,

\[
w(\emptyset) \geq w(b) = \Pr\{G|b\} \cdot P_G. \tag{8}
\]

If this condition is satisfied then all types will discard their name after a failure (except for \( \varepsilon \) which is taken to be 0 for computational convenience) and the conditional probability of a good type running a firm with no history is,

\[
\Pr\{G|\emptyset\} = \frac{\gamma (1 - \delta) + \gamma (1 - P_G)}{\gamma (1 - \delta) + \gamma (1 - P_G) + \beta + \beta (1 - \rho)} = \frac{2\gamma - \gamma \delta - \gamma P_G}{2(1 - \gamma P_G)}. \tag{9}
\]

Using (9) and (7), condition (8) can be replaced by the sufficient condition,

\[
\frac{(2\gamma - \gamma \delta - \gamma P_G)}{2(1 - \gamma P_G)} \geq \frac{\delta (1 - P_G)}{P_G(1 - \delta)}, \tag{10}
\]

which after some algebraic manipulation (10) reduces to,

\[
\delta^2 P_G + \delta [2(P_G - 1) + \gamma P_G(1 + P_G)] + \gamma P_G(2 - P_G) \geq 0. \tag{11}
\]

This is a quadratic inequality in \( \delta \) of the form \( a\delta^2 + b\delta + c \geq 0 \). Clearly, for the range of parameters \( P_G \in (0, 1) \) and \( \gamma \in (0, 1) \), we have \( a > 0, \ b < 0, \) and \( c > 0 \). It is easy to check that the situation where only good types buy \( g \) names violates this condition: plugging \( \delta = P_G \) into the right hand side of (11) causes it to be strictly negative. This implies that there are two roots for the right hand side of (11), \( 0 < \delta_1 < P_G \) and \( \delta_2 > P_G \). Since from market clearing \( \delta_2 \) is irrelevant for the model then letting \( \delta \equiv \delta_1 \) we have that for all \( \delta \in (0, \delta) \) condition (11) is satisfied.

Another condition that must be satisfied for names to be traded is,

\[
v = w(g) - w(\emptyset) \geq 0,
\]

or explicitly,

\[
\frac{P_G + \delta}{2P_G} \geq \frac{(2\gamma - \gamma \delta - \gamma P_G)}{2(1 - \gamma P_G)},
\]
which yields,
\[ \delta \geq P_G(2\gamma - 1) . \]

Therefore, there exists \( \hat{\delta} = \max\{P_G(2\gamma - 1), 0\} \) such that \( \delta \geq \hat{\delta} \) is necessary for any such equilibrium.

To finally conclude that for \( \delta \in (\hat{\delta}, \bar{\delta}) \) the tuple \( \{\delta, \rho, v, w(g), w(\emptyset), w(b)\} \) (where \( \rho \) is determined by the market clearing condition (6)) is an equilibrium it must be shown that:

(i) \( \hat{\delta} \leq \bar{\delta} \)

(ii) Good types who succeed after their first period will not strictly prefer to sell their name

(iii) All agents who fail will change their name

Condition (i) is clearly satisfied for \( \gamma \leq \frac{1}{2} \) because then \( \hat{\delta} = 0 \). For \( \gamma > \frac{1}{2} \) I have verified (using MATLAB\textsuperscript{©}) that this condition is still satisfied. Condition (iii) is satisfied because \( \delta \in (\hat{\delta}, \bar{\delta}) \) guarantees that condition (8) is satisfied. Finally, condition (ii) is satisfied because good types are indifferent between selling a good name before they retire: by continuing with a \( g \) name when they have one period left to be active they receive \( u(\text{continue}) = w(g) + P_G \cdot v \), while by selling a \( g \) name at that stage they receive \( u(\text{sell}) = v + w(\emptyset) + P_G \cdot v = w(g) + P_G \cdot v \). This concludes the construction of the family of SSE.

Note that this SSE can be replicated by using bounded recall rather than a reputation mapping. If clients have a memory of recall 2 (they observe the last two periods only) then the characterization above would constitute a set of SSE for the model.
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